Contents

1 Discrete Fourier Transform of Distributions 1
1.1 Mathematical definitions and preliminary results 1
1.1.1 From pc-sets to an algebra of distributions 1
1.1.2 Introducing the Fourier transform 4
1.1.3 Basic notions 5
1.2 DFT of subsets 9
1.2.1 What stems from the general definition 9
1.2.2 Application to intervallic structure 14
1.2.3 Circulant matrixes 18
1.2.4 Polynomials 22

2 Homometry and the Phase Retrieval Problem 27
2.1 Spectral units 29
2.1.1 Moving between two homometric distributions 30
2.1.2 Chosen spectral units 31
2.1.3 Rational spectral units with finite order 32
2.1.4 Orbits for homometric sets 40
2.2 Extensions and generalisations 41
2.2.1 Hexachordal theorems 41
2.2.2 Phase retrieval even for some singular cases 44
2.2.3 Higher order homometry 45

3 Nil Fourier Coefficients and Tilings 51
3.1 The Fourier nil set of a subset of $\mathbb{Z}_n$ 52
3.1.1 The original caveat 52
3.1.2 Singular circulating matrixes 55
3.1.3 Structure of the zero set of the DFT of a pc-set 58
3.2 Tilings of $\mathbb{Z}_n$ by translation 61
3.2.1 Rhythmic canons in general 61
3.2.2 Characterisation of tiling sets 63
3.2.3 The Coven-Meyerowitz conditions 65

3.2.4 Inner periodicities . . . 67
3.2.5 Transformations . . . 69
3.2.6 Some conjectures and routes to solve them . . . 77
3.3 Algorithms . . . 80
3.3.1 Computing a DFT . . . 80
3.3.2 Phase retrieval . . . 82
3.3.3 Linear programming . . . 82
3.3.4 Searching for Vuza canons . . . 83

4 Saliency . . . 91
4.1 Generated scales . . . 92
4.1.1 Saturation in one interval . . . 93
4.1.2 DFT of a generated scale . . . 94
4.1.3 Alternative generators . . . 96
4.2 Maximal evenness . . . 99
4.2.1 Some regularity features . . . 100
4.2.2 Three types of ME sets . . . 101
4.2.3 DFT definition of ME sets . . . 104
4.3 Pc-sets with large Fourier coefficients . . . 108
4.3.1 Maximal values . . . 108
4.3.2 Musical meaning . . . 113
4.3.3 Flat distributions . . . 123

5 Continuous Spaces, Continuous FT . . . 135
5.1 Getting continuous . . . 135
5.2 A DFT for *ordered* collections of pcs on the continuous circle . . . 140
5.3 'Diatonicity' of temperaments in archeo-musicology . . . 142
5.4 Fourier vs. voice leading distances . . . 145
5.5 Playing in Fourier space . . . 149
5.5.1 Fourier scratching . . . 149
5.5.2 Creation in Fourier space . . . 151
5.5.3 Psycho-acoustic experimentation . . . 153

6 Phases of Fourier Coefficients . . . 157
6.1 Moving one Fourier coefficient . . . 157
6.2 Focusing on phases . . . 159
6.2.1 Defining the torus of phases . . . 160
6.2.2 Phases between tonal or atonal music . . . 166
6.3 Central symmetry in the torus of phases . . . 170
6.3.1 Linear embedding of the T/I group . . . 170
6.3.2 Topological implications . . . 173
6.3.3 Explanation of the quasi-alignment of major and minor triads 177

7 Conclusion . . . 179

8 Annexes and Tables 183
8.1 Solutions to some exercises 183
8.2 Lewin's 'special cases' 188
8.3 Some pc-sets profiles 189
8.4 Phases of major/minor triads 196
8.5 Very symmetrically decomposable hexachords 197
8.6 Major Scales Similarity 197

References 199

Index 205

http://www.springer.com/978-3-319-45580-8

Music Through Fourier Space
Discrete Fourier Transform in Music Theory
Amiot, E.
2016, XV, 206 p. 129 illus., 45 illus. in color., Hardcover
ISBN: 978-3-319-45580-8