Preface

> It is reasonable to hope that the relationship between computation and mathematical logic will be as fruitful in the next century as that between analysis and physics in the last. The development of this concern demands a concern for both applications and mathematical elegance.

(John McCarthy 1963)

The dozen students or so gathered in the lovely Italian town of Perugia were amazed when Martin David Davis first showed up, wearing heavy shoes and short pants seemingly more apt for trekking than for teaching a graduate level course. His accent from the Bronx, together with a little hole, may be produced by the ember of a cigarette, over one shoulder of his red T-shirt, seized the attention of the class. His hair *à la* Queen of Sheba further increased the mismatch between Martin as a person and the stereotype unavoidably associated with his reputation as a distinguished scholar.

Admiration quickly prevailed over astonishment when Martin began his exposition of computability. For the entire one-month duration of his course, concepts remained clear and accessible. At times, when confronted with some odd question coming from his audience, Martin turned his hands upward and disarmingly said "I cannot understand"; far more often, he answered with extreme precision. He indulged in vivid images, such as "brand-new variable" or "crystal-clear proof," but his repertoire of idiomatic expressions also included "gory detail," when technicalities were inescapable.

Through mathematics, the class felt, Martin was also addressing issues of philosophical relevance and depth. On one memorable occasion, the philosophical side of his scientific inquiry showed through a lesson boldly offered in Italian. Forty years have elapsed since, and alas, the tape recording of that lesson, dealing with Turing machines and universality, has by now faded away.

Once Martin was invited to an assembly of all students participating in the Perugia summer school. Unhesitatingly, he joined the crowd, coming hand in hand with his wife Virginia; he even took the floor at some point, rational and quiet, not in the least dismayed by the excited, somehow "revolutionary," atmosphere of the event.

One of the editors of this volume dedicated to Martin was a student in his computability course in Perugia, and this explains why such an anecdotal episode has been recounted here. Many similar fascinating stories could certainly be reported by many: Over the years, Martin lectured in several countries (to cite a few: Japan, India, England, Russia, and Mexico—see Fig. 1), and they have—along

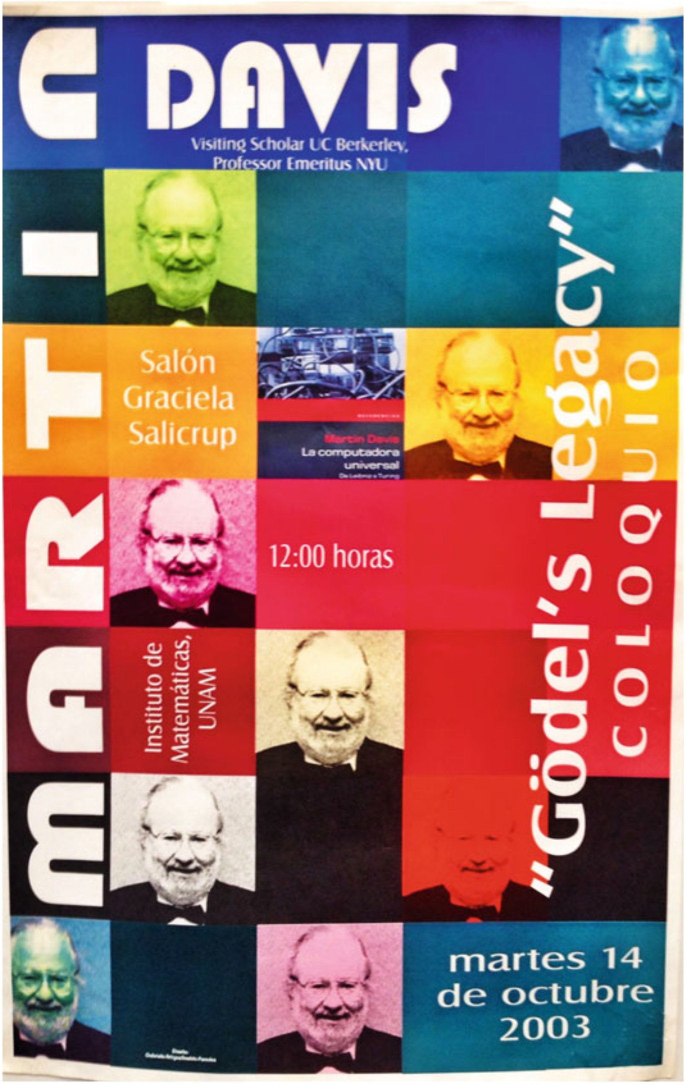
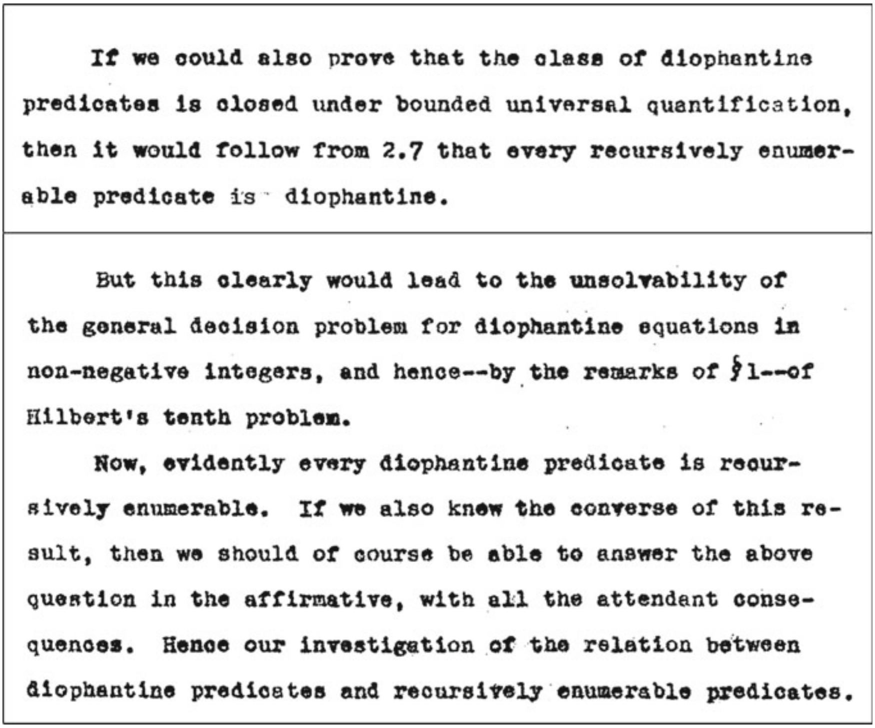


Fig. 1 Poster announcing Martin's first lecture in Mexico

with his publications—exerted a wide influence. This book will testify to this influence by focusing on scientific achievements in which Martin was involved in the first person and on further achievements, studies, and reflections in which work and vision consonant with his have played a role. Our task has been to collect testimonies of Martin's contributions to computability, computational logic, and mathematical foundations.

Three chapters are devoted to a problem that Martin said he found "irresistibly seductive" when still an undergraduate (Fig. 2) and which progressively became his "lifelong obsession": Hilbert's tenth problem—H10 for short. One of the three contains a narrative essay by the Mexican mathematician Dr. Laura Elena Morales Guerrero, telling us how a negative solution to H10 came to light through the joint effort of four protagonists (one being Martin, of course). There are two epic events in that story: One is when Julia Bowman Robinson eliminates "a serious blemish" from a proof by Martin and Hilary Putnam by showing how to avoid a hypothesis that was unproved at the time; the other is when, in 1970, notes of a talk given in Novosibirsk reach Martin in New York. Laura Elena reports to the reader the key equations in those notes based on Yuri Matiysevich's decisive work, which Martin echoed a few months later by his own use of these newly developed methods to obtain an alternative system of equations leading to the same result. See Fig. 3.



If we could also prove that the class of diophantine predicates is closed under bounded universal quantification, then it would follow from 2.7 that every recursively enumerable predicate is diophantine.

But this clearly would lead to the unsolvability of the general decision problem for diophantine equations in non-negative integers, and hence--by the remarks of §1--of Hilbert's tenth problem.

Now, evidently every diophantine predicate is recursively enumerable. If we also knew the converse of this result, then we should of course be able to answer the above question in the affirmative, with all the attendant consequences. Hence our investigation of the relation between diophantine predicates and recursively enumerable predicates.

Fig. 2 Are all recursively enumerable sets Diophantine? (From Martin's Ph.D. thesis)

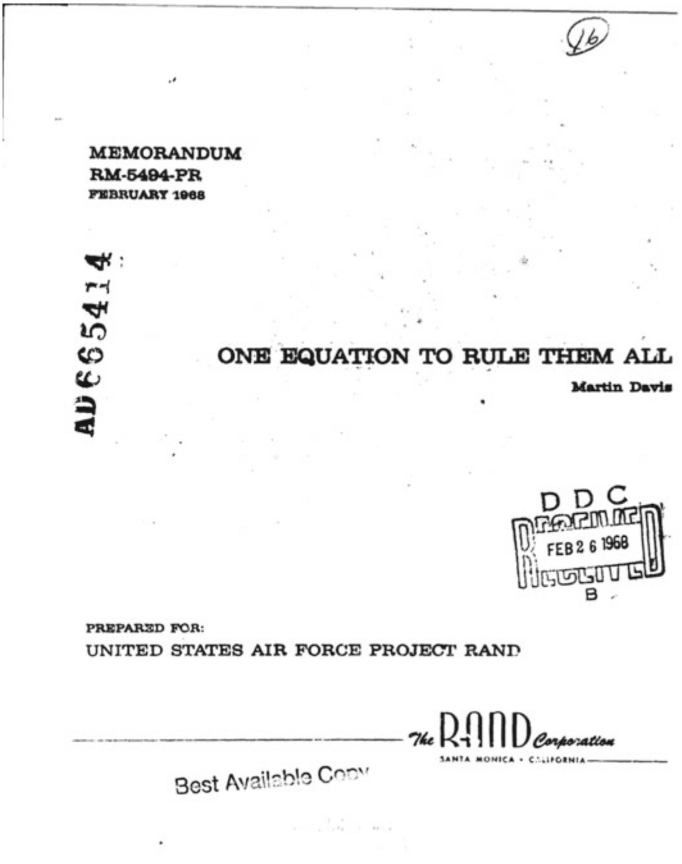
Fig. 3 In all of these equations, variables range over $\mathbb{N}$. Equations (I)–(X) with parameters u, v, a have a solution for $a > 1$ if and only if $v = y_u(a)$, where $X = x_u(a), Y = y_u(a)$ is the $u+1$st solution, over $\mathbb{N}$, of the Pell equation $X^2 - (a^2 - 1)Y = 1$. Equations (I)–(XV) with parameters α, β, u have a solution for $\beta \geq 1$ if and only if $\alpha = \beta^u$

$$
\begin{array}{rr@{\;=\;}l}
\text{(I)} & u+j & v \\
\text{(II}_\text{a}\text{)} & p+(a-1)\,q & v+r+1 \\
\text{(II}_\text{b}\text{)} & g & v+t+1 \\
\text{(III)} & p^2-(a^2-1)\,q^2 & 1 \\
\text{(IV}_\text{a}\text{)} & h+(a+1)\,g & b\,\big(p+(a+1)\,q\big)^2 \\
\text{(IV}_\text{b}\text{)} & h+(a-1)\,g & c\,\big(p+(a-1)\,q\big)^2 \\
\text{(V)} & h^2-(a^2-1)\,g^2 & 1 \\
\text{(VI)} & m & \big(h+(a+1)\,g\big)\,z+a \\
\text{(VII)} & m & \big(p+(a-1)\,q\big)\,f+1 \\
\text{(VIII)} & x^2-(m^2-1)\,y^2 & 1 \\
\text{(IX)} & y & d\,\big(p+(a-1)\,q\big)+u \\
\text{(X)} & y & e\,\big(h+(a+1)\,g\big)+v \\
\hline
\text{(XI)} & w^2-(a^2-1)\,v^2 & 1 \\
\text{(XII)} & \big(w-(a-\beta)\,v-\alpha\big)^2 & \gamma^2\,(2\,a\beta-\beta^2-1)^2 \\
\text{(XIII)} & \alpha+\tau+1 & 2\,a\beta-\beta^2-1 \\
\text{(XIV)} & \eta\;=\;\beta+\zeta+1 & u+\xi+1 \\
\text{(XV)} & a^2-(\eta^2-1)\,(\eta-1)^2\,(\delta+1)^2 & 1
\end{array}
$$

Another chapter on H10 is by Yuri V. Matiyasevich himself, the "clever young Russian" whose appearance Martin had predicted and who, by producing a Diophantine predicate of exponential growth, first obtained that negative solution. Yuri explains how Julia Robinson, Martin Davis, and Hilary Putnam had—through extraordinary insights—paved the way for his decisive mathematical contribution. Far from being exhausted, the field of research triggered by H10 abounds today with unanswered questions, some fairly old (e.g., does the equation of the report shown in Fig. 4 admit finitely many solutions?), other quite contemporary, and in fact, Yuri's article, after disclosing a formidable landscape of open issues in front of us, terminates with a conjecture raised by Martin in 2010.

A third related contribution is by Prof. Alexandra Shlapentokh, who enriches the landscape with extensions of H10 to recursive rings. When referring his Tenth Problem to integers, in 1900, Hilbert may have thought that he was posing the most difficult among variants of the same problem with respect to other rings. Nowadays, we know that H10 as originally posed is unsolvable, but we are in the difficult position of not being able to draw any conclusion about the analog of this problem for, say, the ring $\mathbb{Q}$ of rational numbers.

Martin has been a trailblazer of the field today known as "automated reasoning." The summer of 1954 sees him at work on a JOHNNIAC machine, implementing a logical decision procedure for integer arithmetic. In the late 1950s, a seminal report on computational methods in the propositional calculus arises from his collaboration with Hilary Putnam, the brilliant philosopher with whom Martin enjoyed



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MEMORANDUM
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FEBRUARY 1968

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ONE EQUATION TO RULE THEM ALL
Martin Davis

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FEB 26 1968
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PREPARED FOR:
UNITED STATES AIR FORCE PROJECT RAND

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Fig. 4 A report on an intriguing equation

discussing "all day long about everything under the sun, including Hilbert's tenth problem." The Davis–Putnam–Logemann–Loveland procedure, to date so basic in the architecture of fast Boolean satisfiability solvers, was rooted in that study, and Donald Loveland, who contributed to its pioneering implementation in the early 1960s, coauthors in this book, with Ashish Sabharwal and with Professor Bart Selman, a paper reviewing historical developments and the state of the art of propositional theorem provers.

It is slightly less known that in the early 1960s, the *most-general unification* mechanism for first-order logic was available in the working implementation of Martin's *linked conjunct* proof procedure, a forerunner of decadelong efforts to automatize reasoning in quantification theory. Unification has evolved, subsequently, into a well-established theory that proceeds hand in hand with the topic of rewriting systems. This is why dedicating a paper on a new trend in this field to

Martin seemed appropriate: Jörg Siekmann coauthors in this book, with Peter Szabó and Michael Hoche, a survey on "essential unification."

Udi Boker and Nachum Dershowitz, who dedicate an essay on "*honest computability*" to Martin, contend that "a nefarious representation can turn ... the intractable into trivial," whereas "demanding of an implementation that it also generates its internal representations of the input from an abstract term description of that input ... obviates cheating on complexity problems by giving away the answer in the representation." This attitude is akin to considerations made in the above-cited report by Davis and Putnam (1958) concerning propositional satisfiability, e.g.: "Even if the system has hundreds or thousands of formulas, it can be put into conjunctive normal form 'piece by piece', without any 'multiplying out.' This is a feasible (if laborious) task even for *hand* computation ⋯"

The centennial of Frege's *Begriffsschrift*, Martin reports, "fundamentally changed the direction of my work": Being invited to place some contemporary trends in a proper historical context, he finds "trying to trace the path from ideas and concepts developed by logicians ⋯ to their embodiment in software and hardware ⋯ endlessly fascinating."

Martin actually cultivated, since long, a keen interest in the history and philosophy of computing: The first edition of *The Undecidable*, his anthology of basic papers on unsolvable problems and computable functions, is dated 1965. One of Martin's heroes is Alan Mathison Turing; he also devoutly edited the collected works of Emil Leon Post, who had supervised his beginnings in logic at City College. In a recent paper, Martin and Wilfried Sieg have discussed a *conceptual confluence* between Post and Turing in 1936; in this book, Sieg coauthors with Máté Szabó and Dawn McLaughin, a paper addressing the question: *Did Post have Turing's Thesis?*

Yiannis N. Moschovakis unravels the history of another crucial confluence of ideas. Stephen C. Kleene, Emil Leon Post, and Andrzej Mostowski had raised questions which would influence profoundly the development of the theory of unsolvability when Martin, in the central part of his Ph.D. thesis, moved "*on into the transfinite!*", thus playing a very important role in defining natural extensions of the arithmetical hierarchy. The author skillfully alternates notes about the historical development of the subject with some carefully chosen technical details. This makes for a paper which is really a pleasure to read.

Martin once called "attention to the relevance for the foundational problems in quantum theory of some recent mathematical discoveries" arisen from logic. One of the diverse contributions dedicated to Martin, by Andreas Blass and Yuri Gurevich, aims at explaining certain sorts of *anyons*, "rather mysterious physical phenomena" which may provide a basis for quantum computing, by means of category theory.

Don Perlis's contribution speculates on the concept of infinity and distinguishes several modes of use of infinities in physics. In particular, quantum mechanics, he observes, provides intriguing examples on the subject. Nonstandard analysis—on which Martin wrote a classic—appears to shed light on some such phenomena.

"*Banishing ultrafilters from our consciousness*," the title of the paper contributed by Domenico Cantone with the editors of this book, echoes a comment by Martin in his *Applied Nonstandard Analysis* (1977). Martin then pointed out that the intricacies of the ultrapower construction of a nonstandard universe can be completely forgotten in favor of a few principles relating standard/nonstandard, internal/external objects. Bearing Martin's motto in mind, this paper recounts, and aided by a proof-checker embodying constructs for proof engineering, the authors have undertaken a verification of key results of the nonstandard approach to analysis.

The reader will also attend, inside this volume, Martin Davis and Hilary Putnam resuming some threads of their juvenile philosophical discussion. Martin has recently written about realism in mathematics (partly because Harvey Friedman had judged him "an extreme Platonist"), and Hilary cannot resist to amicably respond to his fascinating essay *Pragmatic Platonism* (also included in this book) and to discuss the relation between Martin's view and the views Hilary defends. In his turn, Martin comments on Hilary's remarks on his essay and takes the opportunity to say a little more about his view about certain topics such as mathematics and natural science, and new axioms for set theory.

The first chapter is an autobiographic essay by the eminent logician to whom the entire book is devoted. An earlier version of this essay, published in 1999, was titled "From Logic to Computer Science and Back." Martin reports that his debut as a computer programmer takes place in 1951, "without [him] realizing it," while he —a recent Ph.D. from Princeton, teaching recursive function theory at Champaign–Urbana—is designing Turing machines; he then gets recruited for a project on an automated system for navigating airplanes, with the task of writing code for an ORDVAC machine. Short afterward, Martin conceives the idea of writing his first book on computability; then, planning an extended visit to the Institute for Advanced Study in Princeton, he proposes to work on connections between logic and information theory. In the following decades, he frequently moves across the USA, teaching in various academic institutions and working on computability, on Hilbert's tenth problem, on computational logic, etc.

Multifaceted life and publications, but a substantial unity: in the new title chosen for his enriched autobiography, Martin regards himself simply as a logician.

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