

Preface

Water and air behave in a similar way and their flow can be described in a very accurate way by the Navier–Stokes equations. Others fluids behave differently: they cannot be described by the Navier–Stokes equations. Fluids that can be described by the Navier–Stokes equations are called Newtonian fluids, and all the others are called non-Newtonian or complex fluids. Most common fluids, such as toothpaste, hair gel, mayonnaise, liquid foam, cement, and blood are complex fluids. Molten plastics are also complex fluids. The success of the forming process depends on the knowledge of the material’s properties. Geophysical fluids, such as mud flows, volcanic lava, glaciers, and snow avalanches are also complex fluids and the hazard assessment uses softwares that are based on complex fluid models. In biology, the optimization of arterial graft requires the knowledge of the behavior of the flow of blood, also a complex fluid.

Goals The object of this book is twofold: modeling and algorithms. The first goal of this book is to present in a comprehensive text the modeling of complex fluids. Complex fluid models are introduced by increasing the level of complexity, and the relations between all the models are addressed using hierarchical diagrams. The second goal is to present an up-to-date mathematical and numerical analysis of the corresponding equations and to propose several practical numerical algorithms for the approximation of the solutions. The various numerical methods presented here are then suitable for implementation on computers, using some finite element libraries. These numerical methods are able to compute velocities, pressures, and stresses at each position and at each time when explicit computations are no more possible for complex geometries and flow conditions. Numerous examples of practical flow computations are presented along with this book. Software implementations are based on the Rheolef finite element library [283–285] developed by the author. Rheolef is a free software available as a standard package under the Debian and Ubuntu GNU/Linux systems and that can be installed from source code on others systems.

Audience This book is primarily intended for undergraduate students and researchers in applied mathematics, engineering sciences, computational mechanics, and physics. The reader is assumed to be familiar with computational methods

such as finite differences and finite elements, together with the corresponding variational formulations of partial differential equations. Special care has been devoted to making the material as much self-contained as possible. The general level of the book is best suited for an undergraduate-level course which can be built by drawing on some of the present chapters. The material is actually an elaboration on the lecture notes by the author for an undergraduate course on complex fluids at Grenoble University and ENSIMAG. Some of the topics covered in this book stem from recent work by the authors, e.g., on the logarithm of conformation tensor formulation of viscoelastic fluids and the joint work with Bernabeu on inertia effects in viscoplastic flows and also the joint work with Cheddadi and Graner for the elaboration of elastoviscoplastic models and their numerical resolution.

Outline Newtonian fluids and the Navier–Stokes equations are presented in Chap. 1. Newtonian fluids constitute the basis on which are developed more complex models. This first chapter also introduces some concepts, notations, and numerical methods that are reused all along this book: the continuum mechanics theory (Sects. 1.1–1.3), the method of characteristics for the time discretization and two algorithms for the resolution of the underlying linear system, the so-called Stokes problem. Chapter 2 covers various quasi-Newtonian fluids: it is a first step inside complex fluids. Quasi-Newtonian fluids are defined by a nonconstant viscosity law that depends upon the velocity gradient. Two numerical algorithms are presented in order to solve this nonlinearity: the fixed point method and the Newton method. Chapter 3 addresses viscoplastic fluids: these complex fluids are defined by a non-differentiable energy of dissipation function and the corresponding constitutive equations are highly nonlinear. Two main classes of numerical algorithms are presented: the regularization method and the augmented Lagrangian algorithm. This second approach uses some convex analysis tools introduced in the chapter. Chapter 4 is dedicated to viscoelastic fluids. This chapter starts with the concept of frame-invariant tensor derivatives. An operator splitting method allows to develop a second order in time algorithm, the so-called θ -scheme. An alternative numerical approach is also presented: it is based on a change of variable, with the new variable defined in terms of the logarithm of the conformation tensor. Chapter 5 deals with elastoviscoplastic fluids that combines nonlinearities presented in the two previous chapters. Elastoviscoplastic fluid models are obtained with the help of a thermodynamical framework with internal variables. These complex fluid models could be organized in a hierarchical view, shown on Fig. 1. This diagram also draws the dependencies upon the chapters of the book. Chapter 1 contains prerequisites (Sects. 1.1–1.4) for all others chapters. Then Chaps. 2, 3 and 4 could be read independently. Chapter 5 presents a synthesis of all complex fluid models of the previous chapters together with a new one that combines their complexities. All chapters share a similar outline: introductory sections leads to the problem statement. Then, the fluid model is presented through Poiseuille and Couette flows, which can be solved explicitly. Next, numerical methods are presented together with examples of computations dealing with complex flow conditions for which no explicit solution is available. Some concluding remarks close each chapter. These notes furnish references and historical complements together with some alternative

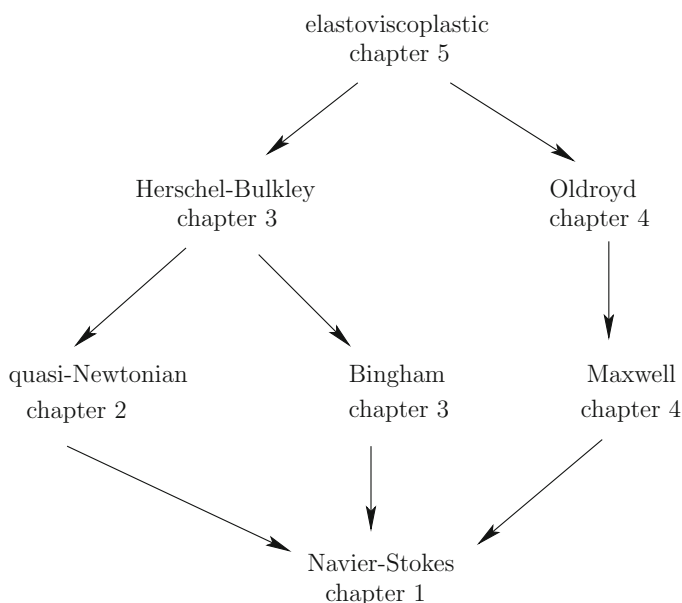


Fig. 1 Complex fluids: a hierarchy of models

methods when available. A bibliography of more than 300 entries and an index closes the book. The literature on complex fluids is so vast that this bibliography is by no means exhaustive. We hope that the selected entries provide the reader with additional information to examine in more deep the topics covered herein and to explore new ones.

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Modeling and Algorithms

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