Preface

This book highlights the latest advances in engineering mathematics with a main focus on the mathematical models, structures, concepts, problems and computational methods and algorithms most relevant for applications in modern technologies and engineering. In particular, it features mathematical methods and models of applied analysis, probability theory, differential equations, tensor analysis and computational modelling used in applications to important problems concerning electromagnetics, antenna technologies, fluid dynamics, material and continuum physics and financial engineering.

The individual chapters cover both theory and applications, and include a wealth of figures, schemes, algorithms, tables and results of data analysis and simulation. Presenting new methods and results, reviews of cutting-edge research, and open problems for future research, they equip readers to develop new mathematical methods and concepts of their own, and to further compare and analyse the methods and results discussed.

Chapters 1–10 are concerned with applied mathematics methods and models applied in electrical engineering, electromagnetism and antenna technologies. Chapter 1 by Dragan Poljak is concerned with applications of integro-differential equations and numerical analysis methods to the analysis of grounding systems important in the design of lightning protection systems. The analysis of horizontal grounding electrodes has been carried out using the antenna theory approach in the frequency and time domain respectively. The formulation is based on the corresponding space-frequency and space-time Pocklington integro-differential equations. The integro-differential relationships are numerically handled via the Galerkin–Bubnov scheme of the Indirect Boundary Element Method. Frequency domain and time domain analysis is illustrated by computational examples. Chapter 2 by Silvestar Šesnić and Dragan Poljak deals with the use of analytical methods for solving various integro-differential equations in electromagnetic compatibility, with the emphasis on the frequency and time domain solutions of the thin-wire configurations buried in a lossy ground. Solutions in the frequency domain are carried out via certain mathematical manipulations with the current function appearing in corresponding integral equations. On the other hand, analytical solutions in the time domain are undertaken using the Laplace transform and Cauchy residue theorem. Obtained analytical results are compared to those calculated using the numerical solution of the frequency domain Pocklington equation, where applicable. Also, an overview of analytical solutions to the Grad-Shafranov equation for tokamak plasma is provided. In Chap. 3 by Milica Rančić, Radoslav Jankoski, Sergei Silvestrov and Slavoljub Aleksić, a new simple approximation that can be used for modelling one type of Sommerfeld integrals typically occurring in the expressions that describe sources buried in the lossy ground, is proposed. The proposed approximation has a form of a weighted exponential function with an additional complex constant term. The derivation procedure for this approximation is explained in detail, and the validation is supplied by applying it to the analysis of a bare conductor fed in the centre and immersed in the lossy ground at arbitrary depth. In Chap. 4 by Radoslav Jankoski, Milica Rančić, Vesna Arnautovski-Toseva and Sergei Silvestrov, high frequency analysis of a horizontal dipole antenna buried in lossy ground is performed. The soil is treated as a homogenous half-space of known electrical parameters. The authors compare the range of applicability of two forms of transmission line models, a hybrid circuit method, and a point-matching method in this context. Chapter 5 by Pushpanjali G. Metri pertains to an experimental implementation and evaluation of geometrically designed antennas. A novel design for an equilateral triangular microstrip antenna is proposed and tested. The antenna is designed, fabricated and tested for single and multiband operation. A theory for such antennas based on the experimental results is also considered. Chapter 6 by Nenad Cvetković, Miodrag Stojanović, Dejan Jovanović, Aleksa Ristić, Dragan Vučković and Dejan Krstić provides a brief review of the derivation of two groups of approximate closed form expressions for the electrical scalar potential Green's functions that originates from the current of the point ground electrode in the presence of a spherical ground inhomogeneity, proposes approximate solutions and considers known exact solutions involving infinite series sums. The exact solution is reorganized in order to facilitate comparison to the closed form solutions, and to estimate the error introduced by the approximate solutions, and error estimation is performed comparing the results for the electrical scalar potential obtained applying the approximate expressions and the accurate calculations. This is illustrated by a number of numerical experiments. In Chap. 7 by Mario Cvetković and Dragan Poljak, the electromagnetic thermal dosimetry model for the human brain exposed to electromagnetic radiation is developed. The electromagnetic model based on the surface integral equation formulation is derived using the equivalence theorem for the case of a lossy homogeneous dielectric body. The thermal dosimetry model of the brain is based on the form of Pennes' equation of heat transfer in biological tissue. The numerical solution of the electromagnetic model is carried out using the Method of Moments, while the bioheat equation is solved using the finite element method. The electromagnetic thermal model developed here has been applied in internal dosimetry of the human brain to assess the absorbed electromagnetic energy and consequent temperature rise. In Chap. 8 by Mirjana Perić, Saša Ilić and Slavoljub Aleksić, multilayered shielded structures are analysed using the hybrid boundary element method. The approach is based on the equivalent electrodes method, on the point-matching method for the potential of the perfect electric conductor electrodes and for the normal component of electric field at the boundary surface between any two dielectric layers. In order to verify the obtained results, they have been compared with the finite element method and results that have already been reported in the literature. In Chap. 9 by Vesna Javor, new engineering modified transmission line models of lightning strokes are presented. The computational results for lightning electromagnetic field at various distances from lightning discharges are in good agreement with experimental results that are usually employed for validating electromagnetic, engineering and distributed-circuit models. Electromagnetic theory relations, thin-wire antenna approximation of a lightning channel without tortuosity and branching, as well as the assumption of a perfectly conducting ground, are used for electric and magnetic field computation. An analytically extended function, suitable for approximating channel-base currents in these models, is also considered. Chapter 10 by Karl Lundengård, Milica Rančić, Vesna Javor and Sergei Silvestrov explores the properties of the multi-peaked analytically extended function for approximation of lightning discharge currents. According to experimental results for lightning discharge currents, they are classified into waveshapes representing the first positive, first and subsequent negative strokes, and long-strokes. A class of analytically extended functions is presented and used for the modelling of lightning currents. The basic properties of this function with a finite number of peaks are examined. A general framework for estimating the parameters of the analytically extended function using the Marquardt least-squares method for a waveform with an arbitrary (finite) number of peaks as well as for the given charge transfer and specific energy is described and used to find parameters for some common single-peak waveforms.

In turn, Chaps. 11–15 address the mathematical modelling and optimisation of technological processes with applications of partial differential equations, ordinary differential equations, numerical analysis, perturbation methods and special functions in fluid mechanics models that are important in engineering applications and technologies. Chapter 11 by Jüri Olt, Olga Liivapuu, Viacheslav Maksarov, Alexander Liyvapuu and Tanel Tärgla, is devoted to the mathematical modelling of the process system which paves the way for research on the selection and optimisation of machining conditions. The subject of this chapter is the method of dynamic process approximation method, which makes it possible to analyse the behaviour of the machining process system in the process of chip formation at a sufficient level of accuracy. In Chap. 12 by Prashant G. Metri, Veena M. Bablad, Pushpanjali G. Metri, M. Subhas Abel and Sergei Silvestrov, a mathematical analysis is carried out to describe mixed convection heat transfer in magnetohydrodynamic non-Darcian flow due to an exponential stretching sheet embedded in a porous medium in the presence of a non-uniform heat source/sink. Approximate analytical similarity solutions of the highly nonlinear momentum and energy equations are obtained. The governing system of partial differential equations is first transformed into a system of nonlinear ordinary differential equations using similarity transformation. The transformed equations are nonlinear coupled differential equations and are solved very efficiently by employing a fifth order Runge–Kutta–Fehlberg method with shooting technique for various values of the governing parameters. The numerical solutions are obtained by considering an exponential dependent stretching velocity and prescribed boundary temperature on the flow directional coordinate. The computed results are compared with the previously published work on various special cases of the problem and are in good agreement with the earlier studies. The effects of various physical parameters, such as the Prandtl number, the Grashof number, the Hartmann number, porous parameter, inertia coefficient and internal heat generation on flow and heat transfer characteristics are presented graphically to reveal a number of interesting aspects of the physical parameter. Chapter 13 by Prashant G. Metri, M. Subhas Abel and Sergei Silvestrov presents an analysis of the boundary layer flow and heat transfer over a stretching sheet due to nanofluids with the effects of the magnetic field, Brownian motion, thermophoresis, viscous dissipation and convective boundary conditions. The transport equations used in the analysis take into account the effect of Brownian motion and thermophoresis parameters. The highly nonlinear partial differential equations governing flow and heat transport are simplified using similarity transformation, and the ordinary differential equations obtained are solved numerically using the Runge–Kutta–Fehlberg and Newton–Raphson schemes based on the shooting method. The solutions for velocity temperature and nanoparticle concentration depend on parameters such as Brownian motion, thermophoresis parameter, magnetic field and viscous dissipation, which have a significant influence on controlling of the dynamics. In Chap. 14 by Jawali C. Umavathi, Kuppalapalle Vajravelu, Prashant G. Metri and Sergei Silvestrov, the linear stability of Maxwell fluid-nanofluid flow in a saturated porous layer is examined theoretically when the walls of the porous layers are subjected to time-periodic temperature modulations. A modified Darcy-Maxwell model is used to describe the fluid motion, and the nanofluid model used includes the effects of the Brownian motion. The thermal conductivity and viscosity are considered to be dependent on the nanoparticle volume fraction. A perturbation method that is based on a small amplitude of an applied temperature field is used to compute the critical value of the Rayleigh number and the wave number. The stability of the system, characterized by a critical Rayleigh number, is calculated as a function of the relaxation parameter, the concentration Rayleigh number, the porosity parameter, the Lewis number, the heat capacity ratio, the Vadász number, the viscosity parameter, the conductivity variation parameter, and the frequency of modulation. Three types of temperature modulations are considered, and the effects of all three types are found to destabilize the system as compared to the unmodulated system. Chapter 15 by J. Pratap Kumar, Jawali C. Umavathi, Prashant G. Metri and Sergei Silvestrov is devoted to a study of magneto-hydrodynamic flow in a vertical double passage channel taking into account the presence of the first order chemical reaction. The governing equations are solved by using a regular perturbation technique valid for small values of the Brinkman number and a differential transform method valid for all values of the Brinkman number. The results are obtained for velocity, temperature and concentration. The effects of various dimensionless parameters such as the thermal Grashof number, mass Grashof number, Brinkman number, first order chemical reaction parameter, and Hartman number on the flow variables are discussed and presented graphically for open and short circuits. The validity of solutions obtained by the differential transform method and regular perturbation method are in good agreement for small values of the Brinkman number. Further, the effects of governing parameters on the volumetric flow rate, species concentration, total heat rate, skin friction and Nusselt number are also observed and tabulated.

Chapters 16–18 are concerned with mathematical methods of stochastic processes, probability theory, differential geometry, tensor analysis, representation theory, differential equations, algebra and computational mathematics for applications in materials science and financial engineering. In Chap. 16 by Anatoliy Malyarenko and Martin Ostoja-Starzewski, a random field model of the 21-dimensional elasticity tensor is considered, and representation theory is used to obtain the spectral expansion of the model in terms of stochastic integrals with respect to random measures. The motivation for treating this tensor as a random field is that nearly all the materials encountered in nature as well those produced by man, except for the purest crystals, possess some degree of disorder or inhomogeneity. At the same time, elasticity is the starting point for any solid mechanics model. Chapter 17 by Anatoliy Malyarenko, Jan Röman and Oskar Schyberg is devoted to mathematical models for catastrophe bonds which are an important instrument in the fields of finance, insurance and reinsurance, where the natural risk index is described by the Merton jump-diffusion while the risk-free interest rate is governed by the Hull–White stochastic differential equation. The sensitivities of the bond price with respect to the initial condition, volatility of the diffusion component, and jump amplitude are calculated using the Malliavin calculus approach. Lastly, in Chap. 18 by Betuel Canhanga, Anatoliy Malyarenko, Jean-Paul Murara and Sergei Silvestrov, stochastic volatilities models for pricing European options are considered as a response to the weakness of the constant volatility models, which have not succeeded in capturing the effects of volatility smiles and skews. A model with two-factor stochastic volatilities where the correlation between the underlying asset price and the volatilities varies randomly is considered, and the first order asymptotic expansion methods are used to determine the price of European options.

The book consists of carefully selected and refereed contributed chapters covering research developed as a result of a focused international seminar series on mathematics and applied mathematics, as well as three focused international research workshops on engineering mathematics organised by the Research Environment in Mathematics and Applied Mathematics at Mälardalen University from autumn 2014 to autumn 2015: the International Workshop on Engineering Mathematics for Electromagnetics and Health Technology; the International Workshop on Engineering Mathematics, Algebra, Analysis and Electromagnetics; and the 1st Swedish-Estonian International Workshop on Engineering Mathematics, Algebra, Analysis and Applications.

This book project has been realised thanks to the strategic support offered by Mälardalen University for the research and research education in Mathematics, which is conducted by the research environment Mathematics and Applied Mathematics (MAM), in the established research area of Educational Sciences and Mathematics at the School of Education, Culture and Communication at Mälardalen

University. We also wish to extend our thanks to the EU Erasmus Mundus projects FUSION, EUROWEB and IDEAS, the Swedish International Development Cooperation Agency (Sida) and International Science Programme in Mathematical Sciences, Swedish Mathematical Society, Linda Peetre Memorial Foundation, as well as other national and international funding organisations and the research and education environments and institutions of the individual researchers and research teams who contributed to this book.

We hope that this book will serve as a source of inspiration for a broad spectrum of researchers and research students in the field of applied mathematics, as well as in the specific areas of applications of mathematics considered here.

Västerås, Sweden
July 2016

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http://www.springer.com/978-3-319-42081-3

Engineering Mathematics I
Electromagnetics, Fluid Mechanics, Material Physics
and Financial Engineering
Silvestrov, S.; Rančić, M. (Eds.)
2016, XV, 341 p. 135 illus., 67 illus. in color., Hardcover
ISBN: 978-3-319-42081-3