

## Chapter 2

# Economics Theory Basics

In this chapter, we will briefly review several basic theories in economics used for our later analysis, including the non-cooperative game theory, super-modular game theory, evolutionary game theory, contract theory, and Nash bargaining theory. The non-cooperative game theory and super-modular game theory are used in the spectrum trading market for analyzing the end-users' behaviors and the secondary operators' price competition (Sect. 3.2). The contract theory is used in the spectrum trading market for optimizing the white space database's spectrum reservation decision under information asymmetry (Sect. 3.3). The evolutionary game theory is used in the information trading market for analyzing the evolution of white space devices' purchasing behaviors (Chap. 4). The Nash bargaining theory is used in the hybrid spectrum and information market for analyzing the negotiation between the white space database and the spectrum licensee (Chap. 5).

### 2.1 Non-cooperative Game Theory

A game is a formal representation of a situation in which a number of individuals (players) interact with *strategic interdependence*. In other words, each player's pay-off depends not only on his own decision but also on the decisions of other users. Non-cooperative game is one of the basic game form where players make decisions independently and non-cooperatively (see Osborne 2004; Gibbons 1992). We will use the non-cooperative game to model and analyze the strategic interaction between white space end-users and white space access devices (secondary operators) in the spectrum trading market (Sect. 3.2).

To describe a game situation of strategic interactions, we need to define:

- **Players:** Who are involved in the game?

- **Rules:** What actions can players choose? How and when do they make decisions? What information do players know about each other when making decisions?
- **Outcomes:** What is the outcome of the game for each possible action combination chosen by players?
- **Payoffs:** What are the players' preferences over the possible outcomes?

Each player is assumed to be *rational (self-interested)*, with the goal of choosing the actions that produce his most preferred outcome. Under such a situation, a central issue is to identify the stable outcome(s) of the game. There are several concepts characterizing such stable outcomes, one of which is called *Nash equilibrium*, where no player has the incentive to change his strategy unilaterally (see Osborne 2004; Gibbons 1992). In the following of this section, we will introduce the basic concepts in two typical classes of non-cooperative games: Strategic form game (static game) and Extensive form game (dynamic game). For more details, please refer to Osborne (2004) and Gibbons (1992).

### 2.1.1 Strategic Form Game (Static Game)

We first consider the non-cooperative *strategic form games* (also called *static game*) described in Osborne (2004) and Gibbons (1992). In such a game, all players make decisions simultaneously without knowing each other's choices. Thus, we only need to define the player set, the action set for each player, and the payoff for each player.

Formally, Osborne (2004) and Gibbons (1992) give the following definition of a strategic form game.

**Definition 2.1** (*Strategic Form Game*) A non-cooperative strategic form game is a triplet  $\langle \mathcal{I}, (\mathcal{S}_i)_{i \in \mathcal{I}}, (u_i)_{i \in \mathcal{I}} \rangle$  where

1. **Player:**  $\mathcal{I} = \{1, 2, \dots, I\}$  is a finite set of players.
2. **Strategy:**  $\mathcal{S}_i$  is a set of available actions (pure strategies) for player  $i \in \mathcal{I}$ . We further denote by  $s_i \in \mathcal{S}_i$  an action for player  $i$ , and by  $\mathbf{s}_{-i} = (s_j, \forall j \neq i)$  a vector of actions for all players except  $i$ . We let  $\mathbb{S} \triangleq \prod_i \mathcal{S}_i$  denote the set of all action profiles. We further denote  $\mathbb{S}_{-i} \triangleq \prod_{j \neq i} \mathcal{S}_j$  as the set of action profiles for all players except  $i$ . With a slight abuse of notation we use  $\mathbf{s} = (s_i, \mathbf{s}_{-i}) \in \mathbb{S}$  to denote the action profile, where player  $i$  selects action  $s_i \in \mathcal{S}_i$  and the other players choose actions  $\mathbf{s}_{-i} \in \mathbb{S}_{-i}$ .
3. **Payoff:**  $u_i : \mathbb{S} \rightarrow \mathbb{R}$  is the payoff function of player  $i$ , which maps every possible action profile in  $\mathbb{S}$  to a real number.

The Nash equilibrium refers to such a stable game outcome, where no player is willing to change his strategy unilaterally, that is, no player can increase his payoff by changing his strategy solely (see Osborne 2004; Gibbons 1992).<sup>1</sup> Formally,

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<sup>1</sup>Here we discuss the pure-strategy Nash equilibrium, where each player chooses a particular strategy from his strategy set. In the more general sense, a player can also randomly chooses an action from

**Definition 2.2** (*Nash Equilibrium*) A Nash equilibrium of a strategic form game  $\langle \mathcal{I}, (\mathcal{S}_i)_{i \in \mathcal{I}}, (u_i)_{i \in \mathcal{I}} \rangle$  is a strategy profile  $s^* \in \mathbb{S}$  such that for each player  $i \in \mathcal{I}$  the following condition holds

$$u_i(s_i^*, s_{-i}^*) \geq u_i(s_i', s_{-i}^*), \quad \forall s_i' \in \mathcal{S}_i.$$

One important concept in game theory is the *best response correspondence*, which refers to the best strategy for a particular player under a particular strategy profile of all other players.

**Definition 2.3** (*Best Response Correspondence*) For each player  $i$ , the best response correspondence  $B_i(s_{-i}) : \mathbb{S}_{-i} \rightarrow \mathcal{S}_i$  is a mapping from the set  $\mathbb{S}_{-i}$  into  $\mathcal{S}_i$  such that

$$B_i(s_{-i}) = \{s_i \in \mathcal{S}_i \mid u_i(s_i, s_{-i}) \geq u_i(s_i', s_{-i}), \forall s_i' \in \mathcal{S}_i\}.$$

A best response correspondence may contain more than one element. Using best response correspondence, we can restate Nash equilibrium in terms of best-response correspondences:

**Definition 2.4** (*Nash Equilibrium-Restated*) A strategy profile  $s^* \in \mathbb{S}$  is a Nash equilibrium of a strategic form game  $\langle \mathcal{I}, (\mathcal{S}_i)_{i \in \mathcal{I}}, (u_i)_{i \in \mathcal{I}} \rangle$  iff

$$s_i^* \in B_i(s_{-i}^*), \quad \forall i \in \mathcal{I},$$

where  $B_i(\cdot)$  is the best response correspondence defined in Definition 2.3.

### 2.1.2 Extensive Form Game (Dynamic Game)

We now consider the non-cooperative *extensive form games* (also called *dynamic game*), where players engage in sequential decision making (see Osborne 2004; Gibbons 1992). A typical example of extensive form game is the Stackelberg game (see He et al. 2007) widely-used in economics literature, where the “leader” players make decisions first, and then the “follower” players make decisions accordingly. Here we focus on the extensive form games with observed actions, where:

1. All previous actions (called history) are observed, i.e., each player is perfectly informed of all previous events.
2. Some players may move simultaneously within the same stage.

Formally, Osborne (2004) and Gibbons (1992) give the definition of an extensive form game as follows:

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(Footnote 1 continued)

several possible actions from the strategy set, with clearly defined probability for each such action. This will lead to a mixed strategy, and the corresponding equilibrium will be a mixed strategy equilibrium.

**Definition 2.5** (*Extensive Form Game*) An extensive form game consists of four main elements:

1. **Player:** A set of players  $\mathcal{I} = \{1, 2, \dots, I\}$ .
2. **History:** A set  $\mathcal{H}$  of sequences which can be finite or infinite, defined by

$$\begin{cases} h^0 = \emptyset & \text{initial history as an empty set} \\ h^1 = (s^0) & \text{history after stage 0} \\ h^2 = (s^0, s^1) & \text{history after stage 1} \\ \dots & \dots \\ h^{k+1} = (s^0, \dots, s^k) & \text{history after stage } k \end{cases}$$

where  $s^t = (s_i^t, \forall i \in \mathcal{I})$  is the action profile at stage  $t$ .

If the game has a finite  $K + 1$  of stages (i.e., from stage 0 to stage  $K$ ), then it is a finite horizon game. Let  $\mathcal{H}^k = \{h^k\}$  be the set of all possible histories after stage  $k - 1$  (i.e., at stage  $k$ ). Then  $\mathcal{H}^{K+1} = \{h^{K+1}\}$  is the set of all possible terminal histories (after stage  $K$ ), and  $\mathcal{H} = \bigcup_{k=0}^{K+1} \mathcal{H}^k$  is the set of all possible histories.

3. **Strategy:** Each strategy for player  $i$  is defined as a contingency plan for every possible history. Let  $S_i(h^k)$  be the set of actions available to player  $i$  under history  $h^k$ , and  $S_i(\mathcal{H}^k) = \bigcup_{h^k \in \mathcal{H}^k} S_i(h^k)$  be the set of actions available to player  $i$  under all possible histories at stage  $k$ . Let  $a_i^k : \mathcal{H}^k \rightarrow S_i(\mathcal{H}^k)$  be a mapping from  $\mathcal{H}^k$  to  $S_i(\mathcal{H}^k)$  such that  $a_i^k(h^k) \in S_i(h^k)$ . Then a strategy of player  $i$  is a sequence  $s_i = \{a_i^k\}_{k=0}^K$ . The collection of all such sequences  $s_i$  form the set of strategies available to player  $i$ . A strategy profile  $s$  includes the path  $s^0 = a^0(h^0)$ ,  $s^1 = a^1(s^0)$ ,  $s^2 = a^2(s^0, s^1)$ , and so on, where  $a^k(\cdot) = (a_i^k(\cdot), \forall i \in \mathcal{I})$ .
4. **Payoff:** Preferences are defined on the outcome of the game  $\mathcal{H}^{K+1}$  (after stage  $K$ ). We can represent the preferences of player  $i$  by a payoff function  $u_i : \mathcal{H}^{K+1} \rightarrow \mathbb{R}$ . As the strategy profile  $s$  determines the path  $(s^0, \dots, s^K)$ , and hence  $h^{K+1}$ , we will denote the payoff to player  $i$  under strategy profile  $s$  as  $u_i(s)$ .

In an extensive form game, *Subgame perfect equilibrium* (SPE) is a stronger concept (a refinement) than Nash equilibrium, which requires the strategy of each player to be optimal not only at the start of the game but also after every history (see Osborne 2004; Gibbons 1992). Before defining SPE, we first define the subgame.

**Definition 2.6** (*Subgame*) Let  $h^k$  denote a history at stage  $k$ . Then, a *subgame* from history  $h^k$ , denoted by  $G(h^k)$ , can be represented as:

- **Player:** All the players making decisions at and after stage  $k$ ;
- **History:**  $h^{K+1} = (h^k, s^k, \dots, s^K)$ .
- **Strategy:**  $s_i|_{h^k}$  is the restriction of  $s_i$  to histories in  $G(h^k)$ .
- **Payoff:**  $u_i(s_i, s_{-i}|h^k)$  is the payoff of player  $i$  after histories in  $G(h^k)$ .

Accordingly, SPE of extensive form game can be defined as follows (see Osborne 2004; Gibbons 1992).

**Definition 2.7** (*Subgame Perfect Equilibrium—SPE*) A strategy profile  $s^*$  is an SPE for an extensive form game if for every history  $h^k$ , the restriction  $s^*_{i|h^k}$  is an Nash equilibrium of the subgame  $G(h^k)$ .

Take a two-stage Stackelberg game given in He et al. (2007) as an example. Let  $\mathcal{I}_L$  denote the set of leaders (who make decisions in Stage I), and  $\mathcal{I}_F$  denote the set of followers (who make decisions in Stage II). Let  $x_i$  denote the strategy of leader  $i \in \mathcal{I}_L$ , and  $x$  denote the strategy profile of all leaders. Let  $z_i$  denote the strategy of follower  $i \in \mathcal{I}_F$ , and  $z$  denote the strategy profile of all followers. Then, an SPE of a Stackelberg game can be characterized as follows (see He et al. 2007).

**Lemma 2.1** (SPE of Stackelberg Game) A strategy profile  $(x^*, z^*) \in \mathbb{S}$  is an SPE of a Stackelberg game  $\langle \mathcal{I} = \mathcal{I}_L \cup \mathcal{I}_F, (\mathcal{S}_i)_{i \in \mathcal{I}}, (u_i)_{i \in \mathcal{I}} \rangle$  iff

$$\text{Stage I: } u_i(x^*, z^*(x^*)) \geq u_i(x'_i, x^*_{-i}, z^*(x'_i, x^*_{-i})), \quad \forall x'_i \in \mathcal{S}_i, i \in \mathcal{I}_L,$$

$$\text{Stage II: } u_i(z^*_i, z^*_{-i}, x^*) \geq u_i(z'_i, z^*_{-i}, x^*), \quad \forall z'_i \in \mathcal{S}_i, i \in \mathcal{I}_F.$$

The differences between the leaders' strategies in Stage I and the followers' strategies in Stage II are as follows. In Stage I, leaders choose the best strategies  $x^*$ , taking the followers' best responses  $z^*(x^*)$  into consideration. Namely,  $z^*(x)$  are functions of the leaders' strategy choices  $x$ . In Stage II, followers choose the best strategies  $z^*$ , treating the leaders' best strategies  $x^*$  as given parameters. Namely,  $x^*$  is a given history at Stage II. Therefore, for a Stackelberg game, we can solve the SPE effectively by backward induction (see He et al. 2007).

## 2.2 Super-Modular Game Theory

Super-modular game usually refers to a non-cooperative strategic form (static) game where each player's payoff is a super-modular function (see Chambers and Echenique 2009). Using the nice properties of super-modular functions (see Topkis 1998), we can easily identify the existence and uniqueness of Nash equilibrium in a super-modular game. We will use the super-modular game to model and analyze the price competition among white space access devices (secondary operators) in the spectrum trading market (Sect. 3.2).

Formally, a super-modular function is defined as follows (see Topkis 1998).

**Definition 2.8** (*Super-Modular Function*) A function  $f(x)$  is super-modular if

$$f(x \uparrow y) + f(x \downarrow y) \geq f(x) + f(y),$$

for all  $x, y \in \mathbb{X}$ , where  $x \uparrow y$  denotes the componentwise maximum and  $x \downarrow y$  the componentwise minimum of  $x$  and  $y$ .

It is easy to show that if  $f(\cdot)$  is twice continuously differentiable, then super-modularity is equivalent to the condition (see Topkis 1998):

$$\frac{\partial^2 f(x)}{\partial x_i \partial x_j} \geq 0, \quad \text{for all } i \neq j.$$

Intuitively, super-modularity reflects the property of *increasing difference* of function  $f(\cdot)$ . Namely, for any  $x'_i \geq x_i$  and  $x'_j \geq x_j$ ,<sup>2</sup>

$$f(x'_i, x'_j) - f(x_i, x'_j) \geq f(x'_i, x_j) - f(x_i, x_j).$$

For a super-modular function, we have the following Topkis' Theorem (see Topkis 1998):

**Theorem 2.1** (Topkis' Theorem) *Suppose  $f(x)$  is a super-modular function. Let  $x_i^*(x_{-i}) \triangleq \arg \max_{x_i} f(x_i, x_{-i})$  denote the set of  $x_i$  that maximizes the function  $f(\cdot)$  under  $x_{-i}$ . Then*

- $x_i^*(x_{-i})$  is nonempty, and has a greatest and least element, denoted by  $\bar{x}_i^*(x_{-i})$  and  $\underline{x}_i^*(x_{-i})$ , respectively;
- $\bar{x}_i^*(x_{-i}) \geq \bar{x}_i^*(x'_{-i})$  and  $\underline{x}_i^*(x_{-i}) \geq \underline{x}_i^*(x'_{-i})$ , for any  $x_{-i} \geq x'_{-i}$ .

From the Topkis' theorem, we can easily find that in a super-modular game, each player's best response correspondence is increasing in the actions of other players. Formally,

**Lemma 2.2** *Suppose  $\langle \mathcal{I}, (\mathcal{S}_i)_{i \in \mathcal{I}}, (u_i)_{i \in \mathcal{I}} \rangle$  is a super-modular game. Let  $B_i(s_{-i})$  denote the best response correspondence of player  $i$  under other players' strategy profile  $s_{-i}$ . Then,*

- $B_i(s_{-i})$  has a largest element  $\bar{B}_i(s_{-i})$  and a smallest element  $\underline{B}_i(s_{-i})$ ;
- $\bar{B}_i(s_{-i}) \geq \bar{B}_i(s'_{-i})$  and  $\underline{B}_i(s_{-i}) \geq \underline{B}_i(s'_{-i})$ , for any  $s_{-i} \geq s'_{-i}$ .

Based on the above lemma, we can further derive the following nice properties for a super-modular game (see Chambers and Echenique 2009).

**Theorem 2.2** *Suppose  $\langle \mathcal{I}, (\mathcal{S}_i)_{i \in \mathcal{I}}, (u_i)_{i \in \mathcal{I}} \rangle$  is a super-modular game. Then,*

1. *There exists at least one Nash equilibrium;*
2. *When multiple Nash equilibriums exist, there is a componentwise largest (or smallest) Nash equilibrium where each player's strategy is larger (or smaller) than those in all other equilibriums;*
3. *When a unique Nash equilibriums exists, it is dominance solvable (and lots of adjustment rules converge to it, e.g., the best response dynamics).*

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<sup>2</sup>Here omit the parameters  $x_{-i,j}$  in function  $f(\cdot)$  for writing convenience.

These nice properties of a super-modular game can greatly facilitate the game-theoretical analysis of a practical problem. In particular, if we can formulate a practical problem into a super-modular game, we immediately get the existence and sometimes the uniqueness of the game equilibrium. Later in Sect. 3.2, we will show that the price competition among white space access devices (secondary operators) is indeed a super-modular game.

## 2.3 Evolutionary Game Theory

Evolutionary game theory is a theory rooted in Biology (see Smith and Price 1973). Evolutionary game theory considers a population players, wherein the payoff of a particular player is related to the frequency with which the possible decision is made in the whole population (rather than the decision of every other player as in a classic game), and such a frequency can evolve with time (i.e., it can be time varying). In this sense, an evolutionary game is usually a non-cooperative extensive form game with an infinite number of players (population) and an infinite number of stages (evolution). We will use the evolutionary game to model and analyze the evolution of white space devices' behaviors in the information trading market (Chap. 4).

Comparing with the classic games mentioned above, an evolutionary game has the following novel features (see Vincent and Brown 2005):

- **Player:** An infinite population of individuals;
- **Strategy:** Each player can choose a pure strategy  $s$  from a set of strategies  $\mathcal{S}$ , or a mixed strategy  $\sigma$  over  $\mathcal{S}$  (i.e., an assignment of probability to choose each strategy in  $\mathcal{S}$ );
- **Payoff:** Each player's payoff depends on the frequency  $x(s)$  with which a particular strategy  $s \in \mathcal{S}$  is played in the population.

The third feature implies that the decision/payoff of a player is related to how the whole population makes decisions, but not the decision of every particular (other) player. This is critical for an evolutionary game, as there is an infinite population of players (hence it is difficult to characterize the best response to other players' choices). Furthermore, it implies that all the players are symmetric in the game, in the sense that exchanging the strategies of multiple players will not change the payoffs of other players.

There are several important problems for an evolutionary game: What is the stable game outcome (equilibrium)? Will the population move away from the equilibrium (stability of equilibrium)? Before talking about these problems, we first introduce *population profile*, one of the most important concepts in evolutionary game.

**Definition 2.9** (*Population Profile*) A population profile is a vector that gives a frequency or probability  $x(s)$  with which each strategy  $s \in \mathcal{S}$  is played in the population, denoted by  $x \triangleq \{x(s), s \in \mathcal{S}\}$ .

Note that the population profile needs not correspond to a strategy adopted by any player of the population. Consider the following example. Suppose a population can choose strategy from  $\mathcal{S} = \{s_1, s_2\}$ . If every player of the population chooses the mixed strategy  $\sigma = \{0.5, 0.5\}$ , then we have:  $x = \{0.5, 0.5\}$ . In this case, the population profile  $x$  is identical to the mixed strategy adopted by all players. In another case, if half of the population adopt the strategy  $s_1$  and other half adopt strategy  $s_2$ , we also have:  $x = \{0.5, 0.5\}$ , which is *not* the same as the strategy adopted by any player of the population.

As mentioned above, each player's payoff depends on the population profile  $x$ . Let  $u(s, x)$  denote the payoff of a player when choosing a particular strategy  $s$  under a population profile  $x$ . Then, the expected payoff of a player when choosing any mixed strategy  $\sigma$  (a pure strategy can be viewed as a special case of mixed strategy) can be defined as follows.

**Definition 2.10** (*Payoff*) Given any population profile  $x$ , the expected payoff of a player when choosing any mixed strategy  $\sigma = \{\sigma_s, s \in \mathcal{S}\}$  is:

$$u(\sigma, x) = \sum_{s \in \mathcal{S}} \sigma_s \cdot u(s, x),$$

where  $\sigma_s$  is the probability of choosing strategy  $s \in \mathcal{S}$ .

We first consider the stable game outcome (equilibrium). Intuitively, a stable game outcome or equilibrium of a population game is the “terminal state” of the evolution of the population (also called stable population or evolutionary stability). Obviously, at the equilibrium, the strategy adopted by each player must be the best response to the population profile (see Vincent and Brown 2005). Formally,

**Definition 2.11** (*Equilibrium*) Let  $\sigma^*$  denote a strategy that generates a population profile  $x^*$ . Then,  $\sigma^*$  leads to an equilibrium if

$$\sigma^* = \arg \max_{\sigma} u(\sigma, x^*).$$

Now we consider the stability of an equilibrium, i.e., the robustness of an equilibrium to a small change on population profile. Intuitively, an equilibrium is stable implies that with a small change on population profile, the population will evolve back to the equilibrium. A mixed strategy that leads to a stable equilibrium is called an *Evolutionary Stable Strategy (ESS)* (see Vincent and Brown 2005). Before providing the formal definition of ESS, we first introduce the concept of Post-Entry Population. Formally,

**Definition 2.12** (*Post-Entry Population*) Consider a population where players adopt strategy  $\sigma^*$ . Suppose a mutation occurs and a small proportion  $\epsilon$  of players adopt some other strategy  $\sigma$ . The new population profile is called the post-entry population and will be denoted as  $x_\epsilon$ .

**Definition 2.13** (*Evolutionary Stable Strategy—ESS*) A mixed strategy  $\sigma^*$  is an evolutionary stable strategy (ESS) if there exists an  $\bar{\epsilon}$  such that for every  $0 < \epsilon < \bar{\epsilon}$  and every  $\sigma \neq \sigma^*$ ,

$$u(\sigma^*, x_\epsilon) > u(\sigma, x_\epsilon),$$

where  $x_\epsilon$  is the post-entry population profile when a small proportion  $\epsilon$  of players mutate by choosing other strategy  $\sigma$ .

Intuitively, a strategy is an ESS if mutants that adopt any other strategy will achieve a worse payoff in the post-entry population, provided that the proportion of mutants is sufficiently small. Later in Chap. 4, we will show how the white space devices' purchasing behaviors in the information trading market gradually evolve to an ESS.

## 2.4 Contract Theory

A contract is an agreement entered into voluntarily by two or more parties with the intention of creating a legal obligation. In economics, contract theory studies how the economic agents construct contractual arrangements, generally in the presence of *asymmetric information* (see Bolton and Dewatripont 2005). Thus, it is closely connected to the truthful (or incentive compatible) mechanism design. Several well known contract models include screening (see Stiglitz 1984), signalling (see Spence 1973), and moral hazard (see Rogerson 1985). The common spirit of these models is to motivate one party (the agent) to act in the best interests of another (the principal) under information asymmetry. In this section, we mainly focus on the **screening contract model** (see Stiglitz 1984), which will be used in our spectrum trading market design. Readers can refer to Bolton and Dewatripont (2005) for more details regarding other contract models. In particular, we will use the screening contract model to formulate and optimize the white space database's spectrum reservation decision in the spectrum trading market (Sect. 3.3).

In a screening model, a principle (player) interacts with multiple agents (players) under information asymmetry, where the principle cannot observe the complete information of agents. The idea is that the principle offers multiple contract options, which are incentive compatible for agents such that every agent is willing to select the option intended for his type (hence reveal his private information) (see Bolton and Dewatripont 2005; Stiglitz 1984). In this sense, a screen model can be viewed as a two-stage extensive form game, where the principle is the leader (whose strategy is to offer the contract options), and the agents are the followers (whose strategies are to choose the best contract option).

To facilitate the understanding, we consider an example where a principle sells certain service to agents under information asymmetry. Let  $\theta \in \Theta$  denote the personal preference of an agent to the principle's service, where  $\Theta$  is the set of all possible agent preferences. That is, a higher  $\theta$  reflects that an agent desires more for the service. For convenience, we refer to  $\theta$  as the "type" of agent. Obviously, the type of

an agent is his *private information*, and cannot be directly observed by others. For convenience, we denote  $u_\theta(q, p)$  as the payoff of an agent with type  $\theta$  for a service with quality  $q$  and price  $p$ .

We first note that if the principle can observe the type  $\theta$  of each agent, he can determine the optimal quality  $q$  and price  $p$  (that maximizes his own payoff) of the service to every agent directly. In the practical scenario, however, the principle *cannot* observe the type  $\theta$  of each agent, and hence cannot determine the optimal quality  $q$  and price  $p$  to every agent directly. Instead, the principle will offers multiple contract options to agents, each intended for one type of agent and consisting of a quality and a price. We refer to such a set of contract options that the principle offers to agents as a *contract*, denoted by

$$\Phi = \{(q_\theta, p_\theta), \forall \theta \in \Theta\},$$

where  $(q_\theta, p_\theta)$  is the contract option intended for agents with type  $\theta$ . It is important to note that this does not mean that an agent with type  $\theta$  will naturally choose the contract option  $(q_\theta, p_\theta)$ . In fact, he is willing to choose the contract option  $(q_\theta, p_\theta)$  only when achieving a higher (and non-negative) payoff under  $(q_\theta, p_\theta)$  than under any other contract options. This is one of the important missions in contract design.

Formally, to ensure that each type- $\theta$  agent will choose the contract option  $(q_\theta, p_\theta)$  intended for his type, we need to guarantee that each type- $\theta$  agent can achieve a higher payoff under  $(q_\theta, p_\theta)$  than under any other contract options. This condition is usually referred to as Incentive Compatibility (IC) (see Bolton and Dewatripont 2005; Stiglitz 1984).

**Definition 2.14** (*Incentive Compatibility—IC*) A contract  $\Phi = \{(q_\theta, p_\theta), \forall \theta\}$  is incentive compatible, if for every type- $\theta$  agents,

$$u_\theta(q_\theta, p_\theta) \geq u_\theta(q_{\theta'}, p_{\theta'}), \quad \forall \theta' \neq \theta.$$

Furthermore, to ensure that each type- $\theta$  agent will choose the contract option  $(q_\theta, p_\theta)$  intended for his type, we further need to guarantee that each type- $\theta$  agent can achieve a non-negative payoff under  $(q_\theta, p_\theta)$ , otherwise the agent will not join the contract. This condition is usually referred to as Individual Rationality (IR) (see Bolton and Dewatripont 2005; Stiglitz 1984).

**Definition 2.15** (*Individual Rationality—IR*) A contract  $\Phi = \{(q_\theta, p_\theta), \forall \theta\}$  is individually rational, if for every type- $\theta$  agents,

$$u_\theta(q_\theta, p_\theta) \geq 0.$$

A contract is called *feasible*, if it is incentive compatible and individually rational. Obviously, with a feasible contract, each agent is willing to choose the contract option intended for his type. Hence, the principle's payoff (revenue) under a feasible contract  $\Phi = \{(q_\theta, p_\theta), \forall \theta\}$  can be computed as:

$$v(\Phi) = \int (p_\theta - c(q_\theta)) \cdot f(\theta) d\theta,$$

where  $f(\theta)$  is the probability distribution function of agent type  $\theta$ , and  $c(q_\theta)$  is the principle's cost of offering a service with quality  $q_\theta$ .

The principle's objective is to design the optimal contract that maximizes his own payoff. By the revelation principle (see Epstein and Peters 1999), for any arbitrary contract (mechanism), we can always find an incentive compatible and individually rational contract (mechanism) that generates the same outcome. This implies that the optimal contract within feasible contracts is equivalent to the globally optimal contract. Formally,

**Theorem 2.3** (Optimal Contract) *A contract  $\Phi^* = \{(q_\theta^*, p_\theta^*), \forall \theta\}$  is optimal, if it solves the following optimization problem:*

$$\begin{aligned} & \max_{\Phi} v(\Phi) \\ & \text{subject to (IC) and (IR)} \end{aligned}$$

The above theorem implies that to find the optimal contract (that maximizes the principle's payoff), the principle can first characterize the feasible contracts, and then find the optimal contract within the feasible contracts. By the revelation principle (see Epstein and Peters 1999), such an optimal feasible contract is equivalent to the globally optimal contract. Later in Sect. 3.3, we will formulate a spectrum reservation (screening) contract for the white space database to optimally reserve spectrum for the spectrum trading market, without knowing the complete demand information of white space devices.

## 2.5 Nash Bargaining Theory

Bargaining problems usually represent situations in which (i) multiple players with specific objectives search for a mutually agreed outcome (agreement), (ii) no agreement may be imposed on any player without his approval, i.e., the disagreement is possible, (iii) players have the possibility of reaching a mutually beneficial agreement, and (iv) there is a conflict of interests among players about agreements (see Muthoo 1999). Bargaining theory is a theory of understanding whether and how players should reach an agreement through proper negotiations. Solutions to bargaining come in two flavors: *Axiomatic approach* (see Nash 1950) and *Strategic approach* (see Rubinstein 1982). Specifically,

- **Strategic approach** (see Nash 1950) models the bargaining procedure explicitly as a sequential game (extensive form game), and considers the game outcome (equilibrium) that results from the players' strategic interactions;

- **Axiomatic approach** (see Rubinstein 1982) abstracts away the details of the process of bargaining, and considers only the set of outcomes or agreements that satisfy “reasonable” properties (axioms).

In this section, we mainly focus on **Nash bargaining model** (see Nash 1950), a typical bargaining problem with the axiomatic approach. Readers can refer to Muthoo (1999) for details regarding other bargaining models. In particular, we will use the Nash bargaining model to analyze the commission negotiation between the white space database and the spectrum licensee in the hybrid spectrum and information market (Chap. 5).

As a bargaining problem with the axiomatic approach, the first question to Nash bargaining model is: *what are the reasonable axioms?* Nash (1950) provided 4 axioms for the outcome of the negotiation among 2 players: (i) Pareto Efficiency, (ii) Symmetry, (iii) Invariant to Affine Transformations (or Invariance to Equivalent Payoff Representations), and (iv) Independence of Irrelevant Alternatives. A Nash Bargaining Solution (NBS) is defined as an outcome that satisfies the above 4 axioms.

To facilitate the understanding, we consider a general 2-person Nash bargaining model as an illustrative example, where

- **Player:** The set of bargaining players  $\mathcal{I} = \{1, 2\}$ ;
- **Agreement:** The feasible set  $\mathcal{U} = \{(u_1, u_2) \in \text{bounded convex set}\}$ , where  $u_i$  is the payoff of player  $i$  in an agreement;
- **Disagreement:** The outcome of disagreement  $D = (d_1, d_2)$ , where  $d_i$  is the payoff of player  $i$  in the disagreement;

For convenience, we denote  $(u_1^*, u_2^*) \in \mathcal{U} \cup \{D\}$  as an NBS, i.e., an outcome that satisfies all of the above Nash’s 4 axioms. Before deriving the NBS  $(u_1^*, u_2^*)$ , we first characterize the above axioms formally (see Nash 1950).

**Definition 2.16** (*Pareto Efficiency*) None of the players can be made better off without making at least one player worse off.

**Definition 2.17** (*Symmetry*) If two players are indistinguishable, the solution should not discriminate between them.

**Definition 2.18** (*Invariant to Affine Transformations*) An affine transformation of the payoff functions and disagreement points should not alter the outcome of the bargaining process.

**Definition 2.19** (*Independence of Irrelevant Alternatives*) If the solution  $(u_1^*, u_2^*)$  chosen from a feasible agreement set  $\mathcal{U}$  is an element of a subset  $\mathcal{A} \subseteq \mathcal{U}$ , then  $(u_1^*, u_2^*)$  must be chosen from the agreement set  $\mathcal{A}$ .

Nash (1950) has shown that there exists a unique NBS, i.e., a unique outcome  $(u_1^*, u_2^*) \in \mathcal{U} \cup \{D\}$  that satisfies all of the above Nash’s 4 axioms. More specifically, the unique NBS  $(u_1^*, u_2^*)$  can be solved in a very simply way as shown in the following theorem.

**Theorem 2.4** (Nash Bargaining Solution—NBS) *There is a unique outcome  $(u_1^*, u_2^*) \in \mathcal{U} \cup \{D\}$  that satisfies all Nash’s 4 axioms. Such a unique outcome  $(u_1^*, u_2^*)$  is called Nash Bargaining Solution and is given by:*

$$\begin{aligned} &\max_{u_1, u_2} (u_1 - d_1) \cdot (u_2 - d_2) \\ &s.t. \ u_1 \geq d_1, u_2 \geq d_2, (u_1, u_2) \in \mathcal{U}. \end{aligned}$$

From the above theorem, we can observe that an NBS depends greatly on the disagreement point  $D$ . Intuitively, when increasing the disagreement point of a player, he can achieve a higher payoff under the NBS. Later in Chap. 5, we will formulate the commission negotiation between the white space database and the spectrum licensee in the hybrid market as a Nash bargaining model, and solve the NBS systematically.

2.6 Summary

In this chapter, we introduce several methodologies widely-used in economics, including the non-cooperative game theory, super-modular game theory, evolutionary game theory, contract theory, and Nash bargaining theory. Figure 2.1 illustrates the relationship among the various methodologies and their applications in this book.

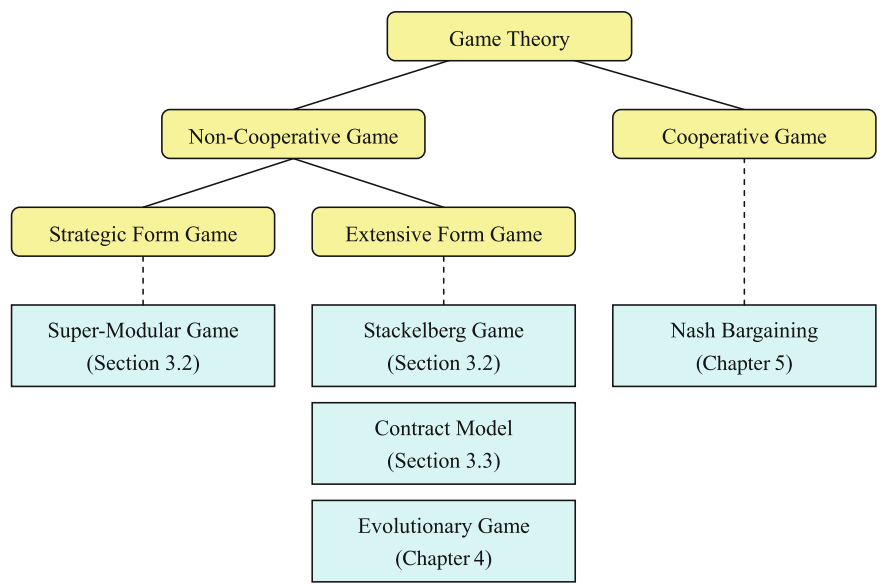


Fig. 2.1 Summary of this chapter

Specifically, the non-cooperative Stackelberg game is used to analyze the strategic interaction between white space end-users and white space access devices (secondary operators) in the spectrum trading market (Sect. 3.2). The super-modular game is used to model and analyze the price competition among white space access devices in the spectrum trading market (Sect. 3.2). The contract theory is used to optimize the white space database's spectrum reservation decision under information asymmetry in the spectrum trading market (Sect. 3.3). The evolutionary game is used to model and analyze the evolution of white space devices' behaviors in the information trading market (Chap. 4). The Nash bargaining is used to analyze the commission negotiation between the white space database and the spectrum licensee in the hybrid market (Chap. 5).

It is notable that the purpose of this chapter is to facilitate readers to understand a limited number of microeconomics tools used in our later analysis. For more in-depth discussions regarding general microeconomics theory especially game theory, we refer the readers to Osborne (2004), Gibbons (1992), He et al. (2007), Chambers and Echenique (2009), Vincent and Brown (2005), Bolton and Dewatripont (2005), Stiglitz (1984), Muthoo (1999) and Nash (1950).

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