

Preface

This book is devoted to explaining how the causal action principle gives rise to the interactions of the standard model plus gravity on the level of second-quantized fermionic fields coupled to classical bosonic fields. It is the result of an endeavor which I was occupied with for many years. Publishing the methods and results as a book gives me the opportunity to present the material in a coherent and comprehensible way.

The four chapters of this book evolved differently. Chapters 1 and 2 are based on the notes of my lecture “The fermionic projector and causal variational principles” given at the University of Regensburg in the summer semester 2014. The intention of this lecture was to introduce the basic concepts. Most of the material in these two chapters has been published previously, as is made clear in the text by references to the corresponding research articles. We also included exercises in order to facilitate the self-study. Chapters 3–5, however, are extended versions of three consecutive research papers written in the years 2007–2014 ([arXiv:0908.1542](#) [math-ph], [arXiv:1211.3351](#) [math-ph], [arXiv:1409.2568](#) [math-ph]). Thus the results of these chapters are new and have not been published elsewhere. Similarly, the appendix is formed of the appendices of the above-mentioned papers and also contains results of original research.

The fact that Chaps. 3–5 originated from separate research papers is still visible in their style. In particular, each chapter has its own short introduction, where the notation is fixed and some important formulas are stated. Although this leads to some redundancy and a few repetitions, I decided to leave these introductions unchanged, because they might help the reader to revisit the prerequisites of each chapter.

We remark that, having the explicit analysis of the continuum limit in mind, the focus of this book is on the computational side. This entails that more theoretical questions like the existence and uniqueness of solutions of Cauchy problems or the non-perturbative methods for constructing the fermionic projector are omitted. To the reader interested in mathematical concepts from functional analysis and partial differential equations, we can recommend the book “An Introduction to the

Fermionic Projector and Causal Fermion Systems” [FKT]. The intention is that the book [FKT] explains the physical ideas in a non-technical way and introduces the mathematical background from a conceptual point of view. It also includes the non-perturbative construction of the fermionic projector in the presence of an external potential and introduces spinors in curved space-time. The present book, on the other hand, focuses on getting a rigorous connection between causal fermion systems and physical systems in Minkowski space. Here we also introduce the mathematical tools and give all the technical and computational details needed for the analysis of the continuum limit. With this different perspective, the two books should complement each other and when combined should give a mathematically and physically convincing introduction to causal fermion systems and to the analysis of the causal action principle in the continuum limit.

We point out that the connection to quantum field theory (in particular to second-quantized bosonic fields) is not covered in this book. The reader interested in this direction is referred to [F17] and [F20].

I would like to thank the participants of the spring school “Causal fermion systems” held in Regensburg in March 2016 for their interest and feedback. Moreover, I am grateful to David Cherney, Andreas Grotz, Christian Hainzl, Johannes Kleiner, Simone Murro, Joel Smoller and Alexander Strohmaier for helpful discussions and valuable comments on the manuscript. Special thanks go to Johannes Kleiner for suggesting many of the exercises. I would also like to thank the Max Planck Institute for Mathematics in the Sciences in Leipzig and the Center of Mathematical Sciences and Applications at Harvard University for hospitality while I was working on the manuscript. I am grateful to the Deutsche Forschungsgemeinschaft (DFG) for financial support.

Regensburg, Germany
May 2016

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The Continuum Limit of Causal Fermion Systems
From Planck Scale Structures to Macroscopic Physics

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2016, XI, 548 p. 11 illus., Hardcover

ISBN: 978-3-319-42066-0