

Preface

This book highlights the latest advances in engineering mathematics with a main focus on the mathematical models, structures, concepts, problems and computational methods and algorithms most relevant for applications in modern technologies and engineering. It addresses mathematical methods of noncommutative algebra, applied matrix analysis, operator analysis, probability theory and stochastic processes, geometry, computational mathematics, optimization and operations research with applications in network analysis, ranking in networks, networks in bioinformatics, genetic analysis and cancer research, data mining and classification, production logistics optimization.

The individual chapters cover both theory and applications, and include a wealth of figures, schemes, algorithms, tables and results of data analysis and simulation. Presenting new methods and results, reviews of cutting-edge research, and open problems for future research, they equip readers to develop new mathematical methods and concepts of their own, and to further compare and analyze the methods and results discussed.

Chapter “[Classification of Low Dimensional 3-Lie Superalgebras](#)” by Viktor Abramov and Priit Lätt is concerned with extension of a notion of n -Lie algebra to \mathbb{Z}_2 -graded structures by means of a graded Filippov identity giving a notion of n -Lie superalgebra. Classification of low dimensional 3-Lie superalgebras is proposed, and it is shown that given an n -Lie superalgebra equipped with a supertrace one can construct the $(n+1)$ -Lie superalgebra which is referred to as the induced $(n+1)$ -Lie superalgebra. Based on Clifford algebra which, when endowed with a \mathbb{Z}_2 -graded structure and a graded commutator, can be viewed as the Lie superalgebra and supertrace defined via its matrix representation, the 3-Lie superalgebras are constructed and explicitly described by their ternary commutators. In Chap. “[Semi-Commutative Galois Extension and Reduced Quantum Plane](#)” by Viktor Abramov and Md. Raknuzzaman, it is shown that a semi-commutative Galois extension of associative unital algebra by means of an element τ , which satisfies $\tau^N = 1$ (1 is the identity element of an algebra and $N \geq 2$ is an integer) induces a structure of graded q -differential algebra, where q is a primitive N th

root of unity. The graded q -differential algebra is constructed and its first order noncommutative differential calculus is studied. Moreover, the higher order noncommutative differential calculus induced by a semi-commutative Galois extension of associative unital algebra is studied, and it is shown that a reduced quantum plane can be viewed as a semi-commutative Galois extension of a fractional one-dimensional space. Chapter “[Valued Custom Skew Fields with Generalised PBW Property from Power Series Construction](#)” describes an interesting construction of associative algebras with a number of useful properties. The construction is basically that of a power series algebra with given commutation relation. The constructed algebras have a Poincaré–Birkhoff–Witt type basis, are equipped with a norm (actually an ultranorm) that is trivial to compute for basis elements, are topologically complete, and satisfy their given commutation relation. In addition, parameters can be chosen so that the algebras will in fact turn out to be skew fields and the norms become valuations. Chapter “[Computing Burchnell-Chaundy Polynomials with Determinants](#)” by Johan Richter and Sergei Silvestrov concerned with generalization of a method of computing the Burchnell–Chaundy polynomial of two commuting differential operators based on Burchnell–Chaundy eliminant determinant construction to the class of rings known as Ore extensions. It is shown that the eliminant construction partially generalizes and also counterexamples showing that these generalizations do not always retain all desired properties are provided. In Chap. “[Centralizers and Pseudo-Degree Functions](#)” by Johan Richter, a generalization of a proof of certain results by Hellström and Silvestrov on centralizers in graded algebras is presented, centralizers in certain algebras with valuations are considered and a proof that the centralizer of an element in these algebras is a free module over a certain ring is given. Under further assumptions it is also shown that the centralizer is also commutative. In Chap. “[Crossed Product Algebras for Piece-Wise Constant Functions](#)” by Johan Richter, Sergei Silvestrov, Vincent Ssembatya and Alex Behakanira Tumwesigye, algebras of functions that are constant on the sets of a partition are considered together with their crossed product algebras with the group of integers and the commutant of the function algebra in the crossed product algebra. In Chap. “[Commutants in Crossed Product Algebras for Piece-Wise Constant Functions](#)” by Johan Richter, Sergei Silvestrov and Alex Behakanira Tumwesigye, crossed product algebras of algebras of piece-wise constant functions on the real line with the group of integers are considered, and for an increasing sequence of algebras the set difference between the corresponding commutants is described.

Chapter “[Asymptotic Expansions for Moment Functionals of Perturbed Discrete Time Semi-Markov Processes](#)” by Mikael Petersson is devoted to the study of moment functionals of mixed power-exponential type for nonlinearly perturbed semi-Markov processes in discrete time. Conditions under which the moment functionals of interest can be expanded in asymptotic power series with respect to the perturbation parameter are given and it is shown how the coefficients in these expansions can be computed from explicit recursive formulas. The results of this chapter have applications for studies of quasi-stationary distributions. In Chap. “[Asymptotics for Quasi-Stationary Distributions of Perturbed Discrete](#)

[Time Semi-Markov Processes](#)” by Mikael Petersson, quasi-stationary distributions of nonlinearly perturbed semi-Markov processes in discrete time are studied. This type of distributions is of interest for analysis of stochastic systems which have finite lifetimes but are expected to persist for a long time. Asymptotic power series expansions for quasi-stationary distributions are obtained, it is shown how the coefficients in these expansions can be computed from a recursive algorithm, and a numerical example for a discrete time Markov chain is presented as an illustration of this algorithm. Chapter [“Asymptotic Expansions for Stationary Distributions of Perturbed Semi-Markov Processes”](#) by Dmitrii Silvestrov and Sergei Silvestrov presents new algorithms for computing asymptotic expansions for stationary distributions of nonlinearly perturbed semi-Markov processes based on special techniques of sequential phase space reduction, which can be applied to processes with asymptotically coupled and uncoupled finite phase spaces. Chapter [“PageRank, a Look at Small Changes in a Line of Nodes and the Complete Graph”](#) is about the PageRank algorithm used as part of the ranking process of different Internet pages in search engines, ranking in citation networks as well as other information, communication and big data networks. The chapter focuses on the behavior of PageRank as the system dynamically changes either by contracting or expanding such as when subtracting or adding nodes or links or groups of nodes or links. PageRank is considered as the solution of a linear system of equations and examined in both the ordinary normalized version of PageRank as well as the non-normalized version, and explicit formulas for the PageRank of some simple link structures are obtained. Chapter [“PageRank, Connecting a Line of Nodes with a Complete Graph”](#) is focused on the PageRank algorithm following original definition of PageRank by Sergey Brin and Larry Page as the stationary distribution of a certain random walk on a graph used to rank homepages on the Internet. Specifically, this chapter is concerned with PageRank changes after adding or removing edge between otherwise disjoint subgraphs, for example link structures consisting of a line of nodes or a complete graph and different ways to combine the two. Both the ordinary normalized version of PageRank as well as a non-normalized version of PageRank can be found by solving corresponding linear system, and it is demonstrated that it is possible to find moreover explicit formulas for the PageRank in some simple link structures and using these formulas take a more in-depth look at the behavior of the ranking as the system changes. Chapter [“Graph Centrality Based Prediction of Cancer Genes”](#) by Holger Weishaupt, Patrik Johansson, Christopher Engström, Sven Nelander, Sergei Silvestrov and Fredrik J. Swartling focuses on how graph centralities obtained from biological networks have been used to predict cancer genes. As current cancer therapies including surgery, radiotherapy and chemotherapy are often plagued by high failure rates, designing more targeted and personalized treatment strategies requires a detailed understanding of druggable tumor driver genes. Specifically, the chapter begins with describing the current problems in cancer therapy and the reasoning behind using network based cancer gene prediction, followed by an outline of biological networks, their generation and properties, and finely by a review of major concepts, recent results as well as future challenges regarding the use of graph centralities in

cancer gene prediction. Chapter “[Output Rate Variation Problem: Some Heuristic Paradigms and Dynamic Programming](#)” by Gyan Bahadur Thapa and Sergei Silvestrov is concerned with the output rate variation problem, which is one of the important research directions in the area of multi-level just-in-time production systems. A short survey of the mathematical models of this problem is provided together with consideration of its NP-hardness, a brief review of heuristic approaches to the problem, the discussion on the dynamic programming approach and pegging assumption reducing the multi-level problem to weighted single-level problem as well as some open problems.

In Chap. “ [\$L^p\$ -Boundedness of Two Singular Integral Operators of Convolution Type](#)” by Sten Kaijser and John Musonda, boundedness properties investigated for two singular integral operators defined on L^p -spaces ($1 < p < \infty$) on the real line, both as convolution operators on $L^p(\mathbb{R})$ and on the weighted spaces $L^p(\omega)$, where $\omega(x) = 1/(2 \cosh \frac{\pi}{2}x)$. In the Chap. “[Fractional-Wavelet Analysis of Positive definite Distributions and Wavelets on \$\mathcal{D}'\(\mathbb{C}\)\$](#) ” by Emanuel Guariglia and Sergei Silvestrov, a wavelet expansion theory for positive definite distributions over the real line is considered and a fractional derivative operator for complex functions in the distribution sense is defined. The Ortigueira–Caputo fractional derivative operator is rewritten as a convolution according to the fractional calculus of real distributions, and the fractional derivatives of the complex Shannon wavelet and Gabor–Morlet wavelet are computed together with their plots and main properties. Chapters “[Linear Classification of Data with Support Vector Machines and Generalized Support Vector Machines](#)” and “[Linear and Nonlinear Classifiers of Data with Support Vector Machines and Generalized Support Vector Machines](#)” by Talat Nazir, Xiaomin Qi and Sergei Silvestrov are devoted to support vector machine for linear and nonlinear classification of data. Generalized support vector machine for classification of data is introduced, and it is shown that the problem of generalized support vector machine is equivalent to the problem of generalized variational inequality. Various results for the existence of solutions are established and several examples are constructed. In Chaps. “[Common Fixed Points of Weakly Commuting Multivalued Mappings on a Domain of Sets Endowed with Directed Graph](#)” and “[Common Fixed Point Results for Family of Generalized Multivalued \$F\$ -contraction Mappings in Ordered Metric Spaces](#)” by Talat Nazir and Sergei Silvestrov, the existence of coincidence points and common fixed points for multivalued mappings satisfying certain graphic ψ -contraction contractive conditions with set-valued domain endowed with a graph, without appealing to continuity, is established, the existence of common fixed points of family of multivalued mappings satisfying generalized F -contractive conditions in ordered metric spaces is also investigated.

The book consists of carefully selected and refereed contributed chapters covering research developed as a result of a focused international seminar series on mathematics and applied mathematics and a series of three focused international research workshops on engineering mathematics organized by the Research Environment in Mathematics and Applied Mathematics at Mälardalen University

from autumn 2014 to autumn 2015: the International Workshop on Engineering Mathematics for Electromagnetics and Health Technology; the International Workshop on Engineering Mathematics, Algebra, Analysis and Electromagnetics; and the 1st Swedish-Estonian International Workshop on Engineering Mathematics, Algebra, Analysis and Applications.

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We hope that this book will serve as a source of inspiration for a broad spectrum of researchers and research students in mathematics and applied mathematics, as well as in the areas of applications of mathematics considered in the book.

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