

Chapter 2

Multiple Higgs and Vector Boson Production

2.1 Overview

The discovery of the Higgs in 2012 [1, 2] means that we now have, in some sense, experimental confirmation for the mechanism responsible for electroweak symmetry breaking (EWSB). Data strongly favour a spin 0, CP even particle [3–5], and confirms that its coupling to other particles is related to their mass in the way predicted by the standard model Higgs mechanism, earning the particle the name *the* Higgs boson (as opposed to initial discussions of a “Higgs like particle”). However, despite this success, its couplings are still imprecisely measured. Possible deviations from SM values of Higgs coupling to vector bosons can be as large as 15–40 % and for fermions as large as 30–100 % depending on assumptions and particle type, whilst still being consistent with LHC data¹ [6–8]. As many BSM models predict only small (0–30 %) deviations from SM couplings (when constraints from outside the Higgs sector have been taken into account), these have not been ruled out. In particular, composite Higgs models are models where EWSB is caused by new strong dynamics with the Higgs arising as a pseudo-Goldstone boson (analogous to the pion in QCD). When these models are required to satisfy electroweak precision measurements, then they generally predict deviations from SM couplings $\lesssim 10\%$, and hence are also in agreement with current data.² In fact the Higgs arising in Technicolor models can be exactly SM-like in terms of their couplings to weak bosons, despite their compositeness [9].

To differentiate between the standard model Higgs, and one which arises due to strong dynamics, we note that it has been shown that a hallmark of strong interactions

¹With such large deviations still possible, you may ask why is this hailed as such a success for the standard model. The reason is that the masses and related couplings vary over many orders of magnitude between particles, and in this context a $\sim 30\%$ deviation is very small.

²Although it should be noted that these models are strongly tuned and do not solve the gauge hierarchy problem.

in the EWSB sector is multiple particle production at energies around the EWSB scale [10]. Therefore in a strongly coupled EWSB sector, one would expect copious production of longitudinal gauge bosons as long as enough energy is available to produce them. This is similar to the way large numbers of pions are produced in QCD at high energies. In fact, multi-W production was studied in a simplified scaled-up version of QCD over 20 years ago [11].

Here we study the inelastic production of longitudinally polarised W and Z bosons (denoted collectively by V_L) and Higgs bosons using a non-linear effective Lagrangian, where couplings can differ from their standard model values. As discussed in [12], the scale of new physics is likely to be where inelastic scattering becomes important. This is what occurs in QCD where multiple pion production indicates the scale at which quarks become important individual degrees of freedom. Analogously we are able to estimate the energy scale of new physics for different coupling values in our effective Lagrangian by calculating the energy at which multiple vector boson or Higgs boson processes become relevant.

As will be discussed in Sects. 2.2 and 2.4, the cross section of such multiparticle production should be more sensitive to non-SM couplings than simple $2 \rightarrow 2$ processes. In particular, we're interested in how sensitive these multiparticle production cross section are to deviations from the SM couplings.

We first study as a simple case, unitarity violation in multi- V_L production in the Higgsless model [13], before considering models with partial unitarisation, such as the composite Higgs model. Even with partial unitarisation we show that provided enough energy is available to produce the particles, large enhancements of multiparticle cross section can occur. This effect becomes more acute as the final state multiplicity increases.

2.2 Multiparticle Cross Sections and Unitarity

In an inelastic $2 \rightarrow n$ process, if we assume s -wave dominance, the perturbative unitarity bound on the cross section for a given centre-of-mass energy $\sqrt{s}$ is [13, 14]:

$$\sigma(2 \rightarrow n) < \frac{4\pi}{s}. \quad (2.1)$$

(The derivation is reproduced in Appendix A for convenience.)

This bound subsequently sets stringent constraints on the scattering amplitudes. The relativistic n -body phase space is proportional to s^{n-2} and therefore, taking into account the flux, the unitarity bound requires that the amplitude grows with energy no faster than

$$A(2 \rightarrow n) \sim s^{1-n/2}. \quad (2.2)$$

We can use this result to easily calculate whether scattering in a model will violate the unitarity bound unless there are precise cancellations between amplitudes. As an

example, if we neglect transverse gauge bosons and the Higgs boson, we can describe vector boson scattering with a simple nonlinear sigma model (NL σ M):

$$\mathcal{L}_{NL\sigma M} = \frac{v^2}{4} \text{Tr} [\partial_\mu U \partial^\mu U^\dagger] \quad (2.3)$$

where $v = 246$ GeV is the usual scale of electroweak symmetry breaking and

$$U = e^{\frac{i\vec{r}\cdot\vec{\pi}}{v}}. \quad (2.4)$$

These scalar “pion” fields π^i ($i = 1, 2, 3$) describe massless Goldstone bosons in a Higgsless model. The equivalence theorem [10] shows that in the high energy limit the scattering cross sections of longitudinal vector bosons asymptotes to that of their respective massless Goldstone bosons, allowing these π^i fields to be identified with the longitudinally polarised vector bosons.

Using power-counting, we see that the scattering amplitude in this model grows with energy as

$$A_{NL\sigma M}(2 \rightarrow n) \sim \frac{s}{v^n} \quad (2.5)$$

and hence naively

$$\sigma(2 \rightarrow n) \sim \frac{1}{s} \left(\frac{s}{v^n} \right)^2 s^{n-2}. \quad (2.6)$$

Therefore, we see that the growth of the cross section towards the unitarity bound in this model is faster for larger number of particles, due to the s^{n-2} factor from phase space.

In turn, this means that if we assume that unitarity is restored by new physics, then there must be larger cancellations between scattering amplitudes as the number of final state particles is increased. For example, Eq. 2.2 tells us that unitarity requires that $A(2 \rightarrow 2) \sim \text{constant}$, while $A(2 \rightarrow 4) \sim 1/s$, whereas they both grow as $\sim s$ in the NL σ M (2.5). Therefore, in the absence of a perfect cancellation between amplitudes, cross section will scale with energy more rapidly for multi- V_L production compared to $2 \rightarrow 2$ scattering, and this is likely to have a large impact on multi- V_L production cross sections. The purpose of the work in this chapter is to examine this impact.

2.3 Naive Estimates of Unitarity Violation

The n -body phase space in the relativistic limit, (given for example by [15]) is:

$$R_n(s) = \int \prod_{i=1}^n \frac{d^3 p_i}{(2\pi)^3 (2E_i)^3} (2\pi)^4 \delta^4(\sqrt{s} - \sum_{i=1}^n p_i) = \frac{(2\pi)^{4-3n} (\pi/2)^{n-1}}{(n-1)!(n-2)!} s^{n-2}. \quad (2.7)$$

Rearranging enables us to estimate the energy scale Λ_n at which perturbative unitarity is violated in $2 \rightarrow n$ processes in the NL σ M:

$$\Lambda_n = \left[\frac{2(n-1)!(n-2)!}{(2\pi)^{3-3n}(\pi/2)^{n-1}} \right]^{\frac{1}{2n}} v. \quad (2.8)$$

Comparing different values of n , we see that the lowest limit occurs for $2 \rightarrow 2$ scattering, with unitarity being violated in $2 \rightarrow 4$ at an energy which is 2.4 times higher. In this rough estimate we do not include a proper phase space integration or the growth due to the combinatorial factors, however this estimate is in reasonable agreement with the results of a full numerical calculation given in [13].

2.4 Anomalous Higgs Couplings and Partial Unitarisation

In order to recover unitarity, the non-linear sigma model must have a UV completion. The simplest possibility is the addition of a scalar field. This makes the theory consistent with the Higgs boson discovery, and the scalar can be identified with the Higgs. Given exactly SM couplings, new Feynman diagrams involving the Higgs cause large cancellations between amplitudes, restoring unitarity at all scales. However, if the Higgs arises as a composite particle in a strong theory it may have couplings which differ from the SM values. In this case, if the high scale theory giving rise to the composite Higgs isn't considered, then the theory isn't UV complete, cancellations are incomplete and unitarity is only partially restored. Such a theory can be described by an effective Lagrangian which parameterises the Higgs self-couplings and couplings to longitudinally polarised gauge bosons [16]:

$$\begin{aligned} \mathcal{L}_{\text{eff}} = & \frac{v^2}{4} \left(1 + 2a \frac{h}{v} + b \frac{h^2}{v^2} + b_3 \frac{h^3}{v^3} + \dots \right) \text{Tr} [\partial_\mu U \partial^\mu U^\dagger] \\ & + \frac{1}{2} (\partial_\mu h)^2 - \frac{1}{2} m_h^2 h^2 - d_3 \lambda v h^3 - d_4 \frac{\lambda}{4} h^4 + \dots \end{aligned} \quad (2.9)$$

(couplings to fermions are not relevant to the results presented here).

This parameterisation describes the low energy behaviour of a large class of models, including composite Higgs models, and has been used to study anomalous Higgs couplings in $V_L V_L \rightarrow V_L V_L, hh$ processes at the LHC [16–18]. Unitarity is recovered for the SM values:

$$\begin{aligned} a &= b = d_3 = d_4 = 1 \\ b_3 &= 0 \end{aligned} \quad (2.10)$$

while for different values of these parameters the usual cancellation provided by the Higgs is incomplete. With SM coupling values, we can embed the h in the multiplet,

$$\Phi \equiv \left(1 + \frac{h}{v}\right) U \quad (2.11)$$

from which we can recover the usual linear sigma model.

Whilst in general, the parameters in the Lagrangian can be independent, many models predict relations between them. For example, in Minimal Composite Higgs Model (MCHM4),³ the couplings of the “pions” with the Higgs boson follows from an expansion around the vacuum $h(x) = 0$ of the effective Lagrangian [21]

$$\frac{f^2}{4} \sin^2 \left(\theta + \frac{h(x)}{f} \right) \text{Tr} [\partial_\mu U \partial^\mu U^\dagger] \quad (2.12)$$

with $v = f \sin \theta$, which comes from the mass term for the gauge fields. In the MCHM4 we therefore have a relation between the coupling, which can ultimately be parameterised in terms of a single variable ξ ,

$$a = \sqrt{1 - \xi}; \quad b = 1 - 2\xi; \quad b_3 = -\frac{4}{3}\xi\sqrt{1 - \xi}; \quad \dots \quad (2.13)$$

Although not obvious from the Lagrangian in Eq. 2.9, in a physically equivalent set of coordinates, this Lagrangian has a discrete symmetry under the parity transformation $h \rightarrow -h$ and $\pi \rightarrow -\pi$ [21].

In order to study the $2 \rightarrow 4$ scattering, we must expand each field U to order $\mathcal{O}(\pi^6)$:

$$\begin{aligned} \frac{v^2}{4} \text{Tr} [\partial_\mu U \partial^\mu U^\dagger] &= \frac{1}{2} (\partial^\mu \vec{\pi} \cdot \partial_\mu \vec{\pi}) + \left[1 - \frac{2}{15v^2} \vec{\pi} \cdot \vec{\pi} \right] \\ &\times \frac{1}{6v^2} \left[(\vec{\pi} \cdot \partial_\mu \vec{\pi})^2 - (\vec{\pi} \cdot \vec{\pi}) (\partial^\mu \vec{\pi} \cdot \partial_\mu \vec{\pi}) \right] + \mathcal{O}(\vec{\pi}^8) \end{aligned} \quad (2.14)$$

The number of diagrams increases considerably with the number of final state particles, making it impractical to perform an analytic computation. Therefore we implemented the Lagrangian given in Eq. 2.9 both in FormCalc [22] and MadGraph [23] using FeynRules [24] (with UFO output [25] for the higher dimensional operators) and in CalcHEP [26] using LanHEP package [27] with the help of auxiliary fields.

In the remainder of this section the $\pi^{0,+,-}$ fields will be referred to as “pions”, but it should be remembered that these are identified with the Goldstones from the Higgs field and ultimately the longitudinally polarised vector bosons, V_L .

In the simplest case of $2 \rightarrow 2$ scattering, amplitudes only depend on the Mandelstam variables s and t . For instance if we assume the pions are massless, which is valid in the high energy scenario, the $\pi^0 \pi^0 \rightarrow \pi^+ \pi^-$ amplitude arising from only 2 diagrams, one with a 4 point interaction and one with an s-channel Higgs, is given by:

³In the MCHM the fermions can be embedded in either the spinorial or fundamental representation of $SO(5)$, denoted by MCHM4 [19] and MCHM5 respectively [20].

$$\mathcal{M}_{\pi^0\pi^0;\pi^+\pi^-} = \frac{s[(1-a^2)s - m_h^2]}{v^2(s - m_h^2)} \xrightarrow{s \gg m_h^2} (1-a^2) \frac{s}{v^2}. \quad (2.15)$$

Equation 2.2 tells us that unitarity requires in this case that the amplitude to be at most constant with s at high energies. We see therefore that there is a violation of unitarity even with the presence of the Higgs boson if its coupling is not SM-like, i.e., $a \neq 1$. This also demonstrates that in the SM the amplitude is constant at high energies as required by unitarity.

The $2 \rightarrow 4$ amplitudes are far more complicated. They containing of the order of 100 diagrams and with multiple combinations of the scalar products of the different 4-momenta involved. However, we can elucidate the high energy behaviour of the cross section by focusing on a given point in phase space, with the assumption that the overall behaviour will be the same in general. In this particular case, all the particles lie in the same plane, in which case we obtain,

$$\begin{aligned} \mathcal{M}_{\pi^0\pi^0;\pi^0\pi^0\pi^+\pi^-} &\propto \frac{1}{v^4} [72s(13a^4 - a^2(7b+5) - 1) + \\ &3m_h^2(1580a^4 - 378a^3d_3 - 3a^2(245b+131) - 74) + \\ &\frac{m_h^4}{s}(9774a^4 - 3087a^3d_3 - a^2(4494b+1289) + 52) + \\ &\dots] \end{aligned} \quad (2.16)$$

Once more, we see that it's leading term grows with s , as expected from power counting. However, in the SM (where $a = b = d_3 = 1$, see Eq. 2.10) cancellations occur and the first two terms in powers of s vanish. In the $s \gg m_h^2$ limit we obtain:

$$\mathcal{M}_{\pi^0\pi^0;\pi^0\pi^0\pi^+\pi^-} \propto \frac{1}{s} \frac{m_h^4}{v^4} \quad (2.17)$$

demonstrating the behaviour change from $\sim s$ to $\sim 1/s$ required for unitarity. Equation 2.16 shows that the triple Higgs anomalous coupling parameterised by d_3 does not enter in the dominant contribution, and therefore in the following we will take $d_3 = 1$. Also note that the d_4 and b_3 couplings do not contribute to the above processes.

For $2 \rightarrow 3$ processes a similar analysis can be performed. For $\pi^0\pi^0 \rightarrow hhh$, again for a given configuration in phase space, the result is

$$\begin{aligned} \mathcal{M}_{\pi^0\pi^0;hhh} &\propto \frac{1}{4v^3} [s(-4a^3 + 4ab - 3b_3) - \\ &m_h^2(-8a^3 + 8ab + 3b_3) + \\ &\frac{4m_h^4}{s}(a^3 + ab - 6b_3 - 3a^2d_3) + \dots], \end{aligned} \quad (2.18)$$

while for $\pi^0\pi^0 \rightarrow \pi^+\pi^-h$ for a configuration where the 2 final state pions are collinear with each other but back-to-back with the Higgs boson we find

$$\begin{aligned} \mathcal{M}_{\pi^0\pi^0;\pi^+\pi^-h} \propto & \frac{a}{192v^3} \left[s(-1 + 2a^2 - b) + \right. \\ & \frac{m_h^2}{4} (-164 + 386a^2 - 213b - 9ad_3) - \\ & \left. \frac{3m_h^4}{2s} (-262 + 291a^2 - 93b + 81ad_3) + \dots \right] \end{aligned} \quad (2.19)$$

Once more we find that for the SM, the first two terms in these amplitudes vanish as required. It is also worth noticing that the $\mathcal{M}_{\pi^0\pi^0;hhhh}$ amplitude depends on b_3 , being the lowest multiplicity process which is sensitive to this coupling. In addition, if we substitute the values for the coupling in terms of ξ so that they obey the MCHM4 relations in Eq. 2.13, we find that cancellations occur and the highest power of s in the amplitude of these $2 \rightarrow 3$ processes vanishes. In fact, this can be anticipated from the parity of the MCHM4 class of theories, under which $\pi \rightarrow -\pi$ and $h \rightarrow -h$ [21].

In summary, we see that neglecting any special symmetries such as in the MCHM4, if the couplings do not have their SM values, the amplitude always grows as s regardless of the number of final state particles. However the requirement for unitarity becomes more stringent as the numbers of final state particles increases due to the increase phase space, requiring the amplitude to grow no faster than $s^{1-\frac{n}{2}}$ at high energy for a $2 \rightarrow n$ process. Therefore, as predicted for the Higgsless scenario, the $2 \rightarrow 3$ and $2 \rightarrow 4$ processes have cross sections which increase more quickly with energy than they do for $2 \rightarrow 2$ processes, and depending on total cross sections these higher multiplicity channels may provide more sensitivity at the LHC to non-SM Higgs couplings. In the next section we evaluate these cross sections in order to quantify this sensitivity.

2.5 Sensitivity of $2 \rightarrow 3, 4$ Cross Section to Anomalous Couplings

In this section, we analyse the cross sections for the $2 \rightarrow 2$, $2 \rightarrow 3$ and $2 \rightarrow 4$ processes at the parton level (i.e. for example $\pi^0\pi^0 \rightarrow \pi^+\pi^-\pi^+\pi^-$), with a fixed centre-of-mass energy of $\sqrt{s} = 1$ TeV, and Higgs mass of 125 GeV. We use a model given by the effective Lagrangian in Eq. 2.9 which was implemented in CalcHEP using LanHEP to generate the model file from the Lagrangian.

In the SM, the Higgs does not decay to a pair of on-shell gauge bosons, and therefore is never on-mass-shell when coupled to two gauge bosons. On the other hand, in our effective model, the pions are massless, and so to ensure that the propagating Higgs in this case is also off shell, we implemented an invariant mass cut on our final state pions of $m_{\pi^+\pi^-} > 200$ GeV.

The effect on the cross-section of varying a , (which scales the hV_LV_L coupling, with $a = 1$ being the SM value) is shown in Fig. 2.1, where the y-axis is presented in terms of the ratio of the cross section to the cross section when $a = 1$. In Fig. 2.1a, the parameter a is varied keeping all other parameters fixed, whilst in Fig. 2.1b we

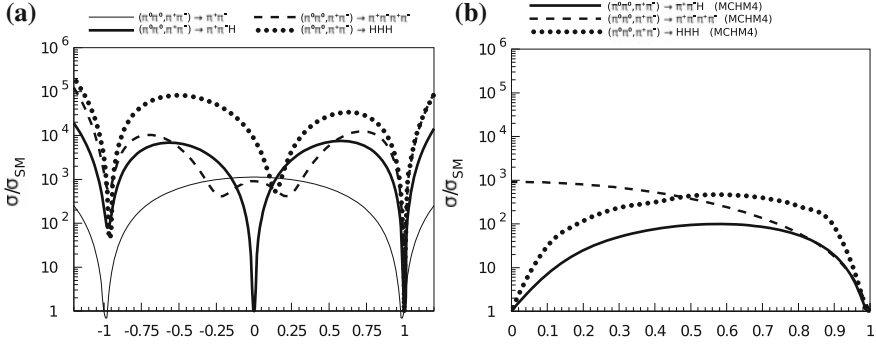


Fig. 2.1 Ratio of the cross sections to the SM cross section as a function of the “ a ” coupling parameter, where $\sqrt{s} = 1$ TeV: **a** the general effective Lagrangian with all other parameters fixed at their SM values; **b** other couplings fixed according to the MCHM4 relations. The different channels are: $(\pi^0\pi^0, \pi^+\pi^-) \rightarrow \pi^+\pi^-\pi^+\pi^-$ (dashed line), $(\pi^0\pi^0, \pi^+\pi^-) \rightarrow \pi^+\pi^-h$ (thick solid line), $(\pi^0\pi^0, \pi^+\pi^-) \rightarrow hhh$ (dotted line), and $(\pi^0\pi^0, \pi^+\pi^-) \rightarrow \pi^+\pi^-$ (thin solid line) for comparison. The notation $(\pi^0\pi^0, \pi^+\pi^-)$ indicates that both $\pi^0\pi^0$ and $\pi^+\pi^-$ initial states were taken into account

model effects in the MCHM4 by altering the other parameters according to Eq. 2.13 along with $d_3 = \sqrt{1 - \xi}$ [17].

We see that very large enhancements of the order of 10^3 – 10^5 with respect to the SM value are obtained, and that the majority of this increase is present even for relatively small deviations, with an $\mathcal{O}(10^2$ – $10^4)$ increase for $a = 0.9$. The largest increases are observed for triple Higgs production. Note that the cross section versus a for $2 \rightarrow 2$, $2 \rightarrow 3$ and $2 \rightarrow 4$ processes have 2, 3 and 4 dips respectively, which can be easily understood by noting from Eqs. 2.15–2.19 that the amplitudes are 2nd, 3rd and 4th order polynomials in a . We see that the enhancements in $2 \rightarrow 2$ processes are modest compared to the large enhancements which occur for higher multiplicities due to the increased phase space (at least at 1 TeV).

When the couplings are related as required for the MCHM4, the increases are smaller as expected from the parity symmetry of the coset. Since the MCHM4 always predicts smaller deviations, in what follows we will consider the more optimistic case where the parameter a varies independently, with the other parameters fixed.

We showed in Sect. 2.4 that in the case of partial unitarisation, $\sigma(2 \rightarrow n) \sim \frac{1}{s} \left(\frac{s}{v^n}\right)^2 s^{n-2}$ as in the Higgsless case. To explore this, we next study the growth of the cross section with centre-of-mass energy for different numbers of final state particles. We consider a few different values of the anomalous coupling, namely $a = 0.9, 0.95$ and 1 (SM), keeping the other couplings at their SM values.

The results are shown in Fig. 2.2a, b, where the cross section of representative processes with 2, 3 and 4 particles in the final state are plotted for different values of the coupling parameter a . The shaded area at the top right of each plot is the unitarity bound from Eq. 2.1. Both (a) and (b), demonstrate that as expected, the SM cross section (with $a = 1$) for each process quickly stabilises at a small value

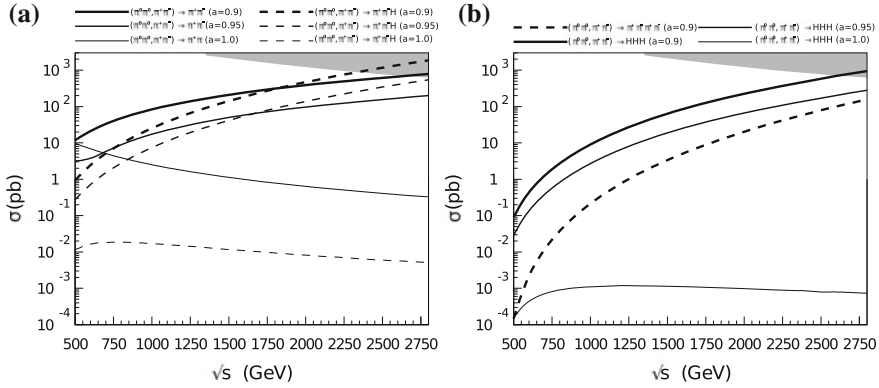


Fig. 2.2 Comparison of cross sections as a function of the centre-of-mass energy for processes with 2, 3 and 4 particles in the final state. **a** the *solid lines* are for $(\pi^0\pi^0, \pi^+\pi^-) \rightarrow \pi^+\pi^-$ for $a = 0.9$ (*thick*), $a = 0.95$ (*medium thick*) and $a = 1$ (*thin*). *Dashed lines* are for $(\pi^0\pi^0, \pi^+\pi^-) \rightarrow \pi^+\pi^-h$, with the same pattern of the thickness of the lines. **b** The same pattern of lines show the results $(\pi^0\pi^0, \pi^+\pi^-) \rightarrow hhh$ and the process $\pi^0\pi^0 \rightarrow \pi^+\pi^-\pi^+\pi^-$ is shown as a *dashed line* for $a = 0.9$. In these plots only the coupling parameter a deviates from the SM value. The unitarity bound is shown as a *shaded area* in the *top right corner*

due to the cancellations between amplitudes, with the precise value depending on the final state. In the non-SM case, without these cancellations, the cross sections grow rapidly with energy, reaching up to order 100 pb and violating unitarity at centre-of mass energies of the order of a few TeV. Also, as anticipated we see that for processes with higher numbers of particles in the final state, the incomplete cancellation between amplitudes allows the increased phase space to lead to a faster growth in cross section with energy. However, what was somewhat unexpected is the relatively low energy scale at which multiparticle cross sections can become comparable to $2 \rightarrow 2$ processes. The $2 \rightarrow 3$ cross sections start to become larger than that of the $2 \rightarrow 2$ process at energies of $\mathcal{O}(1 \text{ TeV})$. This might be signalling the onset of non-perturbative behaviour well before the unitarity bound is reached, and it may be that new physics such as the appearance of new resonances must come in at these scales. However, for this work we assumed that this is not the case. For the $2 \rightarrow 4$ process the cross section grows very rapidly for non-SM couplings, however as it starts off very suppressed, it only surpasses the $2 \rightarrow 2$ at high energies of the order of $\mathcal{O}(5 \text{ TeV})$.

2.6 Cross Sections in the SM with Anomalous Higgs Couplings

So far we've only analysed the scattering of the longitudinally polarised gauge bosons. In an experimental setting, it's difficult to separate out these contributions from the transversely polarised bosons. Therefore, it's important to understand how

Table 2.1 Comparison of $2 \rightarrow 2$ and $2 \rightarrow 3$ cross sections (in picobarns) at $\sqrt{s} = 1 \text{ TeV}$ ($\sqrt{s} = 2 \text{ TeV}$ in parenthesis)

Channel	$a = b = 1$ (SM)	$a = 0.9; b = 1$
$\pi^0 \pi^0 \rightarrow \pi^+ \pi^-$	0.53 (0.13)	66.4 (295)
$ZZ \rightarrow W^+ W^-$	629 (610)	646 (655)
$\pi^0 \pi^0 \rightarrow \pi^+ \pi^- h$	4.6×10^{-3} (2.0×10^{-3})	18.7 (350)
$ZZ \rightarrow W^+ W^- h$	5.49 (10.9)	6.17 (46.2)
$\pi^0 \pi^0 \rightarrow hh$	0.64 (0.18)	43.0 (158)
$ZZ \rightarrow hh$	7.18 (7.61)	4.31 (15.7)
$\pi^0 \pi^0 \rightarrow hhh$	5.6×10^{-4} (4.9×10^{-4})	4.5 (112)
$ZZ \rightarrow hhh$	1.7×10^{-2} (4.7×10^{-2})	0.61 (13.6)

The notation $\pi^0, \pi^\pm$ indicates longitudinally polarised bosons and $Z, W^\pm$ denotes unpolarised gauge bosons

the large enhancements in the scattering of longitudinally polarised will affect the full, unpolarised cross section. To do this, we promote the partial derivatives of our effective Lagrangian (Eq. 2.9) to full covariant derivatives and adopt the unitary gauge ($U = 1$).

The results are presented in Table 2.1, where these unpolarised cross sections are compared to the pure longitudinally polarised vector boson scattering as describe in the previous section. We keep the notation $\pi^0, \pi^\pm$ to indicate longitudinally polarised scattering and $Z, W^\pm$ to denote the unpolarised gauge bosons. We compare results with partonic centre-of-mass energy of both 1 and 2 TeV, both of which are below the unitary bound for the $a = 0.9$ case considered due to partial unitarisation.

The first thing to note is the large degree to which longitudinal polarisations are subdominant in the standard model, with the unpolarised cross section being $\mathcal{O}(10^1\text{--}10^3)$ larger than the purely longitudinally polarised case. However, as discussed, they are greatly enhanced with a 10% deviation where $a = 0.9$, and come to dominate the cross section in all the cases with a Higgs in the final state. As anticipated, the enhancements are larger for larger final state multiplicities, with for example a $\mathcal{O}(10^2)$ enhancement in $\sigma(\pi^0 \pi^0 \rightarrow \pi^+ \pi^-)$ and an $\mathcal{O}(10^{3-4})$ enhancement for $\sigma(\pi^0 \pi^0 \rightarrow \pi^+ \pi^- h)$ at 1 TeV when going from $a = 1$ to $a = 0.9$.

Unfortunately, we also see that when all polarisations are included, the large increase observed in purely longitudinal scattering is often masked. This is due in part to the fact that whilst the longitudinally polarised contribution to the cross section increases when $a(1.0 \rightarrow 0.9)$, the initially larger transverse component reduces. This is to be expected because for processes involving transverse polarisations the couplings are also scaled with a , and hence the amplitude scales with a power of a which depends on the number of relevant vertices in the Feynman diagram. For example for $\sigma(\pi^0 \pi^0 \rightarrow \pi^+ \pi^- h)$, a enters as a^3 for some diagrams. If these were the dominant diagrams, this would naively give us a scaling of the cross section of $a^6 \sim 50\%$ for $a = 0.9$. If we combine this with the fact that the longitudinally polarised cross sections quoted in Table 2.1 assume that both incoming partons are

longitudinally polarised, and so should be divided by 9 due to averaging over spins before comparing to the unpolarised cross sections, we see why many of the cross sections do not increase much. In fact for $ZZ \rightarrow hh$ the cross section decreases. Interference between diagrams involving transversely polarised bosons may also play a role but this was not explored further. As a concrete example, despite the cross section for $\pi^0\pi^0 \rightarrow \pi^+\pi^-h$ being 3 times larger (18.7 pb) than for the unpolarised case, ($\sigma(ZZ \rightarrow W^+W^-h) = 6.17$) when $a = 0.9$, the total increase in the unpolarised cross section is only $\sim 10\%$ at 1 TeV.

On the other hand, in cases where the initial contributions from the transverse polarisations are small, as in $ZZ \rightarrow hhh$, enhancements factors of around 35 and 300 are obtained at 1 and 2 TeV respectively. Its enhanced cross section is however still 1 to 2 orders of magnitude smaller than the other processes considered.

Finally, in all channels, the degree of enhancement for anomalous couplings were larger for 2 TeV than 1 TeV as expected due to the $\mathcal{M} \propto s$.

It is difficult at this point to conclude for certain which process offers the best channel to study anomalous couplings, and the answer is likely to depend on the energy of collision. Triple Higgs production is a promising channel due to the low transverse background and very large factors of increase in cross sections, although the fact that its cross section is still relatively low is against it. $ZZ \rightarrow W^+W^-h$ on the other hand has a more modest factor of increase (at 2 TeV), but a higher overall cross section.

2.7 Impact of Multiparticle Production at the LHC and Future Colliders

Thus far we have studied scattering at parton level, essentially simulating the collision of beams of vector bosons. In order to estimate how these results would manifest at the LHC or other future colliders, we used MadGraph5 (v1.4.8) to perform a full calculation of $pp \rightarrow jj + X$, where $j = u, \bar{u}, d, \bar{d}, s, \bar{s}$ and $X = W^+W^-, W^+W^-h, hhh$. We evaluated tree-level cross sections at $\sqrt{s} = 14$ and 33 TeV,⁴ using the CTEQ6L1 parton density function and the QCD scale equal to M_Z .

Such proton-proton collisions, as well as containing the vector-boson fusion/scattering discussed in the previous section, also contain many additional diagrams leading to the requested final products. Many of these additional diagrams do not contain the anomalous Higgs-vector-vector coupling, and so they will not exhibit the cross section enhancements discussed, likely simply contributing to the overall cross section, acting as a background to our BSM process and obscuring the enhancements we seek. If we are able to select mainly events *with* vector boson fusion (VBF), then we limit ourselves to diagrams which *do* contain the relevant processes discussed

⁴At the time of this study, 33 TeV was being discussed as a possible energy for the High Energy LHC upgrade. Now the possibility of a 100 TeV collider is being considered for which we are updating these results [28].

Table 2.2 Cross section (in fb) for $pp \rightarrow jjW^+W^-$, $pp \rightarrow jjW^+W^-h$ and $pp \rightarrow jjhhhh$ processes evaluated with MadGraph5

Process	14 TeV		33 TeV	
	with (without) VBF cuts		with (without) VBF cuts	
	a = 1.0 b = 1.0	a = 0.9 b = 1.0	a = 1.0 b = 1.0	a = 0.9 b = 1.0
$pp \rightarrow jjW^+W^-$	95.2 (1820)	99.3 (1700)	512 (5120)	540 (5790)
$pp \rightarrow jjW^+W^-h$	0.011 (0.206)	0.0088 (0.172)	0.0765 (0.914)	0.0626 (0.758)
$pp \rightarrow jjhhhh$	1.16×10^{-4} (3.01×10^{-4})	0.0566 (0.0613)	0.00151 (0.00237)	2.02 (2.07)

There are two values of the cross sections for each entry, with the number in parenthesis being the cross section without VBF cuts

previously and we are more likely to observe enhancements. Therefore, we evaluated 2 sets of cross sections, one with and one without cuts selecting for vector boson fusion.

The acceptance cuts which we applied to all events are:

$$\begin{aligned}
 \text{Acceptance cuts:} \quad & p_{T_j} > 30 \text{ GeV} \\
 & |\eta_j| < 5.0 \\
 & \Delta R_{ij} = \sqrt{\Delta\phi_{ij}^2 + \Delta\eta_{ij}^2} > 0.4
 \end{aligned}$$

In addition, we produced a set of events with additional VBF cuts:

$$\text{VBF cuts: [29] } E_j > 300 \text{ GeV} \quad (2.20)$$

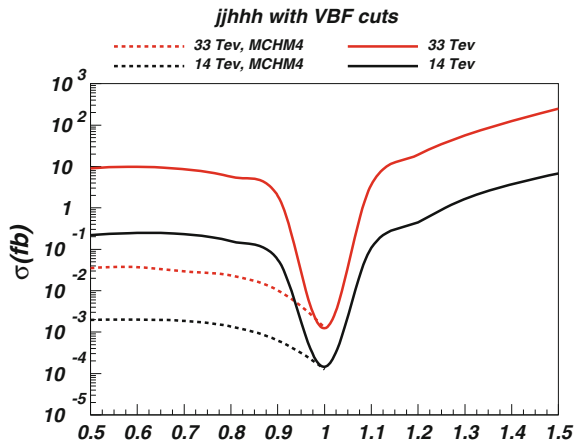
$$\Delta\eta_{ij} > 4. \quad (2.21)$$

The basic idea behind the VBF cuts is that the vector bosons tend to be radiated from a high energy quarks, one from each proton, which then continue with a small angle from the beam pipe. Therefore the signature is of 2 high energy jets which are back to back and therefore have a large rapidity gap between them. The QCD background on the other hand tends to produce more central jets and generally fail to pass these cuts. (For a more detailed motivation of this choice of cuts see e.g. [29].)

In Table 2.2 we present these results both with vector boson fusion cuts, and without these cuts in parenthesis.

The first thing to note is that the overall pattern is similar to that found for the parton level scattering in the previous section. In processes with gauge bosons in the final state, we see either a small increase or a small reduction in cross sections as $a(1.0 \rightarrow 0.9)$, occurring as discussed previously due to the reduction in amplitudes involving transverse bosons. We also see that the VBF cuts (results *not* in parenthesis), successfully isolate a larger proportion of processes involving longitudinal scattering

Fig. 2.3 Cross section for triple Higgs production $pp \rightarrow jjh hh$ with VBF cuts as a function of the anomalous coupling a for LHC14 (dark lines) and LHC33 (light lines). Solid lines are for other parameters fixed to SM values and dashed lines are for parameters given by MCHM4 relations (Eq. 2.13)



for $pp \rightarrow jjW^+W^-$, leading to a small increase in cross section for anomalous coupling instead of a reduction in the more inclusive case. For $pp \rightarrow jjW^+W^-h$ this does not occur, presumably as the overall increase in the longitudinal scattering was too small.

For the triple Higgs production on the other hand, the enhancements remain substantial. With anomalous couplings, there's roughly a factor of 500 increase for $\sqrt{s} = 14$ TeV (LHC14) and 1300 for $\sqrt{s} = 33$ TeV (LHC33), with VBF cuts.

As this remains the only promising channel at the energies considered, we show in Fig. 2.3 the results for the $pp \rightarrow jjh hh$ cross section for both LHC14 and LHC33 with anomalous coupling $0.5 < a < 1.5$. We also include the results for where the other parameters are altered simultaneously according to the MCHM4 relations given in Eq. 2.13. We observe that for anomalous couplings, the enhancements with respect to the SM case ($a = 1$) are large. The majority of this increase occurs by $\Delta a \sim 0.1$, so that there is little advantage in terms of cross section in having deviations $> 10 - 15\%$ from the standard model (which are anyway generally disfavoured in composite Higgs models). As in Sect. 2.5 the increases for the MCHM4 are smaller due to the parity symmetry of the coset.

For the case where other couplings are set to their SM value (solid line), the enhancement can be as large as 10^5 for $a = 1.5$. However even in this extreme case, the absolute value of the cross section is quite low (about 10 fb for $\sqrt{s} = 14$ TeV with VBF cuts) making the study of these processes challenging at the LHC. A dedicated analysis would be required to accurately understand the LHC14 or LHC33 sensitivity, however we can already see that to have any realistic prospect of observing this process we would require high integrated luminosities. There are currently early proposals for a future 100 TeV hadron collider. As the increase in cross section scales rapidly with energy, such a collider seems likely to be able to probe these couplings and processes, but would be the subject of further study [28].

2.8 Conclusion

We have studied multiparticle production in models with anomalous Higgs couplings, such as the composite Higgs models. These modified couplings result in a partial unitarization of the scattering amplitudes. We found that at high energies, the amplitudes scale linearly with centre-of-mass energy squared, s , irrespective of the number of particles in the final state. Therefore, due to the phase space, the cross section increases more rapidly with energy for larger multiplicity processes, and very large enhancements in cross sections compared to the SM can arise. These can be as large as $\mathcal{O}(10^5)$, even for relatively small deviations of the couplings. The increased growth of the cross sections with energy for larger multiplicities is however in competition with the fact that more energy is required to produce the larger number of final state particles, with the results that $2 \rightarrow 4$ processes are less relevant than $2 \rightarrow 3$ processes at the energies investigated. On the other hand $2 \rightarrow 3$ processes can become as important as $2 \rightarrow 2$ even at relatively low energies of the order of 1 TeV, which may be signalling the onset of nonperturbative effects. When accounting for the contributions from the transverse polarisations, the enhancements are somewhat diluted but remain important in some processes, especially triple Higgs production.

We also showed with a realistic calculation that even with these large enhancements the search for multiparticle processes will remain a challenge for the LHC run 2 at 14 TeV and for any future upgrade or new experiment at 33 TeV. On the other hand, the enhancements studied increase rapidly with energy, and multiple gauge and Higgs boson production could be an important way of studying anomalous Higgs couplings in future experimental programs such as a 100 TeV collider.

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