

Preface

During the academic year 2010–2011, the Ohio State University Mathematics Department hosted a special year on geometric group theory. Over the course of the year, four-week-long workshops, two weekend conferences, and a week-long conference were held, each emphasizing a different aspect of topology and/or geometric group theory. Overall, approximately 80 international experts passed through Columbus over the course of the year, and the talks covered a large swath of the current research in geometric group theory. This volume contains contributions from the workshop on “Topology and geometric group theory,” held in May 2011.

One of the basic questions in manifold topology is the Borel Conjecture, which asks whether the fundamental group of a closed aspherical manifold determines the manifold up to homeomorphism. The foundational work on this problem was carried out in the late 1980s by Farrell and Jones, who reformulated the problem in terms of the K -theoretic and L -theoretic Farrell–Jones Isomorphism Conjectures (FJIC). In the mid-2000s, Bartels, Lück, and Reich were able to vastly extend the techniques of Farrell and Jones. Notably, they were able to establish the FJICs (and hence the Borel Conjecture) for manifolds whose fundamental groups were Gromov hyperbolic. Lück reported on this progress at the 2006 ICM in Madrid. At the Ohio State University workshop, **Arthur Bartels** gave a series of lectures explaining their joint work on the FJICs. The write-up of these lectures provides a gentle introduction to this important topic, with an emphasis on the techniques of proof.

Staying on the theme of the Farrell–Jones Isomorphism Conjectures, **Daniel Juan-Pineda** and **Jorge Sánchez Saldaña** contributed an article in which both the K - and L -theoretic FJIC are verified for the braid groups on surfaces. These are the fundamental groups of configuration spaces of finite tuples of points, moving on the surface. Braid groups have been long studied, both by algebraic topologists, and by geometric group theorists.

A major theme in geometric group theory is the study of the behavior “at infinity” of a space (or group). This is a subject that has been studied by geometric

topologists since the 1960s. Indeed, an important aspect of the study of open manifolds is the topology of their ends. The lectures by **Craig Guilbault** present the state of the art on these topics. These lectures were subsequently expanded into a graduate course, offered in Fall 2011 at the University of Wisconsin (Milwaukee).

An important class of examples in geometric group theory is given by $\text{CAT}(0)$ cubical complexes and groups acting geometrically on them. Interest in these has grown in recent years, due in large part to their importance in 3-manifold theory (e.g., their use in Agol and Wise's resolution of Thurston's virtual Haken conjecture). A number of foundational results on $\text{CAT}(0)$ cubical spaces were obtained in Michah Sageev's thesis. In his contributed article **Daniel Farley** gives a new proof of one of Sageev's key results: any hyperplane in a $\text{CAT}(0)$ cubical complex embeds and separates the complex into two convex sets.

One of the powers of geometric group theory lies in its ability to produce, through geometric or topological means, groups with surprising algebraic properties. One such example was Burger and Mozes' construction of finitely presented, torsion-free simple groups, which were obtained as uniform lattices inside the automorphism group of a product of two trees (a $\text{CAT}(0)$ cubical complex!). The article by **Pierre-Emmanuel Caprace** and **Bertrand Rémy** introduces a geometric argument to show that some nonuniform lattices inside the automorphism group of a product of trees are also simple.

An important link between algebra and topology is provided by the cohomology functors. Our final contribution, by **Peter Kropholler**, contributes to our understanding of the functorial properties of group cohomology. He considers, for a fixed group G , the set of integers n for which the group cohomology functor $H^n(G, -)$ commutes with certain colimits of coefficient modules. For a large class of groups, he shows this set of integers is always either finite or cofinite.

We hope these proceedings provide a glimpse of the breadth of mathematics covered during the workshop. The editors would also like to take this opportunity to thank all the participants at the workshop for a truly enjoyable event.

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