Knowledge Extraction from L-Fuzzy Hypercontexts

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Abstract. As a generalization of the L-fuzzy contexts, we propose the study of the L-fuzzy hypercontexts where the relation R between the objects X and the attributes Y takes as values other L-fuzzy relations. In this work, we propose the study of these structures using OWA operators in different situations. Finally, the practical case that has motivated this paper is analyzed.

Keywords: L-fuzzy contexts · L-fuzzy concepts · L-fuzzy hypercontexts · OWA operators

1 Introduction

The L-fuzzy concept analysis provides a tool for the extraction of knowledge from tables (L-fuzzy contexts) using L-fuzzy concepts. These L-fuzzy contexts are tuples (L, X, Y, R), with L a complete lattice, X and Y the object and attribute sets and $R \in L^{X \times Y}$ an L-fuzzy relation between the objects and the attributes.

L-fuzzy contexts establishing frameworks that allow us to represent $R(x, y)$, $x \in X$ and $y \in Y$, as a collection of values that has the structure of L-fuzzy context with a set of objects Q_x associated with x and a set of attributes S_y associated with y. These sets Q_x are related to each other, and the same happens with the S_y.

For example, consider a chain of supermarkets that has several establishments in different cities. We want to study the evolution in time Y of the sales of different articles in each of the cities X where they are sold.

Our hypercontexts are the extensions to the fuzzy case of certain multicontexts of Wille [13] verifying some properties with respect to the objects and the attributes. On the other hand, the approach given in this work, using OWA operators, will also be different from the topic discussed by Wille.

The main goal of the paper is the study of the values of R taking into account that for every $x \in X$, we have a set of objects Q_x that can be different in every case and for every $y \in Y$, a different set of attributes S_y.

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Although this problem is related to the construction of formal contexts developed in [9], we extend our study to new structures with values in a complete lattice L. Then, fuzzy logic tools and specifically OWA operators can be applied for obtaining information.

As some of the practical cases are represented in a natural way by a fuzzy relation, we consider the new framework as a good contribution because it increases the range of application of formal and fuzzy concept analysis.

Firstly, we will see some results about L-fuzzy concept analysis and OWA operators [11]:

1.1 Formal Concept Analysis and L-Fuzzy Concept Analysis

The Formal Concept Analysis of R. Wille [12] extracts information from a binary table that represents a formal context (X, Y, R) with X and Y finite sets of objects and attributes respectively and $R \subseteq X \times Y$. The hidden information consists of pairs (A, B) with $A \subseteq X$ and $B \subseteq Y$, called formal concepts, verifying $A^* = B$ and $B^* = A$, where $(\cdot)^*$ is the derivation operator that associates the attributes related to the elements of A to every object set A, and the objects related to the attributes of B to every attribute set B. These formal concepts can be interpreted as a group of objects A that shares the attributes of B.

In previous works [6,7] we have defined the L-fuzzy contexts (L, X, Y, R), with L a complete lattice, X and Y sets of objects and attributes respectively and $R \in L^{X \times Y}$ a fuzzy relation between the objects and the attributes. This is an extension of Wille's formal contexts to the fuzzy case when we want to study the relations between the objects and the attributes with values in a complete lattice L, instead of binary values.

In our case, to work with these L-fuzzy contexts, we have defined the derivation operators 1 and 2 given by means of these expressions:

$\forall A \in L^X, \forall B \in L^Y$

$$A_1(y) = \inf_{x \in X} \{\mathcal{I}(A(x), R(x, y))\}$$

$$B_2(x) = \inf_{y \in Y} \{\mathcal{I}(B(y), R(x, y))\}$$

with $\mathcal{I}$ a fuzzy implication operator defined in the lattice $(L, \leq)$.

The information stored in the context is visualized by means of the L-fuzzy concepts that are pairs $(A, A_1) \in L^X \times L^Y$ with $A \in fix(\varphi)$, set of fixed points of the operator φ, being defined from the derivation operators 1 and 2 as $\varphi(A) = (A_1)_2 = A_{12}$. These pairs, whose first and second components are said to be the fuzzy extension and intension respectively, represent a group of objects that share a group of attributes.

Using the usual order relation between fuzzy sets, that is,

$$\forall A, C \in L^X, \quad A \leq C \iff A(x) \leq C(x) \quad \forall x \in X,$$

we define the set $\mathcal{L} = \{(A, A_1)/A \in fix(\varphi)\}$ with the order relation $\preceq$ defined as:

$$\forall (A, A_1), (C, C_1) \in \mathcal{L}, (A, A_1) \preceq (C, C_1) \text{ if } A \leq C (\text{ or } A_1 \geq C_1)$$

As φ is an order preserving operator, by the theorem of Tarski, the set $fix(\varphi)$ is a complete lattice and then $(\mathcal{L}, \preceq)$ is also a complete lattice that is said to be [6,7] the L-fuzzy concept lattice.

On the other hand, given $A \in L^X$, (or $B \in L^Y$) we can obtain the associated L-fuzzy concept applying twice the derivation operators. In the case of using a residuated implication, as we do in this work, the associated L-fuzzy concept is (A_{12}, A_1) (or (B_2, B_{21})).

Other important papers that generalize the Formal Concepts Analysis using residuated implication operators are due to R. Belohlavek [4,5]. Moreover, extensions of Formal Concept Analysis to the interval-valued case are in [1,8] and to the fuzzy property-oriented and multi-adjoint concept lattices framework in [10].

1.2 OWA Operators

Families of OWA operators were introduced by Yager [11] as a new aggregation technique based on the ordered weighted averaging. This is the definition of these operators:

Definition 1. *A mapping F from $L^n \longrightarrow L$, where $L = [0,1]$ is called an OWA operator of dimension n if associated with F is a weighting $n-$tuple $W = (w_1, w_2 \dots w_n)$ such that $w_i \in [0,1]$ and $\sum_{1 \leq i \leq n} w_i = 1$, where $F(a_1, a_2, \dots a_n) = w_1.b_1 + w_2.b_2 + \cdots + w_n.b_n$, with b_i the ith largest element in the collection $a_1, a_2, \dots a_n$.*

There are two particular cases of special interest:

W_* defined by the weighting n-tuple with $w_n = 1$ and $w_j = 0, \forall j \neq n$, and W^* defined by the weighting n-tuple such that $w_1 = 1$ and $w_j = 0, \forall j \neq 1$.

It is proved that $F_*(a_1, a_2, \dots a_n) = \min_j(a_j)$ and $F^*(a_1, a_2, \dots a_n) = \max_j(a_j)$. These operators are said to be *and* and *or*, respectively.

To study the L-fuzzy hypercontexts, we are interested in the use of operators close to *or*. To measure this proximity the orness degree can be used [11].

Furthermore, the use of different weighting vectors provides different results as we will see in the paper.

2 *L*-Fuzzy Hypercontexts

We will begin defining the framework:

Definition 2. *We denote the tuple $(L, X, Y, (Q_x)_{x \in X}, (S_y)_{y \in Y}, R)$ by an L-fuzzy hypercontext, with $L = [0,1]$, X and Y sets of objects and attributes respectively, $(Q_x)_{x \in X}$ and $(S_y)_{y \in Y}$ families of sets associated with the elements of X and Y, and R such that $R(x, y)$ is also a new relation $R_{xy} \in L^{Q_x \times S_y}$, for every $(x, y) \in X \times Y$. This relation R_{xy} defines a new L-fuzzy context (L, Q_x, S_y, R_{xy}).*

Remark 1. The objects Q_x associated with $x \in X$ do not have to be elements of X. Neither for S_y.

The main target of the paper is the study of these hypercontexts when $L = [0, 1]$. In some cases, we want to make the study only in respect to the elements of X or Y but, in other cases, we will also be interested in analyzing the results based on the families $(Q_x)_{x \in X}$ and $(S_y)_{y \in Y}$.

We begin with the general case. It is not easy to work with the original R from the point of view of the L-fuzzy concept analysis as it does not represent an L-fuzzy context, the initial idea is to transform this structure in a derived L-fuzzy context $(L, \breve{X}, \breve{Y}, \breve{R})$. Then we can extract the information by means of the construction of its L-fuzzy concepts.

After that, we analyze the particular case where $Q_x = Q, \forall x \in X$ (or $S_y = S, \forall y \in Y$). In this case, the relation values can be studied from the values of an only set Q (or S respectively). Furthermore, if $Card(X)$=1, we have an L-fuzzy sequence studied in [3].

Let us see an example:

Example 1. We want to study the evolution in time Y of the sales of some articles in a supermarket chain in the different cities X in which it works. We define an L-fuzzy hypercontext $(L, X, Y, (Q_x)_{x \in X}, (S_y)_{y \in Y}, R)$ with X and Y sets of objects and attributes respectively. The establishment chain $(Q_x)_{x \in X}$ has different values according to the cities and the family of products $(S_y)_{y \in Y}$ changes over time Y (months). Finally, we define $R_{xy} \in L^{Q_x \times S_y}$, for every (x, y), as a relation that recover the sales of the different products S_y in a month $y \in Y$, in the different establishments Q_x of the city $x \in X$.

R	y_1				y_2				y_3		
x_1	$R_{x_1y_1}$	$s_{y_{1_1}}$	$s_{y_{1_2}}$	$s_{y_{1_3}}$	$R_{x_1y_2}$	$s_{y_{2_1}}$	$s_{y_{2_2}}$	$s_{y_{2_3}}$	$R_{x_1y_3}$	$s_{y_{3_1}}$	$s_{y_{3_2}}$
	$q_{x_{1_1}}$	0.3	1	0.1	$q_{x_{1_1}}$	0.6	0.9	1	$q_{x_{1_1}}$	0.4	0.5
	$q_{x_{1_2}}$	0.7	0.3	0.8	$q_{x_{1_2}}$	1	0	0.2	$q_{x_{1_2}}$	0	1
	$q_{x_{1_3}}$	0.9	0.2	0	$q_{x_{1_3}}$	0.6	0.8	1	$q_{x_{1_3}}$	0.9	1
x_2	$R_{x_2y_1}$	$s_{y_{1_1}}$	$s_{y_{1_2}}$	$s_{y_{1_3}}$	$R_{x_2y_2}$	$s_{y_{2_1}}$	$s_{y_{2_2}}$	$s_{y_{2_3}}$	$R_{x_2y_3}$	$s_{y_{3_1}}$	$s_{y_{3_2}}$
	$q_{x_{2_1}}$	0.7	0.8	1	$q_{x_{2_1}}$	0.3	0.9	0	$q_{x_{2_1}}$	0.4	0.2
	$q_{x_{2_2}}$	1	0	0.2	$q_{x_{2_2}}$	1	0.2	0.8	$q_{x_{2_2}}$	0	0.6

2.1 General Study

We analyze different ways to transform the L-fuzzy hypercontext in an L-fuzzy context.

(1) In this first case, we are interested in keeping the complete information that we have. So, we give the following definition:

Definition 3. *The L-fuzzy hypercontext* $(L, \breve{X}, \breve{Y}, \breve{R})$ *derived in a natural way from the tuple* $(L, X, Y, (Q_x)_{x\in X}, (S_y)_{y\in Y}, R)$, *is:*

- $\breve{X} = \bigcup \dot{Q}_x$ *with* $\dot{Q}_x = \{x\} \times Q_x$
- $\breve{Y} = \bigcup \dot{S}_y$ *with* $\dot{S}_y = \{y\} \times S_y$
- $\breve{R}((x,q),(y,s)) = R_{xy}(q,s)$, $\forall x \in X, \forall y \in Y, \forall q \in Q_x, \forall s \in S_y$.

Example 2. For our example, this will be our relation $\breve{R}$:

$\breve{R}$	$(y_1, s_{y_{11}})$	$(y_1, s_{y_{12}})$	$(y_1, s_{y_{13}})$	$(y_2, s_{y_{21}})$	$(y_2, s_{y_{22}})$	$(y_2, s_{y_{23}})$	$(y_3, s_{y_{31}})$	$(y_3, s_{y_{32}})$
$(x_1, q_{x_{11}})$	0.3	1	0.1	0.6	0.9	1	0.4	0.5
$(x_1, q_{x_{12}})$	0.7	0.3	0.8	1	0	0.2	0	1
$(x_1, q_{x_{13}})$	0.9	0.2	0	0.6	0.8	1	0.9	1
$(x_2, q_{x_{21}})$	0.7	0.8	1	0.3	0.9	0	0.4	0.2
$(x_2, q_{x_{22}})$	1	0	0.2	1	0.2	0.8	0	0.6

In this case, the derivation operators have the following expression:

Proposition 1. $\forall A \in L^{\breve{X}}, \forall B \in L^{\breve{Y}}, \forall x \in X, \forall y \in Y, \forall q \in Q_x, \forall s \in S_y$

$$\begin{aligned} A_1(y,s) &= \inf_{(x,q)\in\breve{X}} \{\mathcal{I}(A(x,q), \breve{R}((x,q)(y,s))\} \\ &= \inf_{(x,q)\in\breve{X}} \{\mathcal{I}(A(x,q), R_{xy}(q,s)\} \end{aligned}$$

$$\begin{aligned} B_2(x,q) &= \inf_{(y,s)\in\breve{Y}} \{\mathcal{I}(B(y,s)), \breve{R}((x,q)(y,s))\} \\ &= \inf_{(y,s)\in\breve{Y}} \{\mathcal{I}(B(y,s)), R_{xy}(q,s)\} \end{aligned}$$

with $\mathcal{I}$ *an L-fuzzy implication operator defined in* $(L, \leq)$ *and where* A_1 *represents the attributes related to the objects of* A *and* B_2*, to the objects related to the attributes of* B.

In this case, every pair (x,q) behaves as an object and every pair (y,s) as an attribute.

This definition does not lose the original information but the size of the obtained context is large.

(2) In some situations, it can be interesting to try to reduce the size of the L-fuzzy context although we lose information related to $Q_x, S_y, x \in X, y \in Y$. We have three possibilities:

(a) Aggregate all the values for every R_{xy}.

Definition 4. *We define the derived L-fuzzy context* $(L, \breve{X}, \breve{Y}, \breve{R})$*:*

- $\breve{X} = X$
- $\breve{Y} = Y$
- $\breve{R} : \breve{X} \times \breve{Y} \longrightarrow L$ *such that* $\forall x \in X, \forall y \in Y$:

$$\begin{aligned}\breve{R}(x,y) =& F_{w_{xy}}(R_{xy}(q_{x_1}, s_{y_1}), R_{xy}(q_{x_1}, s_{y_2}) \dots R_{xy}(q_{x_{|Q_x|}}, s_{y_{|S_y|}})) = \\ & w_1.b_1 + w_2.b_2 + \dots + w_{|Q_x|.|S_y|}.b_{|Q_x|.|S_y|},\end{aligned}$$

where $F_{w_{xy}}$ *is an OWA operator with the associated weighting vector* $w_{xy} = (w_1, w_2, \dots w_{|Q_x|.|S_y|})$ *and* b_i *the ith largest element of the collection* $R_{xy}(q_{x_1}, s_{y_1}), R_{xy}(q_{x_1}, s_{y_2}) \dots R_{xy}(q_{x_{|Q_x|}}, s_{y_{|S_y|}})$.

This aggregation allows to establish an study of the elements of X respect to Y. There is not information about the set Q_x neither about S_y.

Let see an example.

Example 3. Suppose that in example 1 we want to give more relevance to the closest to 1 observations, then the use of OWA operators can be a good election.

We can use weights w_{xy} (see Sect. 1.2) such that

$$w_i = \frac{2(n-i+1)}{n(1+n)}, \forall i \in \{1, \dots, n\}.$$

In this case, we obtain the result applying the definition:

$\breve{R}_{F_{w_{xy}}}$	y_1	y_2	y_3
x_1	0.66	0.84	0.80
x_2	0.80	0.71	0.40

For the extraction of the information, we can now take a set that represents the interest of study and calculate the associated L-fuzzy concept using the Lukasiewicz implication operator.

For instance, if we want to study the second city, then we take $\{x_1/0, x_2/1\}$ and we obtain the following result:

$$(\{x_1/0.86, x_2/1\}, \{y_1/0.8, y_2/0.71, y_3/0.4\})$$

We can interpret this L-fuzzy concept saying that there are important sales in both cities during the first two months.

(b) Aggregate for every $x \in X$ the values of R_{xy} associated with the different Q_x.

Definition 5. *We define the derived L-fuzzy context* $(L, \breve{X}, \breve{Y}, \breve{R})$*:*

- $\breve{X} = X$
- $\breve{Y} = \bigcup \dot{S}_y$ *with* $\dot{S}_y = \{y\} \times S_y, \forall y \in Y$
- $\breve{R} : \breve{X} \times \breve{Y} \longrightarrow L$ *such that* $\forall x \in X, \forall y \in Y, \forall s \in S_y$*:*

$$\breve{R}(x,(y,s)) = F_{w_x}(R_{xy}(q_{x_1}, s), R_{xy}(q_{x_2}, s) \dots R_{xy}(q_{x_{|Q_x|}}, s))$$

where F_{w_x} *is an OWA operator with the associated weighting vector* $w_x = (w_1, w_2, \dots w_{|Q_x|})$ *and* b_i *the ith largest element in the collection* $R_{xy}(q_{x_1}, s)$*,* $R_{xy}(q_{x_2}, s) \dots R_{xy}(q_{x_{|Q_x|}}, s)$*.*

In this case, we can analyze the elements of X although there is not information about those of Q_x.

Example 4. We are going to prioritize the membership degrees closest to 1 by means of w. Then, to aggregate the values, we use $w_{x_1} = (3/6, 2/6, 1/6)$ and $w_{x_2} = (2/3, 1/3)$. The result is:

$\breve{R}_{F_{w_x}}$	y_1			y_2			y_3	
	$s_{y_{1_1}}$	$s_{y_{1_2}}$	$s_{y_{1_3}}$	$s_{y_{2_1}}$	$s_{y_{2_2}}$	$s_{y_{2_3}}$	$s_{y_{3_1}}$	$s_{y_{3_2}}$
x_1	0.73	0.63	0.43	0.8	0.72	0.87	0.58	0.92
x_2	0.9	0.53	0.73	0.77	0.67	0.53	0.27	0.47

In this example, what is sold in every city x_i in time can be studied although the establishments q_{x_i} information is missing.

For instance, if we want to study the second city, we take $\{x_1/0, x_2/1\}$ and, applying the derivation operator, obtain the following result:

$$\{y_1/(0.9, 0.53, 0.73), y_2/(0.77, 0.67, 0.53), y_3/(0.27, 0.47)\}$$

We can *highlight the sales of* $s_{y_{1_1}}$ *product in all establishments of the second city in the first month.*

We can conclude that *in the second city there are important sales mainly in the first two months* (y_1, y_2).

Moreover, we can also see details of sales of products: there is a larger sale of the first product in the first month followed by the sale, also of the first product, in the second month.

We can establish different nuances that otherwise would not be possible using other OWA operators. For instance, with the minimum ($w_{x_1} = (0, 0, 1)$ and $w_{x_2} = (0, 1)$), and also for the second city, the result is:

$$\{y_1/(0.7, 0, 0.2), y_2/(0.3, 0.2, 0), y_3/(0, 0.2)\}$$

We can *highlight the sales of* $s_{y_{1_1}}$ *product in all establishments of the second city in the first month.*

(c) Aggregate for every $y \in Y$ the values R_{xy} associated with the different S_y.

Definition 6. *We define the derived L-fuzzy context* $(L, \breve{X}, \breve{Y}, \breve{R})$*:*

- $\breve{X} = \bigcup \dot{Q}_x$ *with* $\dot{Q}_x = \{x\} \times Q_x, \forall x \in X$
- $\breve{Y} = Y$
- $\breve{R} : \breve{X} \times \breve{Y} \longrightarrow L$ *such that* $\forall x \in X, y \in Y, q \in Q_x$

$$\breve{R}((x,q),y) = F_{w_y}(R_{xy}(q, s_{y1}), R_{xy}(q, s_{y2}) \dots R_{xy}(q, s_{y|S_y|}))$$

where F_{w_y} *is an OWA operator with the associated weighting vector* $w_y = (w_1, w_2, \dots w_{|S_y|})$ *and* b_i *the ith largest element in the collection* $R_{xy}(q, s_{y_1})$, $R_{xy}(q, s_{y_2}) \dots R_{xy}(q, s_{y_{|S_y|}})$.

This aggregation allows the study of the elements of Y (there is not information about the products).

Example 5. We are going to see the evolution of the sales over time (Y). In this case, it is also interesting for us the study of the observations with membership degrees closest to 1 and next to the current instants of time. For the aggregation of the values, we will use $w_{y_1} = (3/6, 2/6, 1/6)$ and $w_{y_2} = (2/3, 1/3)$. The obtained result is:

$\breve{R}_{F_{w_y}}$		y_1	y_2	y_3
x_1	$q_{x_{11}}$	0.62	0.9	0.47
	$q_{x_{12}}$	0.68	0.57	0.67
	$q_{x_{13}}$	0.52	0.87	0.97
x_2	$q_{x_{21}}$	0.88	0.55	0.33
	$q_{x_{22}}$	0.57	0.8	0.4

We can here study the evolution in time of the sales in every supermarket $q_{x_{ij}}$ of every city x_i.

For instance, if we look at the third month (y_3), we can take $\{y_1/0, y_2/0, y_3/1\}$ and obtain the result:

$$\{x_1/(0.47, 0.67, 0.97), x_2/(0.33, 0.4)\}$$

Then, we can conclude that *there are lower sales values in the second city (x_2) and many differences among the establishments of the first one (x_1) with higher sales in the last two.*

Also in this case the results are different if we use other OWA operators. For instance, using the minimum ($w_{y_1} = (0, 0, 1)$ and $w_{y_2} = (0, 1)$) and also for the third month, the result is:

$$\{x_1/(0.4, 0, 0.9), x_2/(0.2, 0)\}$$

Then, we can *highlight the sales of all products of establishment $q_{x_{13}}$ of the city x_1 in the third month.*

2.2 Study of a Particular Case

In this case, when one of the sets (Q_x or S_y) is fixed, we can perform more complete studies. For instance, suppose that $S_y = S, \forall y \in Y$.

For the example:

Example 6. In this case, the articles $S = \{s_1, s_2, s_3\}$ are the same in all the establishments but there are different establishments in the different cities.

R		y_1			y_2			y_3		
		s_1	s_2	s_3	s_1	s_2	s_3	s_1	s_2	s_3
x_1	$q_{x_{11}}$	0.3	1	0.1	0.6	0.9	1	0.4	0.5	0.9
	$q_{x_{12}}$	0.7	0.3	0.8	1	0	0.2	0	1	0.2
	$q_{x_{13}}$	0.9	0.2	0	0.6	0.8	1	0.9	1	0.2
x_2	$q_{x_{21}}$	0.7	0.8	1	0.3	0.9	0	0.4	0.2	1
	$q_{x_{22}}$	1	0	0.2	1	0.2	0.8	0	0.6	0.3

Then, we can apply all the results obtained in the general case and, as the set S is fixed, we can also perform a more individualized study for each one of its values.

Definition 7. *We define the derived L-fuzzy context* $(L, \breve{X}, \breve{Y}, \breve{R})$ *(Aggregating the values of* Y*) in this way:*

- $\breve{X} = \bigcup \dot{Q}_x$ *with* $\dot{Q}_x = \{x\} \times Q_x, \forall x \in X$
- $\breve{Y} = S$
- $\breve{R} : \breve{X} \times \breve{Y} \longrightarrow L$ *such that* $\forall x \in X, q \in Q_x, s \in S$:

$$\breve{R}((x,q),s) = F_{w_Y}(R_{xy_1}(q_x,s), R_{xy_2}(q_x,s) \ldots R_{xy_{|Y|}}(q_x,s))$$

where F_{w_Y} *is an OWA operator with the associated weighting vector* $w_Y = (w_1, w_2, \ldots w_{|Y|})$ *and* b_i *the ith largest element in the collection* $R_{xy_1}(q_x,s)$, $R_{xy_2}(q_x,s) \ldots R_{xy_{|Y|}}(q_x,s)$.

Example 7. In our example and using $w_Y = (3/6, 2/6, 1/6)$, we obtain the following table:

We can answer the question about which are the different cities with good sales of the different products over time.

For instance, if we take as the set $\{s_1/0, s_2/1, s_3/0\}$, we obtain the result:

$$\{x_1/(0.88, 0.6, 0.8), x_2/(0.75, 0.37)\}$$

Hence, we can conclude that *there are good sales, mainly of* s_2 *in all the establishments of* x_1.

However, if we do the same for the product s_1 we have:

$$\{x_1/(0.48, 0.73, 0.85), x_2/(0.53, 0.83)\}$$

and now we have fewer sales only in the last establishment.

$\breve{R}_{F_{w_Y}}$		s_1	s_2	s_3
x_1	$q_{x_{11}}$	0.48	0.88	0.82
	$q_{x_{12}}$	0.73	0.6	0.5
	$q_{x_{13}}$	0.85	0.8	0.57
x_2	$q_{x_{21}}$	0.53	0.75	0.83
	$q_{x_{22}}$	0.83	0.37	0.53

Similar developments can be obtained for $Q_x = Q, \forall x \in X$.

Definition 8. *We define the derived L-fuzzy context $(L, \breve{X}, \breve{Y}, \breve{R})$ (Aggregating the values of X) in this way:*

- $\breve{X} = Q$
- $\breve{Y} = \bigcup \dot{S}_y$ *with* $\dot{S}_y = \{y\} \times S_y, \forall y \in Y$
- $\breve{R} : \breve{X} \times \breve{Y} \longrightarrow L$ *such that* $\forall x \in X, q \in Q, s \in S_y$:

$$\breve{R}(q,(y,s)) = F_{w_X}(R_{x_1 y}(q,s_y), R_{x_2 y}(q,s_y) \ldots R_{x_{|X|} y}(q,s_y))$$

where F_{w_X} is an OWA operator with the associated weighting vector $w_X = (w_1, w_2, \ldots w_{|X|})$ and b_i the ith largest element in the collection $R_{x_1 y}(q,s_y)$, $R_{x_2 y}(q,s_y) \ldots R_{x_{|X|} y}(q,s_y)$.

In this case, we can perform a study in depth of the elements of Q although it has no interest for our example.

3 Conclusions and Future Lines

This paper introduces the study of L-fuzzy hypercontexts by using OWA operators in a complete lattice L with the aim of obtaining the relevant information. These new frameworks allow us to work with L-fuzzy contexts where the set of objects and attributes are variable.

Firstly we have developed a general study in two different situations: the first one without lost of information and the second one, reducing the size of the context and losing the information of the not prioritized set.

Finally, we analyze a particular case where the set of objects or attributes is fixed. Then a more individualized study can be performed.

In all the cases, the use of OWA operators is an interesting tool for obtaining the relevant information and establishing different nuances in our study.

In future works we will use linguistic variables [14] for the representation of the points of interest following the ideas of [2].

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