

Preface

This book is the first, basic, part of a more advanced textbook *Symmetric Spaces of Measurable Functions*. It contains an introduction to the theory, including a detailed study of Lorentz, Marcinkiewicz, and Orlicz spaces.

The theory of symmetric (rearrangement invariant) function spaces goes back to the classical spaces L_p , $1 \leq p \leq \infty$. The theory was intensively developed during the last century, mainly in the context of general Banach lattices. It presents many interesting and deep results having important applications in various areas of function theory and functional analysis. The theory has a great many applications in interpolation of linear operators, ergodic theory, harmonic analysis, and various areas of mathematical physics.

The authors of this book (at different years and in different countries) have studied and taught the theory of symmetric spaces. They discovered independently the following surprising fact: despite the abundance of monographs, there was no book suitable for our purposes either in the Russian mathematical literature or in the mathematical literature of the rest of the world.

In fact, we wished to have a book with a relatively small volume that met the following criteria:

1. The book should contain basic concepts and results of the general theory of symmetric spaces with the main focus on a detailed exposition of classical spaces L_p , $1 \leq p \leq \infty$, and Lorentz, Marcinkiewicz, and Orlicz spaces, as well.
2. The book should be accessible to master's students, doctoral students, and researchers in mathematics and physics departments who are familiar with the basics of the measure theory and functional analysis in the framework of standard university courses.
3. The material of the book should correspond to a one-semester special course of lectures (about 4 months or 17–18 weeks).
4. The presentation should not require any additional source except standard references on basic concepts and theorems of measure theory and functional analysis.

In our opinion, this book, offered now to the reader, completely meets the above requirements.

We can point out three main sources from which the material of the book was adopted.

First is a monograph by S. G. Krein, J. I. Petunin, E. M. Semenov, *Interpolation of Linear Operators*.

The second source is two volumes of J. Lindenstrauss, L. Tzafriri, *Classical Banach Spaces I. Sequence Spaces* and *Classical Banach Spaces II. Function Spaces*.

Third, the part devoted to Orlicz spaces is based on a nice exposition of this theme in the book by G. A. Edgar, L. Sucheston, *Stopping Times and Directed Processes*.

Our book includes four parts comprising seventeen chapters. This allows us to divide the corresponding one-semester lecture course into 4 months or 17 weeks, and rigorously restricts, in turn, the volume of material.

As a result a great many important related topics have not been included in the main part of the book. The reader can find this additional material in the exercises at the end of each part and in the section called “Complements” at the end of the book. Throughout the main exposition, we deal only with symmetric spaces on the half-line $\mathbb{R}^+ = [0, \infty)$, while the symmetric spaces on the interval $[0, 1]$ and the symmetric sequence spaces are considered in the exercises and complements.

Each of the four parts begins with an overview and then is divided into chapters. Each part concludes with exercises and notes. Complements are located at the end of the book together with references and an index.

Complements and exercises are intended for independent study.

The list of references contains some historical material, the books and articles from which we took terminology, results, and their proofs, and also a bibliography for further reading. The list of references is not, of course, comprehensive, but it points out, we hope, the most of important directions of the theory.

Be'er Sheva, Israel
New York, NY, USA
Simferopol, Russia
Simferopol, Russia

Ben-Zion A. Rubshtein
Genady Ya. Grabarnik
Mustafa A. Muratov
Yulia S. Pashkova

Foundations of Symmetric Spaces of Measurable
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Lorentz, Marcinkiewicz and Orlicz Spaces

Rubshtein, B.-Z.A.; Grabarnik, G.Y.; Muratov, M.A.;
Pashkova, Y.S.

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