

Preface

I first heard about long-range dependence while working on a book on stable processes with Murad Taqqu. Initially, the notion did not seem to stand out among other notions I was familiar with at the time. It seemed to describe simply situations in which covariance functions (or related functions) decayed at a slow rate. Why were so many other people excited about long-range dependence? At best, it seemed to require us to prove some more theorems. With time, I came to understand that I was wrong, and the people who got excited about long-range dependence were right. The content of this phenomenon is truly special, even if somewhat difficult to define precisely. This book is a product of many years of thinking about long memory (this term is synonymous with long-range dependence). It is my hope that it will serve as a useful complement to the existing books on long-range dependence such as Palma (2007), Giraitis et al. (2012), and Beran et al. (2013), and numerous surveys and collections.

I firmly believe that the main importance of the notion of long-range dependence is in statistical applications. However, I think of long-range dependence as a property of stationary stochastic processes, and this book is, accordingly, organized around probabilistic properties of stationary processes that are important for the presence or absence of long memory. The first four chapters of this book are therefore not really about long-range dependence, but deal with several topics in the general theory of stochastic processes. These chapters provide background, language, and models for the subsequent discussion of long memory. The subsequent five chapters deal with long-range dependence proper. This explains the title of the book: *Stochastic Processes and Long-Range Dependence*.

The four general chapters begin with a chapter on stationarity and invariance. The property of long-range dependence is by definition a property of stationary processes, so including such a chapter is necessary. Information on stationary processes is available from many sources, but some of the material in this chapter is less standard. The second chapter presents elements of ergodic theory of stationary processes. Ergodic theory intersects our journey through long-range dependence multiple times, so this chapter is also necessary. There are plenty of books on ergodic theory, but this literature is largely disjoint from books on stochastic

processes. Chapter 3 is a crash course on infinitely divisible processes. These processes provide a crucial source of examples on which to study the presence or absence of long memory. Much of the material in this chapter is not easily available from a single alternative source. Chapter 4 presents basic information on heavy tailed models. There is significant difference in the way long-range dependence expresses itself in stationary processes with light tails and those with heavy tails, particularly processes with infinite second moment. Therefore, including this chapter seems useful.

Chapter 5 is the first chapter specifically on long-range dependence. It is of an introductory and historical character. The best-known approach to long-range dependence, applicable to stationary processes with a finite second moment, is presented in Chapter 6. The vast majority of the literature on long-memory processes falls within this second-order approach. The chapter we include contains results not easily available elsewhere. Long-range dependence is sometimes associated with fractional integration, and Chapter 7 discusses this connection in some detail. Long-range dependence is also frequently associated with self-similarity. The connection is deep, and much of its power is due to the Lamperti theorem, which guarantees self-similarity of the limit in certain functional limit theorems. Chapter 8 presents the theory of self-similar processes, particularly self-similar processes with stationary increments. Finally, Chapter 9 introduces a less-standard point of view on long memory. It is the point of view that I have come to adopt over the years. It views the phenomenon of long-range dependence as a phase transition. In this chapter, we illustrate the phenomenon in a number of situations. Some of the results in this chapter have not appeared before.

The book concludes with an appendix. I have chosen to include it for convenience of the reader. It describes a number of notions and results belonging to the topics used frequently throughout this book.

The book can be used for a one-semester graduate topics course, even though the amount of material it contains is probably enough for a semester and a half, so the instructor has to be selective. There are exercises at the end of each chapter.

Writing this book took me a long time. I started working on it during my sabbatical in the Department of Mathematics of the University of Copenhagen and finished it during my following sabbatical (!) in the Department of Statistics of Columbia University. Most of it was, of course, written between those two visits, in my home department, School of Operations Research and Information Engineering of Cornell University. I am grateful to all these institutions for providing me with wonderful facilities and colleagues that greatly facilitated writing this book.

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