

# Preface

The investigation of the Taylor coefficients of functions in various spaces or classes of analytic functions is a rather old and well-studied topic. The subject finds its real beginnings in the classical theorems of Hardy, Littlewood and Paley [1], [2]; *cf.* also [3] for more information. However, the concept of coefficients multipliers was neither defined nor mentioned explicitly in the original papers. The explicit study of multipliers actually began seriously in the 1960s with the works of Hardy's student Flett and was continued by Duren and Shields, their coauthors and various followers who used extensively the methods of functional analysis in function theory.

Over the last 50 years, many authors have worked on multipliers, often arriving at similar results from a different point of view. Thus, an enormous material on the subject has accumulated in the literature without sufficient cross-referencing. Some new related topics, such as the multipliers on very large spaces of functions (with fast-decaying weights) or the Hilbert matrix on spaces of analytic functions, have only appeared in the literature very recently. It appears that there is a need for all relevant information to be presented in one place and treated in a systematized way.

One purpose of this book is to give a systematic overview of the field in a unified and organized way. Thus, in part this monograph is intended as a relatively complete reference book on the subject. We consider a variety of results concerning both the Taylor coefficients of functions in various classes or classical spaces of analytic functions and the coefficient multipliers between different pairs of such spaces, starting with the pioneering works mentioned earlier and ending with a number of new results. Some of these results are of quite recent date.

Our exposition also includes a number of practical applications and related topics that experts in other subjects may find useful for their own purposes. Thus, another purpose of this book is to communicate to specialists interested in other questions some concrete facts that may be of interest for their own work.

It should be noted that many multiplier spaces have not yet been described so the subject is by no means closed and there are many interesting questions that may be worth further study. Thus, the monograph should also be understood as an invitation to the specialists to work on open problems and thus contribute further to the field.

This book is intended for, among others, those researchers interested in multipliers who may want to systematize their knowledge on the subject and further deepen their understanding. It is our hope that they will find this book useful as a reference.

Specialists in related fields (*e.g.*, in complex, functional, or harmonic analysis) may have come across various questions in their own work that can actually be rephrased in terms of Taylor coefficients and studied from our point of view. These colleagues may need a ready-made product for use in their own research without a thorough study of the multipliers. With this in mind, we have prepared a final section in several chapters of this book that should provide the reader with sufficiently many practical examples of analytic functions or sequences with certain required properties or simple examples of operators with given features, with complex analysis proofs of some facts, etc.

Another target group is that of advanced graduate students. In their graduate work they have probably read several monographs on complex analysis and function spaces but may have not studied this very subject in depth. It is our hope that they also may find this book useful, not only the early chapters but also several of those that come later.

In summary, it is our hope that both experienced researchers in complex analysis and graduate students may find it useful, either as a reference book or as a source of information for specific results that they may need in their own research.

The reader is assumed to be familiar with a number of topics such as real analysis (measure, integration, and  $L^p$  spaces), basic complex analysis (including the basics of analytic functions, integration theorems, normal families, and conformal mappings), as well as some rudiments of functional analysis (basic spaces and operators, the three basic principles of functional analysis, linear functionals, and dual spaces), and Fourier series (Fourier coefficients and basic convergence).

In order to make the text as self-contained as possible, we have tried to present as much of the standard material on the Hardy spaces of the disk, for example. The reader is probably aware that there are wonderful monographs devoted to this subject (for example, those by Hoffman, Duren, Koosis, Fisher, or Garnett) and whatever is missing in our exposition can be found in at least one of these texts.

We have opted for *listing the authors according to their contribution to the project rather than in alphabetic order*. While all authors have written some portions of the book, supplied some proofs and done further proofreading, the first-listed author has been the main driving force behind this project and has prepared most of the material presented and the second author is responsible for reorganizing various parts of the monograph and introducing different novelties in the presentation.

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