

Contents

| | | |
|----------|--|----------|
| 1 | Introduction | 1 |
| 1.1 | Towards Absolute Zero | 1 |
| 1.1.1 | Discovery of Superconductivity and Superfluidity | 2 |
| 1.1.2 | Bose–Einstein Condensation | 3 |
| 1.2 | Ultracold Quantum Gases | 4 |
| 1.2.1 | Laser Cooling and Magnetic Trapping | 4 |
| 1.2.2 | Bose–Einstein Condensate à la Einstein | 5 |
| 1.2.3 | Degenerate Fermi Gases | 6 |
| 1.3 | Quantum Fluids Today | 6 |
| | References | 8 |
| 2 | Classical and Quantum Ideal Gases | 9 |
| 2.1 | Introduction | 9 |
| 2.2 | Classical Particles | 10 |
| 2.3 | Ideal Classical Gas | 10 |
| 2.3.1 | Macrostates, Microstates and the Most Likely State of the System | 11 |
| 2.3.2 | The Boltzmann Distribution | 12 |
| 2.4 | Quantum Particles | 14 |
| 2.4.1 | A Chance Discovery | 14 |
| 2.4.2 | Bosons and Fermions | 15 |
| 2.4.3 | The Bose–Einstein and Fermi–Dirac Distributions | 16 |
| 2.5 | The Ideal Bose Gas | 17 |
| 2.5.1 | Continuum Approximation and Density of States | 17 |
| 2.5.2 | Integrating the Bose–Einstein Distribution | 19 |
| 2.5.3 | Bose–Einstein Condensation | 20 |
| 2.5.4 | Critical Temperature for Condensation | 21 |
| 2.5.5 | Condensate Fraction | 21 |
| 2.5.6 | Particle-Wave Overlap | 22 |
| 2.5.7 | Internal Energy | 23 |
| 2.5.8 | Pressure | 24 |

| | | |
|----------|--|-----------|
| 2.5.9 | Heat Capacity | 24 |
| 2.5.10 | Ideal Bose Gas in a Harmonic Trap | 25 |
| 2.6 | Ideal Fermi Gas. | 27 |
| 2.7 | Summary | 28 |
| | Problems | 29 |
| | References | 30 |
| 3 | Gross-Pitaevskii Model of the Condensate. | 33 |
| 3.1 | The Gross-Pitaevskii Equation. | 33 |
| 3.1.1 | Mass, Energy and Momentum | 35 |
| 3.2 | Time-Independent GPE | 35 |
| 3.3 | Fluid Dynamics Interpretation | 36 |
| 3.4 | Stationary Solutions in Infinite or Semi-infinite Homogeneous Systems. | 38 |
| 3.4.1 | Uniform Condensate | 38 |
| 3.4.2 | Condensate Near a Wall | 39 |
| 3.5 | Stationary Solutions in Harmonic Potentials | 40 |
| 3.5.1 | No Interactions | 40 |
| 3.5.2 | Strong Repulsive Interactions | 41 |
| 3.5.3 | Weak Interactions | 43 |
| 3.5.4 | Anisotropic Harmonic Potentials and Condensates of Reduced Dimensionality | 44 |
| 3.6 | Imaging and Column-Integrated Density | 47 |
| 3.7 | Galilean Invariance and Moving Frames | 47 |
| 3.8 | Dimensionless Variables. | 48 |
| 3.8.1 | Homogeneous Condensate | 49 |
| 3.8.2 | Harmonically-Trapped Condensate | 50 |
| | Problems | 50 |
| | References | 52 |
| 4 | Waves and Solitons | 53 |
| 4.1 | Dispersion Relation and Sound Waves | 53 |
| 4.1.1 | Dispersion Relation | 53 |
| 4.1.2 | Sound Waves | 55 |
| 4.2 | Landau's Criterion and the Breakdown of Superfluidity | 56 |
| 4.3 | Collective Modes. | 58 |
| 4.3.1 | Scaling Solutions | 58 |
| 4.3.2 | Expansion of the Condensate. | 62 |
| 4.4 | Solitons | 63 |
| 4.5 | Dark Solitons | 64 |
| 4.5.1 | Dark Soliton Solutions | 64 |
| 4.5.2 | Particle-Like Behaviour | 66 |
| 4.5.3 | Collisions | 67 |
| 4.5.4 | Motion in a Harmonic Trap | 68 |
| 4.5.5 | Experiments and 3D Effects | 70 |

| | | |
|----------|---|------------|
| 4.6 | Bright Solitons | 71 |
| 4.6.1 | Collisions | 72 |
| 4.6.2 | Experiments and 3D Effects | 74 |
| | Problems | 75 |
| | References | 77 |
| 5 | Vortices and Rotation | 79 |
| 5.1 | Phase Defects | 79 |
| 5.2 | Quantized Vortices. | 80 |
| 5.3 | Classical Versus Quantum Vortices | 81 |
| 5.4 | The Nature of the Vortex Core | 82 |
| 5.5 | Vortex Energy and Angular Momentum | 84 |
| 5.6 | Rotating Condensates and Vortex Lattices. | 86 |
| 5.6.1 | Buckets | 86 |
| 5.6.2 | Trapped Condensates | 89 |
| 5.7 | Vortex Pairs and Vortex Rings | 92 |
| 5.7.1 | Vortex-Antivortex Pairs and Corotating Pairs | 92 |
| 5.7.2 | Vortex Rings. | 93 |
| 5.7.3 | Vortex Pair and Ring Generation by a Moving Obstacle | 94 |
| 5.8 | Motion of Individual Vortices | 96 |
| 5.9 | Kelvin Waves | 97 |
| 5.10 | Vortex Reconnections | 98 |
| 5.11 | Sound Emission. | 100 |
| 5.12 | Quantum Turbulence | 100 |
| 5.12.1 | Three-Dimensional Quantum Turbulence | 102 |
| 5.12.2 | Two-Dimensional Quantum Turbulence | 103 |
| 5.13 | Vortices of Infinitesimal Thickness | 104 |
| 5.13.1 | Three-Dimensional Vortex Filaments | 104 |
| 5.13.2 | Two-Dimensional Vortex Points | 106 |
| | Problems | 107 |
| | References | 109 |
| | Appendix A Simulating the 1D GPE | 111 |
| | Index | 117 |

<http://www.springer.com/978-3-319-42474-3>

A Primer on Quantum Fluids

Barenghi, C.F.; Parker, N.G.

2016, XIII, 119 p. 56 illus., 34 illus. in color., Softcover

ISBN: 978-3-319-42474-3