Preface

This is a textbook on *Quantum Zero-Error Information Theory*. The reader will find an approachable introduction to this subject, from the building blocks of this theory to the latest contributions to the literature. The contents of several original research papers are introduced in a pedagogical way, making it easier for the reader to learn the concepts and depict the developments.

Our dedication to this subject started almost 15 years ago when we were with the IQuanta—Institute for Studies in Quantum Computation and Information, Campina Grande Federal University, Paraíba, Brazil. This institute was founded considering a multidisciplinary body of research having strong interest in learning and contributing to this amazing field of quantum computation and information. Since the definition of the zero-error capacity of quantum channels in 2005, many interesting contributions to this area were given by some of the most important researches in quantum information around the world. Last year, we decided to write a book that contemplate our seminal contributions, as well as the state of the art in this field.

This book starts with an introduction to the fundamentals of quantum information processing in Chap. 2. The main goal of this chapter is to introduce the basic concepts of quantum mechanics for the reader who is not familiar with them. We show how information is represented, processed, and measured in a quantum domain. We also introduce some of the amazing features of the quantum mechanics theory, such as superposition, parallelism, and entanglement. All the concepts presented in this chapter are essential for understanding further results in quantum information theory.

Chapter 3 revisits elementary concepts of classical and quantum information theory. Initially, we give a mathematical definition of information and define the main measures of information, such as entropy and mutual information. We characterize information sources and communication channels in order to enunciate two important theorems proposed by Shannon—the source coding and channel coding theorems. Concerning the quantum information theory, we explore the quantum counterpart of classical concepts already presented. Initially, we define what are quantum states and how information can be encoded within them. Then, we introduce measures of quantum information in terms of the von Neumann entropy. Quantum channels are defined, as well as the accessible information of quantum source. These initial concepts, together with basic results such as the Holevo bound, are sufficient to introduce a variety of quantum channels capacity, such as the $C_{1,1}$ capacity, the HSW capacity, the quantum capacity, and the entanglement-assisted capacity. These capacities illustrate how quantum channels can be used to convey information in many different ways.

Once these background skills are established, Chap. 4 gives an overview of the classical zero-error information theory. The chapter examines the results of an important paper written by Claude E. Shannon in 1956, where he demonstrated how classical channels could be used to transmit information in a scenario where no errors are permitted, instead of allowing an asymptotically small probability of error. We show the characterization of such capacity, its relation with graph theory, the Lovász theta function, and the zero-error capacity of sums and products of classical channels.

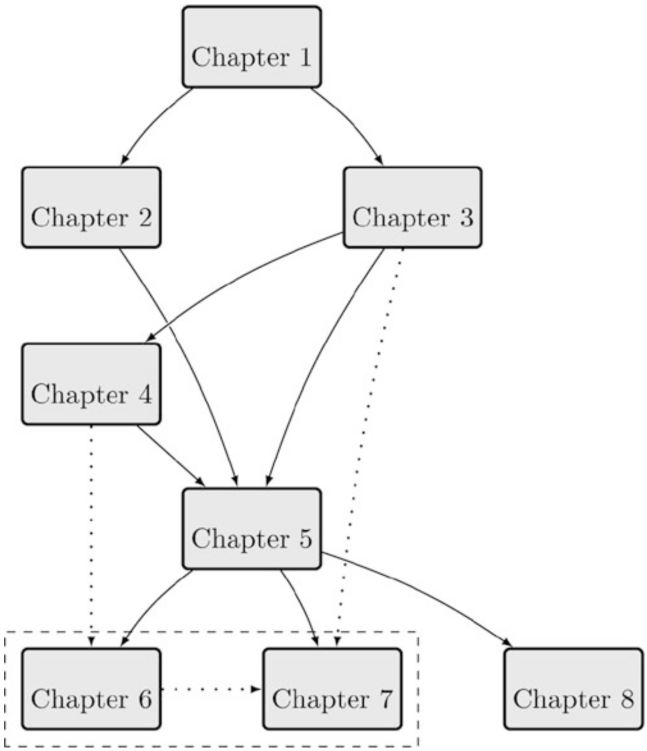
Chapter 5 brings the generalization of zero-error capacity to the quantum scenario. Many of the results presented in this chapter, including the definition of the zero-error capacity of quantum channels, were taken from the PhD thesis of Rex A.C. Medeiros, when he was advised by Francisco Marcos de Assis of Campina Grande Federal University and Gérard Cohen and Romain Alléaume of Télécom ParisTech. The chapter also presents contributions from other researchers; the most impressive of which is the superactivation of the zero-error capacity, a phenomenon that has no counterpart in classical theory.

The next two chapters present contributions to the quantum zero-error information theory developed by Elloá B. Guedes during her doctorate under the advisory of Francisco Marcos de Assis at Campina Grande Federal University. Chapter 6 introduces the notion of zero-error secrecy capacity of quantum channels which puts together both zero-error and secrecy capacities. This is a particular scenario where information can be conveyed not only without errors but also in perfect secrecy among two parties sharing a particular class of quantum channels. The chapter starts with some background concepts regarding decoherence-free subspaces and quantum secrecy capacity. Also, we present a characterization of the quantum zero-error secrecy capacity in terms of graphs. Finally, a security analysis of the proposed protocol is made; some examples and related literature are also included.

Chapter 7 introduces a measure of information of quantum source that is based on the zero-error capacity of classical channels, the zero-error accessible information of quantum sources. It has no classical counterpart and measures the maximum amount of information that can be retrieved from a quantum source after a measurement without decoding errors. Besides introducing this concept, the chapter includes some examples and discusses some related works in the literature.

Finally Chap. 8 includes most of the recent contributions to the literature of zero-error information theory. After some discussion about classical and quantum correlations on the proof of Bell's inequality, Gleason and Kochen-Specker theorems are introduced in order to give a formal definition of the quantum chromatic number. A quantum version of the Wielandt's inequality is discussed, followed by the definition of the entangled assisted zero-error capacity of a quantum channel and a very interesting generalization of the Lovász ϑ functional. The quantum clique problem is defined, and we show that the problem of finding the quantum clique belongs to the $\mathcal{QMA}$-complete complexity class. Some of these topics are currently being actively researched and have strong impact on the development of new results in quantum zero-error information theory.

We recommend our readers to follow the dependency diagram below in order to see how the chapters of the book are interconnected. The most important dependencies are emphasized with a continuous line, while suggested reading is shown with a dotted line.



By reading this book, we believe that the reader will follow the path along the most up-to-date developments of the quantum zero-error information theory. To make this task easier, the reader can find suggestions for further reading at the end of each chapter. The references lead to original works where the reader can retrieve the seminal ideas behind every subject discussed. Within the chapters, the readers can also find many detailed examples to illustrate and help in understanding the multiple concepts presented.

Corrections to this edition and suggestions or comments regarding the topics discussed can be sent to our e-mails (ebgcosta@uea.edu.br, rexmedeiros@ect.ufrn.br, fmarcos@dee.ufcg.edu.br). We would be very glad to hear from you!

Manaus, Brazil — Elloá B. Guedes
Natal, Brazil — Rex A.C. Medeiros
Campina Grande, Brazil — Francisco Marcos de Assis

http://www.springer.com/978-3-319-42793-5

Quantum Zero-Error Information Theory
Guedes, E.B.; de Assis, F.M.; Medeiros, R.A.d.C.
2016, XIX, 189 p. 50 illus., Hardcover
ISBN: 978-3-319-42793-5