Chapter 2
Different Approaches on CDS Valuation—An Empirical Study

Abstract In this chapter we want to discuss several approaches on the calculation of CDS prices. We apply different approaches in the hazard rate term structure, the influence of different interest rate curves used for discounting and the question of the price variation under the consideration of additional information will be discussed in detail. Further, we take a look how we can deal with constant maturity spreads in comparison to fixed maturity CDS Indices. Our benchmark is the iTraxx Europe index with different maturities of three, five, seven and ten years in comparison to its members.

Blythe Masters from J.P. Morgan & Co. invented the CDS in 1994. The market for CDS grew in the following years in a tremendous way. The volume invested in CDS products rose from about $300 billion in 1998 to about $62 trillion at the end of 2007. However, the CDS market is still an important sector in the financial markets. Even after the financial crises in 2007 and the following years, there was still 25 trillion dollars invested in CDS products. The influence of CDS investments was clearly demonstrated in April/May 2012, when JPMorgan Chase & Co., known as one of the top CDS trading banks worldwide, lost about two billion dollars within a few weeks as a result of speculative CDS trading.

In this chapter we take a look at different ways to price a CDS contract and the influence on CDS pricing by loosening some standard assumptions. There are basically two different approaches to evaluate a CDS contract. On the one hand, there is the structural form approach, and on the other, the reduced-form approach.

The structural form uses the option price theory on the company's value to gain the CDS spread. This approach is based on Merton (1974) and Black and Scholes (1973) and is the basis of the KMV model, which is used by some rating agencies. One key fact of this approach is that the probability of default is modelled indirectly. The problem of this model lies within the computation of the firm's volatility. Furthermore, this approach is rather inflexible, since it uses a lot of

---

This chapter is a working paper by Schmidt. See Schmidt (2014) "Different approaches on CDS valuation—an empirical study".

M. Schmidt, *Pricing and Liquidity of Complex and Structured Derivatives*,
SpringerBriefs in Finance, DOI 10.1007/978-3-319-45970-7_2

information based on company actions and numbers that are only published on a few dates each year. In the scientific world, however, this approach is very popular and is used in many papers. There has been some interesting research on methods for improving this approach. Many papers use different kinds of volatilities such as implied volatility from out of the money puts see Cao et al. (2010) or Carr and Wu (2011). Other papers deal with the modelling of the recovery rate such as Li (2009). Even though this is a very interesting approach, we focus on the reduced form approach in this dissertation.

The reduced form approach is basically a method gathering the probability of default from an exogenous data such as ratings, bond prices or CDS spreads. Therefore, in contrast to the structured approach, the reduced form approach models the probability of default directly. The market standard model for the reduced form approach to CDS pricing is described by O'Kane and Turnbull (2003). Their description plays an important role in this dissertation. Pursuant to O'Kane's and Turnbull's assumptions, there are only a few parameters that determine the price of a CDS contract. These parameters are the recovery rate, the interest rate and the term structure of the so-called hazard rate. The hazard rate, or to be more precise the hazard rate term structure, is a method to model the probability of default or, respectively, the survival rate. O'Kane and Turnbull assume a partial constant hazard rate as well as a constant recovery rate. In the past, several research papers have been published on different approaches on a better method to model the recovery rate such as Li (2009), Krekel (2008), Amraoui and Hitier (2008) and Böttger et al. (2008).

In this chapter we discuss several approaches on the calculation of CDS prices and we test these approaches on historical market data quotes. For the empirical test we use a CDS index and we replicate the index by its constituencies. The CDS index we are using is the iTraxx Europe series 15 with different maturities of three, five, seven and ten years. Our ideas on the alternative pricing approaches are dealing with alternative hazard rate term structures, a multi-curve approach for the interest rates and the problem of maturity differences between the index and their member in the market data quotes. We describe each change later in detail.

This chapter comprises the following matters. In Sect. 2.1, we go into the details of the CDS single name and CDS index functionalities as well as the market standards by the International Swaps and Derivate Association (ISDA). For a better understanding we discuss several examples. In Sect. 2.2 we describe the standard method for CDS pricing using a reduced form approach. This chapter is based on O'Kane and Turnbull (2003). Section 2.3 explains how we are able to imply a hazard rate term structure and a probability of default (PD) from market data. Further, we explain the different hazard rate term structure that we apply in our research and how it is different to O'Kane and Turnbull (2003). The influence of the financial crises on the interest rates is specified in Sect. 2.4. We demonstrate the changes between different interest rate constructions and recapture the basic findings of Bianchetti (2010). Further, we explain the different interest rates in our approaches. Section 2.5 mentions the data set for our empirical test and the following Sect. 2.6 pictures our results. In the last Sect. 2.7 we draw a short conclusion on our results.

2.1 How Does a CDS Work?

In a CDS contract the investor, the so-called protection buyer, wants to secure himself against a credit event. This credit event can be attached to a company bond, a government bond or a basket existing of either or both. The protection seller guarantees the protection buyer to pay the outstanding loan in the case of a credit event. The outstanding loan is equal to the residual of the recovery rate times the nominal. The definition of a credit event can vary and needs to be clearly defined. In most cases a credit event is defined as bankruptcy or the failure to pay. In some case restructuring is also defined as a credit event. In return the protection seller receives a risk-adequate payment from the protection buyer. These cash flows are displayed Fig. 2.1.

The protection buyer can settle his obligation towards the protection seller with a single up-front payment at initiation, a regular coupon payment or a combination of both. The advantage of an up-front payment is that no future payments for the protection buyer exist. The size of up-front payment is equal to the present value of the regular coupon payments. However, the problem of the up-front payment is the uncertainty of the time of default. This problem cannot be explained in several notes. If you are more interested in this particular problem, please take a look at O'Kane and Sen (2003). The advantage of a regular payment is that it is a fair price, meaning no initial payment would be necessary, and the payments stop after a credit event. The disadvantage is the lack in tradability and comparison of CDS contracts on the same reference. Imagine two on-going CDS contracts on the same reference and the same maturity. The first contract has a regular payment of 230 bps

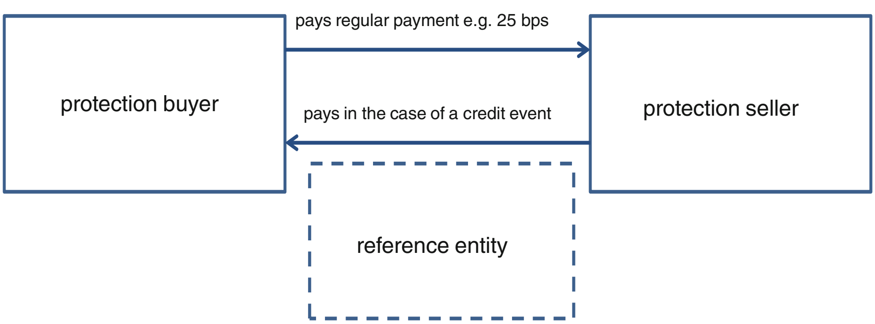

Fig. 2.1 In this figure, we observe the cash flows of a CDS contracts i.e. the regular payments of the protection buyer to the protection seller and the payment of the protection seller to the protection buyer in case of a credit event of the reference entity

and the second of 178 bps. Which one reflects the risk in a more accurate way? What if one of the parties would like to get rid of his position at the market? The advantage of a combination of both ideas is that the contracts are easy to trade, since the size of the regular payment does not change, and the changes in the market are dealt with a minor up-front payment. Minor up-front payment means that this up-front payment is not equal to the size of the single up-front payment as we discussed earlier. In fact, up-front payment does not necessarily have to be paid by the protection buyer, but more to that topic later on. For a detailed discussion about the differences between up-front and regular coupon payment take a look at O'Kane and Sen (2003).

The counterparties can agree on any of these payment schemes, which we just mentioned, since a CDS contract is an over-the-counter (OTC) derivative, meaning it can be tailor made. Due to the ambition of implementing market standards by the ISDA on the OTC markets, the most common way is to pay a regular coupon payment with an additional minor up-front payment. This regular coupon, which we refer to as contractual or deal spread, is denoted in basis points (bps). In case of a credit event the regular coupon payments stop, but the protection buyer has to pay the accrued interest (until the day of the credit event) to the protection seller. In Fig. 2.2, we monitor the cash flows in this particular payment agreement.

Example

Suppose investor "*A*" buys a CDS concerning company "*C*" from company "*B*" with a contractual spread of 100 bps and a nominal of €2,000,000. The payments are made in a quarterly frequency and the maturity is three years. We assume that in the case of a credit event company "*C*" has a recovery rate of 40 %. If there is no credit event until maturity, the investor "*A*" pays "*B*" about €5000 (≈2,000,000 * 0.01 * 3/12)—this amount can vary depending on the day count convention and the actual number of days—each quarter until maturity without any payments from "*B*" to "*A*". Let us assume a credit event occurs one week after a quarterly payment, then "*A*" would have to pay the accrued interest of about €385 (≈2,000,000 * 0.01 * 1/52).

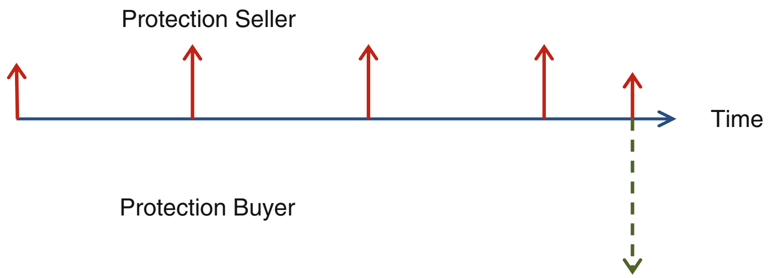

Fig. 2.2 We see the payments made by both parties from the initial until a credit event. The arrows point towards the party that receives the payment. The first payment does not have to be done by the protection buyer sometime the protection seller needs to pay an initial up-front. The payment at default by the protection buyer is the accrued interest

Additionally, "*B*" has to pay the outstanding loan of €1,200,000 € [=2,000,000 * (1 − 0.4)] to investor "*A*".

Non tailor-made CDS deals use the market standards introduced by the International Swaps and Derivative Association (ISDA). These are standards in terms of coupon dates, day count convention, coupons per year, recovery rate, definition of a credit event etc. The coupons are paid on the 20th of March, June, September and December, which are the same dates as for futures and the dates are referred to as the IMM (International Monetary Market) dates. The date differences are calculated with the day count convention ACT/360, where the actual number of days between the dates is divided by 360. The recovery rate depends on the reference, but there are only a few values possible. For more information on the standardisation of CDS contracts take a look at markit.com.

It is very important to mention that we distinguish between the contractual spread and the market spread. The contractual spread is the size of the regular coupon and does not change for an existing contract until maturity. The market spread on the other hand, is the size of the contractual spread that the market believes to be fair for this particular underlying. In comparison to the contractual spread, the market spread change at any minute.

It is almost certain that at the opening of the CDS contract, a difference between market spread and contractual spread will be present. This difference can be priced and this price is what we called the up-front payment earlier. The relation between contractual and market spread with the addition of the accrued interest determines whether the protection seller or protection buyer needs to pay the up-front amount. Let us neglect the accrued interest for the moment. Three different states at the contract opening exist. First the deal spread is equal to the contractual spread. In this very unlikely situation no up-front payment needs to be done, because there is no difference between market and contractual spread. Secondly, the market spread is above the deal spread. This implies that the market believes the risk of the entity to be higher than the regular coupon. In this case the protection buyer has an advantage, since the protection is cheaper. Therefore, the protection buyer needs to pay the up-front, otherwise the protection seller does not agree on the transaction. In the last case, where the market spread is lower the contractual spread, the protection buyer receives the up-front payment, since the protection seller receives more than the market believes to be fair.

In general, the CDS price is notated in per cent just like bonds i.e. a clean price—which means without accrued interests—of 100 means that the contractual spread has the same size as the current market spread. The dirty price is the clean price plus the accrued interests. The up-front payment, from a protection buyer perspective, is then calculated via

$$\frac{100 - price_{dirty}}{100} * nominal$$

where a negative up-front means that the protection buyer receives the amount. The values for the protection seller are the same multiplied by minus one. Even though the standard price notation is in percentage you find the current market quote usually notated in basis points (bps) of the notional. There are some CDS securities that are not notated as a credit spread but instead as clean price. Examples are the CDX HY (Credit Default Swap index for high Yield Companies located in Northern America) and CDX EM (Credit Default Swap index for emerging market government bonds). Due to the ambition of market standards there are only a few contractual spreads used in the market (25, 50, 100 or 500 bps). The following example helps to gain a better understanding for the relations and notations.

Example
We would like to buy a three-year protection with a notional of ten million euros against a default of BASF on 2011-11-11. The current deal spread is 25 bps and the end of the day market quote is 78.3 bps. Then the clean price is about 98.57 and with an accrued interest of about 0.04 we derive a dirty price of 98.71. Then we would have to pay $(1 - 0.9871) * 10{,}000{,}000 = €129{,}000$ to enter this protection. Furthermore, we would have to pay each quarter about €6250 until the default of BASF or until 2014-12-20, whichever happens first.

In the case where a company holds a corporate bond and is secured by a CDS at the same time, the portfolio is not free of any risk. For example, the counterparty risk of the CDS protection seller still exists. For further information take a look at O'Kane and McAdie (2001).

It is possible to buy a CDS without holding a corresponding loan. This is called a naked CDS (sometimes called naked sell CDS or naked selling CDS). In this case the investor uses this position to speculate on the credit worthiness of the reference entity. There are estimations that assume the rate of naked sells to be about 80 % of the total CDS market. Also, it is discussed whether the naked sells support the negative trend of the credit worthiness of corporations or sovereigns. In this regard the Greek crisis is especially interesting. Since the credit spread rose in such an extreme way, which could be an effect from naked selling, the credit worthiness of Greece kept on sinking, also influencing the price of the government's bonds. That meant a cost increase to gather capital for the Greek sovereign on the financial markets. Consequently, the European Parliament has passed a ban on naked sells, where the reference entities are sovereign bonds, in the December of 2011. The EU believes that the dramatic situations as seen in Athens should not be repeated due to speculative investors' behaviour.

CDS indices
In recent years, CDS indices played a major role in the credit derivatives market. A CDS index consists of a number of CDS contracts that are clustered to a specific topic. In general, each index exists with different maturities (three, five, seven and ten years) and is rolled twice a year, meaning a newer version is placed in the market. The liquidity of these indices is always highest in the first six months and the first weeks after the roll (see Fig. 2.1). Within this new index the members can

be exchanged, the deal spread or the number of members can be modified, and so on. The indices can be divided into benchmark indices such as the iTraxx Europe and into topic indices like the CDX high yield (CDX HY). Unlike a single name CDS, which is determined after a credit event of the reference entity, the CDS indices exist further after the credit event of a member. In the case of a credit event in a CDS index, a payment in the default amount, which is equal to $(1 - R) * \frac{nominal}{number\ of\ members}$ and the nominal is reduced by $\frac{nominal}{number\ of\ members}$, will be made and the regular payment will be adjusted for the prospective payments since the face value decreased. The adjusted payments are expressed by a factor that represents the percentage of still existing members in the index. For example, if one member defaults and all other 124 members still exist, then this factor is equal to 0.992. The advantages of CDS indices are the simplification to hedge a portfolio of bonds against the possibility of defaults. Due to their standardisation, the indices are more liquid than single name CDS. Therefore, CDS indices often offer a smaller bid-ask spread leading to lower transaction costs than a single name CDS. Furthermore, the tradability and transparency for these CDS indices are higher than in a basket of cash bond indices or single name CDS.

There are two main CDS index families-the iTraxx and CDX family. In the CDX indices consist of CDS contracts on companies, which are located in North America, or of CDS contracts on sovereign bonds in emerging markets. As mentioned above, there are also some sub-indices like the high volatility index. The CDX indices are notated either in US dollars or in euros. The most common and liquid index within this family is the CDX investment grade (CDX IG), consisting of 125 North American companies with an investment grade rating. In the iTraxx family, CDS members are generally companies located in Asia or in Europe. The currencies within the iTraxx indices are the US dollar, the euro or Japanese yen. There also exist sub-indices like financial, Xover etc. Here, the most known and liquid index is the iTraxx Europe, whose 125 members are the most liquid companies in Europe during the last six months. In both index families a credit event is defined as either bankruptcy or the failure to pay. Additionally, in the iTraxx family a modified restructuring also counts as credit event.[1] The CDS indices play an important role within this chapter, since the index prices are the benchmarks for the different approaches we discuss later.

2.2 The Standard Approach for CDS Pricing

The standard approach for CDS pricing is described by O'Kane and Turnbull (2003). As mentioned above, this method is a reduced form approach. The CDS is divided into two separate legs. The premium leg represents the regular payments

[1]For further information on the CDX and iTraxx indices look at www.markit.com.

made by the protection buyer, and the protection leg simulates the payment by the protection seller in the case of a credit event.

The premium leg is priced like a bond with a fixed coupon. Additionally, we have to consider the probability of default as well as the necessary payment of accrued interest in the case of default. For reasons of simplification, for the pricing O'Kane and Turnbull assume that a credit event occurs always occur in the middle of two coupon dates. We can then derive the price of premium leg at time t as follows

$$\mathrm{premium(t)} = \mathrm{s_c}\frac{1}{2}\sum_{\mathrm{n=1}}^{\mathrm{N}}\Delta(\mathrm{t_n},\mathrm{t_{n-1}})\mathrm{d_n}(\mathrm{Q}(\mathrm{t},\mathrm{t_{n-1}}) - \mathrm{Q}(\mathrm{t},\mathrm{t_n})) =: \mathrm{s_c} * \mathrm{rcf(t)}$$

where N is the number of future coupons, $\mathrm{t_n}$ is the time in years between the n th coupon and t, $\mathrm{d_n}$ the discount factor from t to $\mathrm{t_n}$, the parameter $\mathrm{s_c}$ represents the contractual spread and $\Delta(\mathrm{t_n},\mathrm{t_{n-1}})$ is the time difference between the n th and the $(n-1)$ th coupon according to the agreed day count convention (ACT/360 as mentioned earlier). The function $Q(s,t)$ is the condition survival probability between s and t meaning the probability that there has not been a default until the time t given there was no default up till time s. Consequently we follow $Q(0,t) = SR(t)$, where SR is the survival probability. At this time we do not further discuss the form and derivation of $Q(s,t)$, but we come back to that topic later.

The pricing of the protection leg simulates the discounted cash flow in the case of a credit event. Even though it might take up to 72 calendar days between the notification of the credit event and the settlement of the protection payment, O'Kane and Turnbull assume that the protection payment is made immediately after the incident in order to simplify matters. For the validation of the protection leg, two factors are important, the recovery rate and the timing of the credit event. For the recovery rate, we assume that the historical recovery rate is the "correct" recovery rate. In our calculations, we always apply a recovery rate of 40 % because we only look at companies with an investment grade. Secondly, for the timing of the credit event we assume—without any material loss of accuracy—that the credit event only occurs on a finite number M of discrete points per year. Thus, we divide the maturity in a grid of $[M * ttm]$ discrete time points, where ttm is the time to maturity in years according to the day count convention. With a higher M we are able to gather more accurate validation of the protection leg, but the algorithm takes more time to evaluate. Therefore, like O'Kane and Turnbull we assume that $M = 12$, a simulation of a default once per month, is fine for our purpose. Then we gain at time t

$$protection(t) = (1-R)\sum_{i=1}^{M*ttm} d_i(Q(t,t_{i-1}) - Q(t,t_i))$$

where R is the historical recovery rate, d_i is the discount factor between t and t_i and $Q(s,t)$ is same function as described above in the premium leg i.e. $Q(t,t_{i-1}) - Q(t,t_i)$ represents the probability of default within the ith coupon.

Under the assumption that we know the structure of the probability of default or the probability of survival respectively, we are able to gain the value of both legs. A transaction only takes place, if both legs have the same value. Thus, we can gain the adequate credit spread s for the deal at time t

$$s(t) = \frac{protection(t)}{\mathrm{rcf}(t)}.$$

As we see, the only input parameters varying depending on the respective company are the recovery rate and the probability of survival. Apart from the discussion about the correct risk free rate and the resulting discount factors, we are able to connect our opinion on a probability of default with a corresponding credit spread as seen in the market.

On the other hand, we are able to extract the corresponding probability of default from a market quote only using few assumptions. To imply the probability of default, we need to set the market quote equal to $s(t)$, since we defined the market quote as the contractual spread which the market believes to be fair. Then, we are able to price the CDS with this "implied" probability of default by setting $s(t)$ equal to the contractual spread, which we know in advance. In the following section we discuss the probability of default and the hazard rate term structures.

2.3 Probability of Default and Hazard Rate Structure

In the reduced form approach, a credit event is characterized as the first event of a Poisson counting process. That means we model the probability of a credit event in a time interval $[t, t+dt)$ under the condition that there has not been a default until time t as follows

$$P(\tau < t+dt \,|\, \tau \geq t) = \lambda(t)dt.$$

The function λ(t) is called the hazard rate term structure or just hazard rate. The equation leads us to the following model for the conditional survival probability until time T, if time t has been reached

$$Q(t,T) = \exp\left(- \int_t^T \lambda(s)ds \right).$$

Since we only want to evaluate the CDS at the trading date, meaning no forward CDS evaluation, this equation can be reduced to

$$SR(T) = Q(0,T) = \exp\left(-\int_0^T \lambda(s)ds\right).$$

This term is the same as the survival probability until time T, i.e. the condition dissolves, since we assume that the underlying has not defaulted before the pricing date. The following passages discuss several approaches on the construction of the hazard rate term structure λ(t). The survival probabilities always have to be arbitrage-free survival rates, i.e. then the CDS values in a risk-neutral world and the real world are the same. Therefore, the discounting factor is for our purpose the risk-free interest rate through the whole dissertation. Furthermore, the hazard rates are also arbitrage-free to fit the market values. Hazard rates based on historical data are higher, since they possess a liquidity risk premia, spread risk premia and so on.

2.3.1 Constant Hazard Rate

The first assumption is a constant hazard rate i.e.

$$\lambda \equiv \lambda(t) \text{ for all } t \in [0,T].$$

Due to this simple assumption the survival probability gets even simpler

$$SR(t) = \exp\left(-\int_0^t \lambda ds\right) = \exp(-\lambda t).$$

The parameter λ cannot be negative, because t is always positive and negative probabilities do not exist. Further λ cannot be zero, otherwise $SR(t) = 1$, meaning the probability of default $[P(t) = 1 - SR(t)]$ is zero.

The big advantage of this assumption is its simplicity and ability to gather fast results. In order to get the implied probability of default, we need to find a λ, so that the protection leg is equal to the premium leg, with the market quote being the contractual spread $s(t)$. A possible approximation to find λ is to guess an upper λ_{upper} and a lower λ_{lower} and then to apply a combination of the Newton and bisection method in order to derive the implied λ. The λ_{lower} can be close to zero, which means a high survival rate, and the λ_{upper} should be chosen high enough that $\lambda_{upper} > \lambda$. The approximation stops after a certain precision is reached. We are then able to price the CDS with the implied hazard rate and the contractual spread. The disadvantage of this method is, that it leads to different probabilities of default, if we look at different maturities.

Example

We collected the CDS spreads for BASF for maturities of one, three, five, seven and ten years on 2011-11-11. We then gathered the constant hazard rate like we

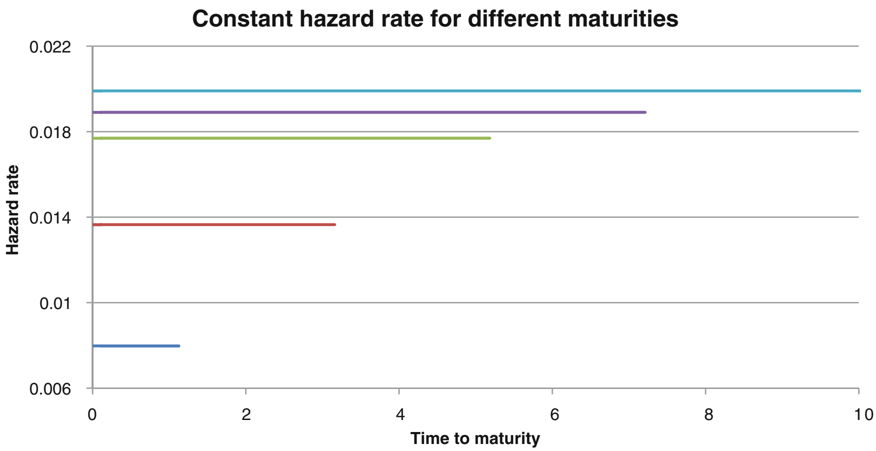

Fig. 2.3 The hazard rate under the assumption of a constant hazard rate term structure implied form the CDS market data for BASF on 2011-11-11

discussed above, see Fig. 2.3, and gained, as expected, higher constant hazard rates for longer life times. From this we can follow, that, depending on the hazard rate, we gather different probability of defaults for the same time period. For example, the probability of default within the first year, implied from the one-year CDS quote, is 0.79 %. Whereas the probability of default within the first year implied from to the three or ten year CDS quote is equal to 1.36 % respectively 1.907 %.[2] The corresponding curves showing the probability of default can be seen Fig. 2.4.

As we have seen in the example using a constant hazard rate can lead to different results for the implied probability of default. For each maturity, we gather different curves for the probability of default.

Why do we not use the additional information of different maturities to find a better approximation for the probability of default? We address with this idea in the next approach. Nevertheless, as we see later on, the constant hazard rate approach leads, on average, to a very good approximation for the index Value. Method and results are explained in more detail in Sect. 2.6.

2.3.2 *Partial Constant Hazard Rate*

In this approach we use all available market quotes from different maturities to bootstrap a unique hazard rate. All market quotes q_i are ordered according to their

[2]The results are based on the constant maturity quoted market spread for BASF CDS and on the interest curve, which was the standard interest curve before the crises. We refer to this interest curve as the single curve approach.

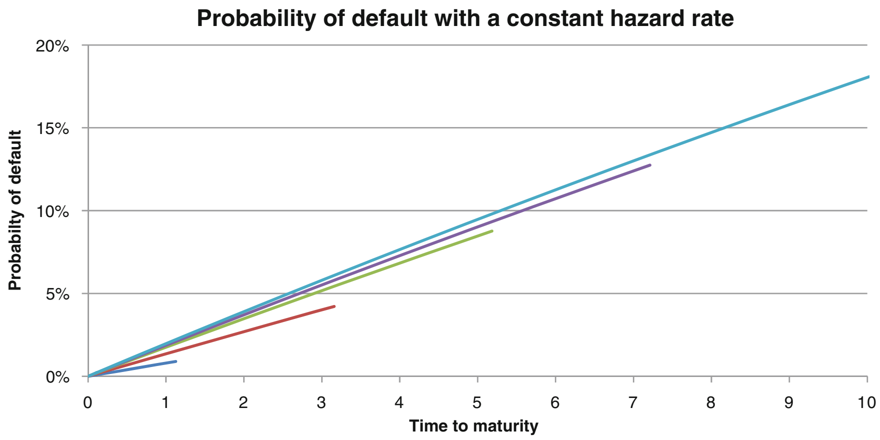

Fig. 2.4 The probability of default under the assumption of a constant hazard rate implied form the CDS market data for BASF on 2011-11-11

maturity t_i starting with the shortest t_1, which is the one-year maturity, to the longest maturity t_K of ten years, where K is the number of available market quotes. The hazard rate term is then built as follows[3]

$$\lambda(t) = \sum_{i=1}^{L} \lambda_i$$

with $L = min\{i | 1 \leq i \leq K \wedge t_i \geq t\}$. Consequently, the survival rate changes to

$$SR(t) = \exp\left(-t \sum_{i=1}^{L} \lambda_i\right).$$

The parameters λ_i are calculated from the market quotes q_i with the following bootstrapping algorithm. First, we determine λ_1 from the market quote q_1 in the same way as in the constant hazard rate case. Then, we extract the next parameter λ_2 and use for the survival rate

$$SR(t) = \exp(-t\lambda_1)$$

if $t \leq t_1$ and

$$SR(t) = \exp(-t(\lambda_1 + \lambda_2))$$

[3]This formula can also be written in the following form $\lambda(t) = \sum_{i=1}^{K} \lambda_i 1_{\{t_i \geq t\}}$, where $1_{\{t_i \geq t\}} =$ 1 iff $t_i \geq t$ and else zero.

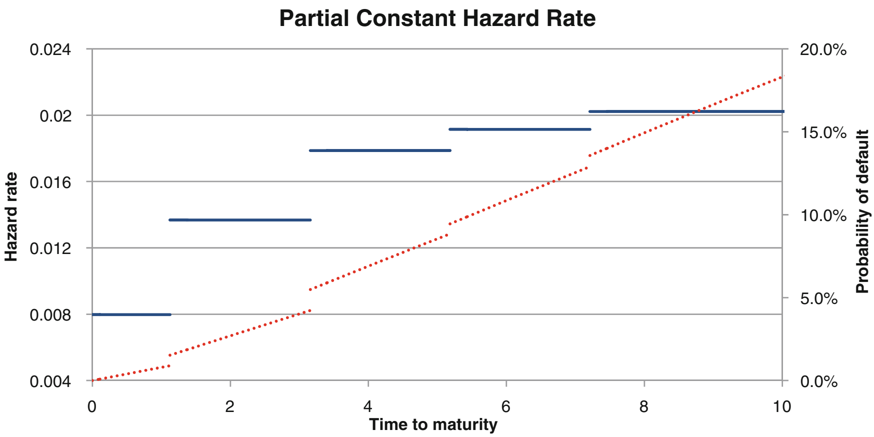

Fig. 2.5 The hazard term structure (*blue straight line*) and the probability of default (*red dotted line*) under the assumption of a partial constant hazard rate implied from the CDS market data for BASF on 2011-11-11

if $t_1 < t \leq t_2$. Like in the constant hazard rate approach, we use a combination of the Newton and the bisection methods to calculate λ_2. For all following maturities, we apply the same strategy as described for the second maturity, until we reach the last maturity K.

Example
We consider the same case as before, i.e. BASF CDSs on 2011-11.11. This time we gain that the probability of default within the first year is 0.79 %. Also, this time only one hazard rate exists. Therefore the corresponding probability of default is unique. The corresponding data is shown in Fig. 2.5.

The advantage of this approach is that all market data are implied and there is only one probability of default curve. Nevertheless, some disadvantages exist. Firstly, the method takes longer computing time to gain results for longer maturities. Secondly, more market data are needed. Thirdly, the quality of the data can be different between maturities, but this disadvantage affects all approaches. At last, it is very likely that the probability of default curve is discontinuous and jumps at each maturity of a market quote.

2.3.3 Linear Hazard Rate

The simplest idea to gain a more realistic and smoother probability of default curve is to use a linear hazard rate. Like in the constant hazard rate case we first take a look at the different CDS maturities on their own, meaning we gain one hazard rate for each maturity. Subsequently the hazard rate term structure is modelled via

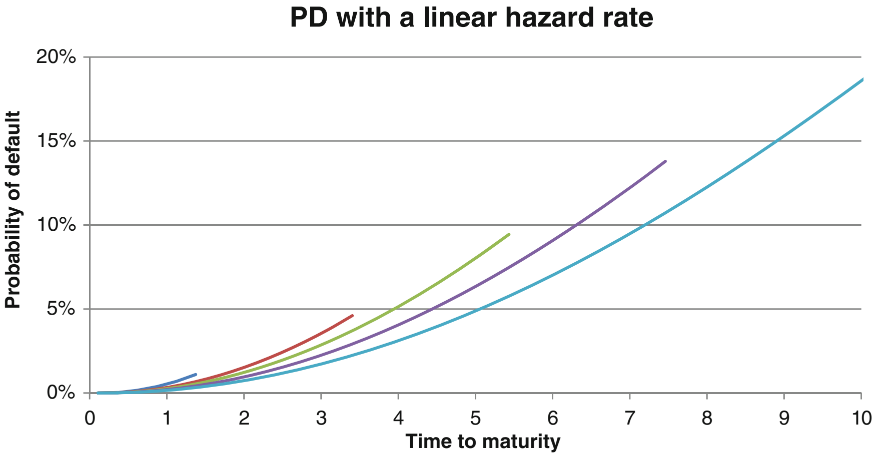

Fig. 2.6 The probability of default under the assumption of a linear hazard rate implied from the CDS market data for BASF on 2011-11-11

$$\lambda(t) = \lambda t$$

and the corresponding survival rate is given by

$$SR(t) = \exp\left(-0.5\lambda t^2\right).$$

The deviation of the parameters λ_i is the same as described in the constant hazard rate but this time with a survival rate modelled like the equation above.

Example

In our standard example this leads us to different hazard rates as well as different probabilities of default. For example, we gain for the probability of default within the first year based on the one-year CDS a value of 0.71 %, based on the three year CDS quote a value of 0.44 % and based on the ten year CDS quote a value of 0.21 %. At first, it is surprising that the probability is decreasing for longer maturities whereas in the constant hazard rate case the probability of default is increasing for longer maturities. This is due to the fact, that in the linear hazard rate case the probability of default curve is not linear but quadratic and therefore the probabilities for longer maturity are more "weighted" than shorter maturities (see Fig. 2.6).

2.3.4 Partial Linear Hazard Rate

Lastly, we want to use a partial linear hazard that combines the linear approach as well as the idea of using all available information from all market quoted CDS

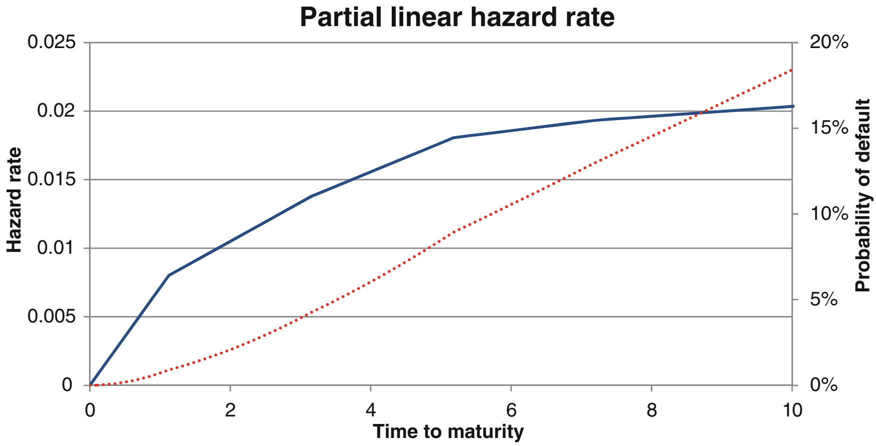

Fig. 2.7 The hazard rate term structure with a partial linear hazard rate (*blue straight line*) and its corresponding probability of default (*red dotted line*) implied form the CDS market data for BASF on 2011-11-11

spreads.[4] We use the same method as before. First, we gather the hazard rate λ_1 following the linear hazard rate case. Then, we drive λ_2 and so on until we reach λ_K. In this model we assume that the hazard is constructed in the following way

$$\lambda(t) = \sum_{i=1}^{L} \lambda_i t$$

with $L = min\{i|1 \le i \le K \wedge t_i \ge t\}$ and the corresponding survival rate changes to

$$SR(t) = \exp\left(-0.5t^2 \sum_{i=1}^{L} \lambda_i\right).$$

Thus, in this model the hazard rate is never the same at two different time points, and if we assume $\lambda_i > 0$ the hazard rate is a monotone increasing function.

Example
If we consider our standard example again, we gain a probability of default of 0.71 % for the first year which, as mentioned above, is the same as in the linear case. The results of this example are plotted in Fig. 2.7.

As we can see from this example, the partial linear smoothly combines many advantages and only one probability of default curve exists. Nevertheless, there are

[4]A similar approach has been demonstrated by O'Kane and Turnbull (2003).

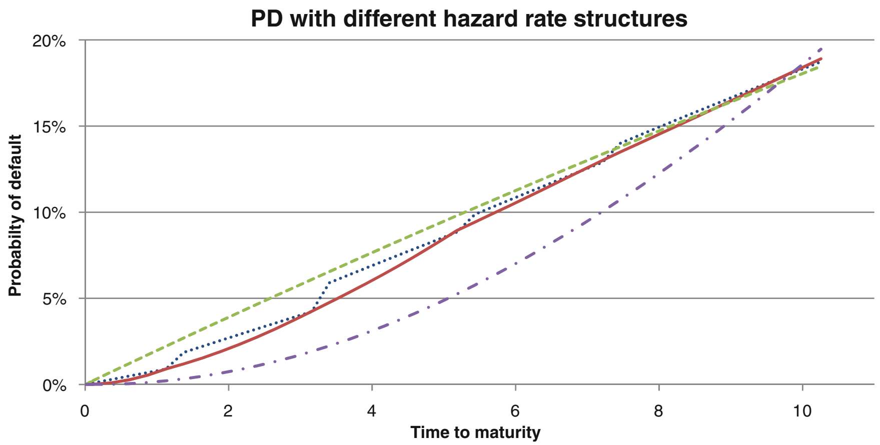

Fig. 2.8 The probability of default with different hazard rate structures, where the *red straight line* stands for the partial linear approach, the *blue dotted line* for the partial constant method, the *green disrupted line* for the constant (for the ten-year contract) and the *purple*, *dashed line with dots* represents the linear methods (again for the ten-year contract) implied from the CDS market data for BASF on 2011-11-11

also some disadvantages. In the linear case, for example, we find the property that $P(t < s | t \geq r) = \exp(-\lambda(s - r))$ i.e. the probability of default in a given time $(r - s)$ always stays the same. This does not apply in any of the other models.

Of course, it is possible to apply other hazard rate structure terms such as a quadratic polynomial and higher. However, the results show that a higher degree does not lead to a more precise pricing.

In the following Fig. 2.8 we combined all different models. Again, we see that the partial linear case is the combination of the linear and the partial constant approach. We discuss the advantages and disadvantages later along with the results.

2.4 Multi Curve Approach

In this chapter, we briefly discuss the differences in the yield curves before and after the financial crises. As an effect of the financial crises of 2007, the need for a multi curve approach grew. In other words, using a single interest rate curve for discounting as well as for forward rate calculation regardless the tenor was not adequate anymore. This was due to the fact that the basis spread quoted on the market had increased tremendously as demonstrated in Fig. 2.9. The plot shows the difference in prices between a basis swap for a three months EURIBOR against a six months EURIBOR basis swap, both with a maturity of five years. We see the price difference was negligible until the 3rd quarter of 2007. However, with the

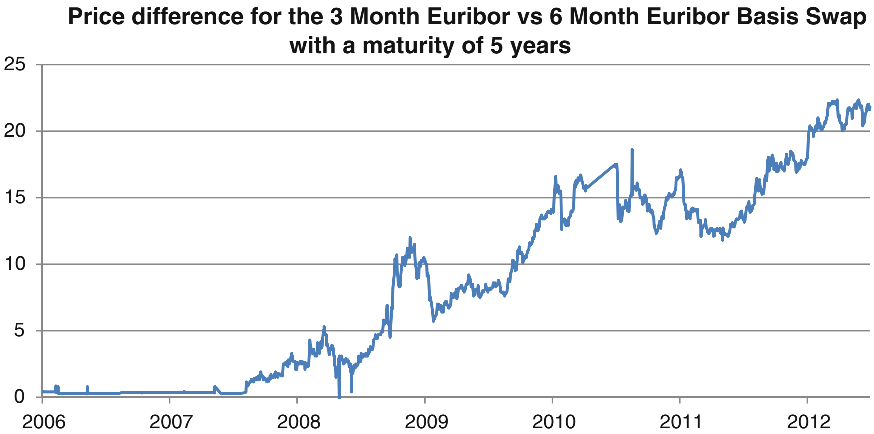

Fig. 2.9 This graph pictures the historical price differences between the three months EURIBOR basis swap with a maturity of five years against the six months EURIBOR basis swap with a five year maturity

beginning of the credit crunch, its value began increasing and has not stopped since, i.e. it shows the increase in liquidity between these two tenors.

This effect can also been observed if we take a look at basis swaps with different tenors and maturities as displayed in Fig. 2.10. We recognise that the basis swaps spread is the highest for short term contracts as well as for contracts of which the

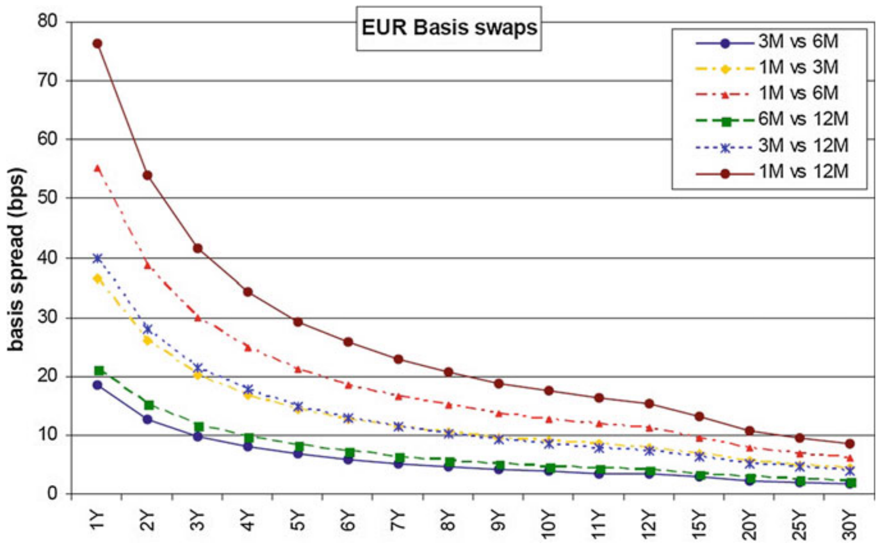

Fig. 2.10 This figure from Bianchetti (2010) demonstrates the difference in the swap rates due to their tenor and maturity

tenors are further apart (time wise) from another. The spreads vary from nearly 80 bps for a one-month versus twelve months (1 M vs. 12 M) basis swap contract with a maturity of one year to about 2 bps for three months against six months (3 M vs. 6 M) basis swap contract with a maturity of thirty years.

All these observable effects are due to a higher counterparty risk as well as a higher liquidity risk after the credit crunch of 2007.

Pre-crises Market Practice for yield curve construction

The standard market practice before the credit crunch was based on a single curve approach. In other words, only one valid yield curve was used for discounting as well as for forward rates calculations. The procedure is to gain one unique yield curve from the most liquid interest rate products via a bootstrapping method. For example, in the Eurozone it was very convenient for the short term to use the EURO deposits, i.e. the EURIBOR spot rates with a maturity up to one year, for the medium term FRAs/futures/swaps on the three months EURIBOR and for the long term, maturities longer than two years, swaps on the six months EURIBOR. Then the forward rates r_f in time t for the time interval $[t_1, t_2]$ could be calculated via

$$r_f(t; t_1, t_2) = \frac{B_z(t, t_1) - B_z(t, t_2)}{\tau_x(t_1, t_2)\, B_z(t, t_1)}$$

where $B_z(t, t_1)$ is the price for a zero bond at time t expiring in time t_1 and $\tau_x(t_1, t_2)$ being the year fraction between t_1 and t_2 using the day count convention x.

Post-crises Market Practice for yield curve construction

The above approach is no longer valid, since it fails to take several effects into consideration. First, the single curve approach does not take the basis swaps into account, which are no longer negligible as seen in Fig. 2.9. Secondly, the approach does not consider the segmentation of the interest rate market into sub-areas that correspond with different instruments with distinct underlying tenors such as one, three, six or twelve months. We still assume the arbitrage free discounting must be unique.

The procedure to gain multiple curves is first to gather a unique discounting curve $\mathrm{r_d}$, which for example can be the same as the curve we gain from the single curve approach. Bianchetti (2010) writes about the construction of the discounting curve as follows "we stress that the key first point in the procedure is much more a matter of art than of science, because there is not a unique financially sound recipe for selecting the bootstrapping instruments and rules". Subsequently, we need to build distinct forwarding curves $\mathrm{r_{f_i}}$ for different tenors i, which typically are one, three, six or twelve months. The bootstrapping can be done for each forward curve by using the interest rate products with the same tenor (as the desired forward rate curve) and under the consideration of the unique discounting curve. Finally the basis swap spreads between the different curves can be calculated as follows

$$BS_{id}(t, t_1, t_2) = \frac{B_{z_d}(t, t_2)}{B_{z_i}(t, t_2)} \frac{B_{z_i}(t, t_1) - B_{z_i}(t, t_2)}{B_{z_d}(t, t_1) - B_{z_d}(t, t_2)}$$

where BS_{id} is the multiplicative basis swap spread between the curve with tenor i and the discounting curve. We gather the additive basis swap spread very easily via

$$BS'_{id}(t, t_1, t_2) = r_{f_d}(t, t_1, t_2)\,[BS_{id}(t, t_1, t_2) - 1]$$

with r_{f_d} as the forward rate using the discounting curve. It is obvious, that in the single curve approach $BS_{id}(t, t_1, t_2) = 1$ and $BS'_{id}(t, t_1, t_2) = 0$.

Because the multi curve approach strongly depends on the discounting curve, several practices for the construction of the discounting curve exist in the market. According to Bianchetti (2010), there are mainly two different methods in the market. On the one hand, some market participants use the "pre-crisis" curve for discount i.e. spot, FRA/futures (three months tenor) and swap (six months tenor) basis, which we refer to as the "single curve". On the other hand, other participants apply the OIS (overnight index swap) curve, which we consider to be the best approximation for a risk free counterpart. In Fig. 2.11 we see the differences between these two curves. With this figure we can first support the "pre-crises" thesis, meaning there was no need for a multiple curve approach. If you compare the top lines in Fig. 2.11, we see that they are nearly the same, i.e. no basis swap spread. Secondly, we can encourage the tremendous effect of the basis swap spread after the crises and the need for an adequate discussion about the proper discount

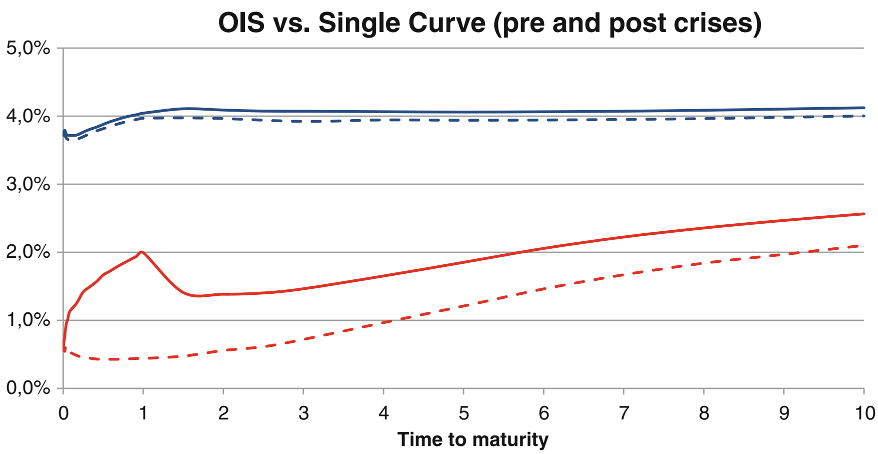

Fig. 2.11 These are examples of interest rate term structures before and after the financial crises with the OIS and the single curve approach, where the pre-crises curves are *blue* ones with the OIS method curve being interrupted and the after crisis are *red* with the OIS method curve again being interrupted

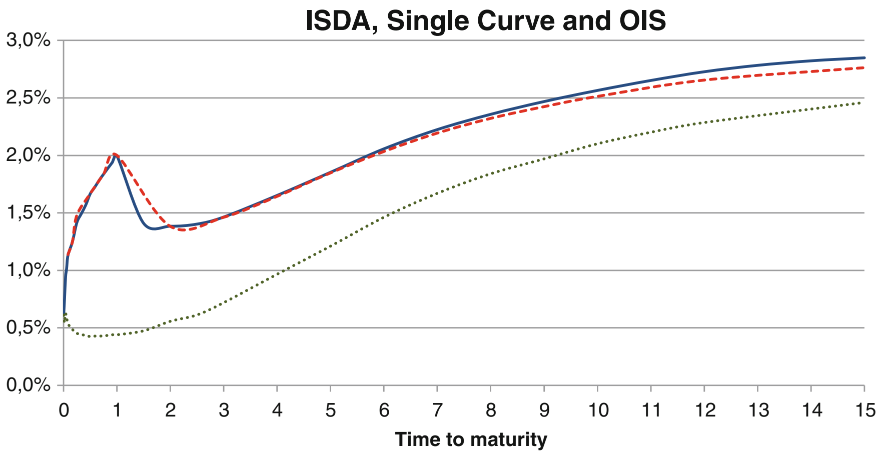

Fig. 2.12 We plotted the different interest rate structures on the 2011-12-21, where the *blue line* is the SC approach, the *red interrupted line* the ISDA curve and the *green dotted* the OIS approach

(or risk-free) yield curve. Furthermore, it is worth noting that the curves after the crises are not monotonic increasing functions anymore.

In this dissertation, we take a look at the influence of different discounting curves on the CDS evaluation. Most CDS deals are based on the ISDA convention with their "own" ISDA interest rate, which is published every day. This curve is based—at least for the euro discounting curve—on the spot rates of the EURIBOR up to one year and on the swap rates for maturities longer than one year. This is similar to the single curve and as we observe in Fig. 2.12, these yield curves are almost the same. Their differences are probably caused by different time points, since the ISDA curve is based on the 11.00 o'clock Frankfurt values for the spot rates and the 16.00 o'clock Frankfurt values for the swap rates and our single curve is based on end of day data from Frankfurt. The OIS curve represents the rate that can be seen as the most *risk-less*[5] curve. Therefore, the OIS curve can be seen as the curve in which the counterpart is a central counterpart. If we compare the derived prices using the same hazard rate structure but different interest rate curves for discounting, we can interpret these differences as the price for the counterparty risk.

2.5 Data Set

We collected end of day CDS quotes from all members or constituencies of the iTraxx Europe series 15 with maturities of one, three, five, seven and ten years. That means we collected end of day quotes from up to 625 securities per day from

[5]Due to the experience of the financial crisis we circumvent the term risk-free.

Fig. 2.13 This graph shows quoted market spreads for the five year iTraxx Europe series 15 with a contractual spread of 100 bps

Thomsen Reuters via Datastream. We gathered the end of day quotes for the iTraxx Europe series 15 indices itself from Markit. Our first day of observation is 2011-03-21 and the last is 2012-02-24, the development of five-year iTraxx Europe series 15 can be seen in Fig. 2.13. On 2011-03-21 the iTraxx indices were set up, meaning we collected nearly the whole first year of all CDS quotes for the indices and their constituencies. End of day quotes means the last traded quote on that particular trading day and thus the timestamps are not identical for all quotes. Until the iTraxx Europe series 16 started on 2011-09-20, the iTraxx Europe Series 15 was the current iTraxx index or as it is called the “on the run” index. It has been mentioned in several papers that the CDS market is the most liquid for contracts with a maturity of five years. In Fig. 2.14 we show the money invested in the 5-year iTraxx Europe series 15 to support the statement that the indices are the most liquid while they are “on the run”.

It is important to mention, that a slight problem concerning the maturity of the CDS deal exists. The maturity date of the indices obviously keeps the same until it is reached, e.g. 2014-06-20 for the indices with a runtime of three years starting on 2011-03-021. The maturity date for the index members always differ, since there is a roll every quarter. In other words, the CDS contracts for the constituencies never reach their maturity date. This phenomenon is also known as constant maturity contracts. For example, let us assume the CDS of a member such as Kraft Food with a runtime of five years. Then its maturity date is 2016-06-20 of the CDS quote during the dates 2011-03-20 and 2011-06-19. However, the maturity date changes to 2016-09-20 for the quotes from 2011-06-20 to 2011-09-19 due to the roll on 2011-06-20. Therefore, only in the first quarter of the index all the CDS quotes from the members have the same maturity as the index. In our case, that is from 2011-03-21 to 2011-06-19. Thus, we have to find a way to build an adequate

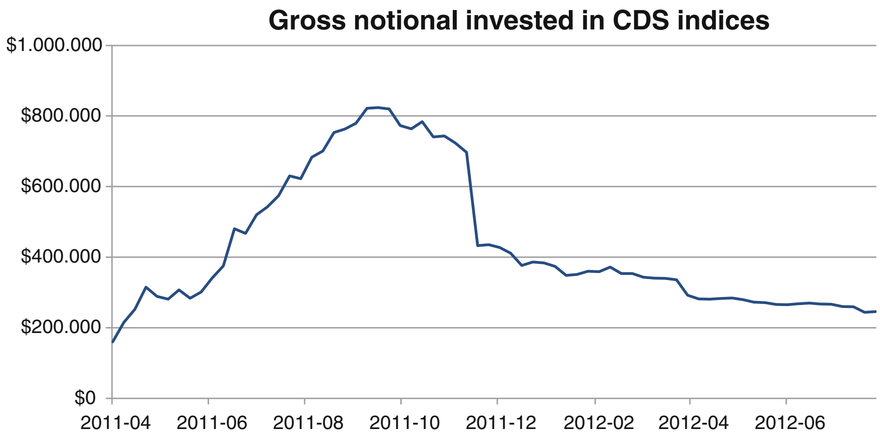

Fig. 2.14 We display the gross notional amount invested in the CDS indices measured in million USD for the iTraxx Europe series 15

market quote for the constituencies for the time interval 2011-06-20 to 2012-02-24. Therefore, we always need a shorter maturity of the index member to be able to interpolate between market quotes. That is the reason why the one-year quote is necessary for each member, even though there is no iTraxx index with one-year maturity.

As noted above, the question of the proper CDS quote rises. For example, what is the proper quote for a contract with a maturity of four years and nine months, if we only have a quote for a maturity of five years and three months and a quote for a maturity of three years and three months? There are several ways to deal with this problem and we try three different approaches to handle it. First, we use the constant maturity quotes to price the CDS even if the maturity of the index and the maturities of the index members differ. Secondly, we apply the constant maturity quote again, but this time we only extract the hazard rate structure from its original maturity and apply it to the maturity of the index. For example, let us assume a constant hazard rate on the three-year index with its maturity on 2014-06-20. On 2011-11-11 the maturity of the constant three-year CDS quote is on 2014-12-20. According to this market data approach we imply a constant hazard rate of the members calculated with their correct maturity (2013-12-20), but we price the CDS by applying the hazard rate to the maturity of the index (2014-06-20). We refer to this market data approach as “the constant maturity applying to the index maturity”. Ultimately, it is possible to interpolate in between the two closest runtimes e.g. using the five and three year constant maturity CDS contract to imply the market quote for a maturity of four years. An interpolation can have different degrees for the polynomial, but for reasons of simplicity, we apply a linear interpolation. Further research could investigate the difference of higher-degree interpolation.

2.6 Results

We calculate the CDS prices for the indices as well as for the members in several ways. We use two different interest rate curves, single curve and OIS as mentioned above. We utilise the following different hazard rate structures: constant, partial constant, linear and partial linear, as described earlier in Sect. 2.3. Since we have an additional problem with the proper market data for the CDS of the index member, we use yet another three different approaches. The market data approaches are (i) the original constant maturity, (ii) constant maturity applying to the index maturity and (iii) the linear interpolation between two constant maturities spreads with different maturities. To sum up, we calculate CDS prices with eight different methods for all indices (two interest rate and four hazard rate structures) and CDS prices with 24 different methods for all members (additionally three methods for the market data). Then we determined the average of all CDS members to gain an index replication. As an alternative index replication we apply the members' median. Since this approach leads to values that are of much greater value in comparison to the average, we do not want to discuss this approach in detail. That means, we calculated the replication as follows

$$R_{index}(t, ir, hr, md) = \frac{1}{125}\sum_{n=1}^{125} CDS_n(t, ir, hr, md),$$

where t is the date, ir the used interest rate, hr the hazard rate structure and md the market data. Then we derived for each day

$$\hat{R}_{index}(t, ir, hr, md) = (Index(t) - R_{index}(t, ir, hr, md))^2$$

and for the whole time of observation

$$\tilde{R}_{index}(ir, hr, md) = \frac{1}{N}\sum_{t=1}^{N} \hat{R}_{index}(t, ir, hr, md).$$

Before we go into detail on the replications results, let us analyse the impacts of the various hazard rates structures as well as the differentiation between the interest rate curves on the price of the CDS indices.

First, let's take a look at the difference hazard rate structures, i.e. the relations between index prices determined with different hazard rate structures. With a larger maturity, the absolute differences to the prices rise between the different structures, this finding is not surprising. Obviously this is due to the fact that up to three years the methods are not so different or even the same as in the constant and the partial constant or as in the linear and the partial linear case. Further, the longer the maturities are, the more noticeable the differences in the approaches become. If we consider our example of BASF on 2011-11-11 again and compare the constant and

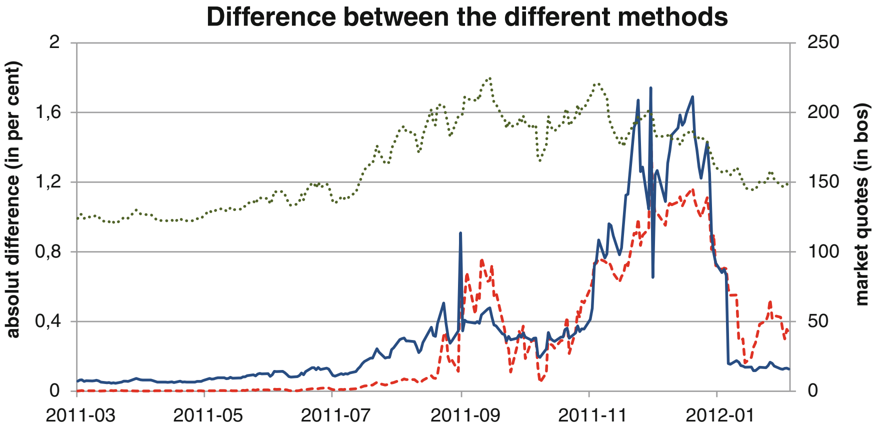

Fig. 2.15 This figure shows the absolute difference between the index calculated with different hazard rates, where the ten year iTraxx Europe difference is represented by the *blue line*, the five year iTraxx Europe difference is displayed by the *red interrupted line* and the *green dotted line* is the market quote of the ten year iTraxx Europe. Note that the CDS prices are denoted in percent. Therefore, this difference is an absolute value and not a relative one

the partial constant hazard rate structure for a ten-year contract, we notice that in case of the constant hazard rate, the probability of default (PD) is about 1.907 % in the first year. But in the partial constant approach the PD over the same time horizon is only 0.79 %. These results show that early payments are weighted more heavily in the constant hazard rate structure. Therefore, their prices are (slightly) higher than in the partial constant approach. Further, in the partial constant hazard rate approach, in the first year, the PD is extracted from the one-year CDS quote and not from the ten-year CDS quote. The following two graphs support this statement. In such graphs, we display the (maximum) absolute difference between the indices prices of the five and ten year contract.

The differences in the iTraxx prices are rather large (up to 1.76 %) as shown in Fig. 2.15. From all hazard rate structures (constant, partial constant, linear, partial linear) we observe, that the constant hazard approach most often (in about 76 % of all results) leads to the highest calculated price, whereas the linear case mostly bears the lowest price (about 62 %). For maturities greater than five years the price always (!) is the highest if we assume a constant hazard rate and if the lowest price for these maturities is calculated either with a linear hazard rate (86 % of all dates) or with a partial linear hazard rate (14 %, or the rest of the dates). This shows that if we only use a single market quote, which is the case for the constant or the linear hazard rate structure, we subsequently gain a greater difference in calculated CDS prices, i.e. the choice of hazard rate term structure becomes more crucial.

In the next step, we examine the different interest rate curve for the iTraxx contracts. First, we draw the attention to the differences within the interest curves. There are two main facts in the behaviour of the curves. First, they differ the most in

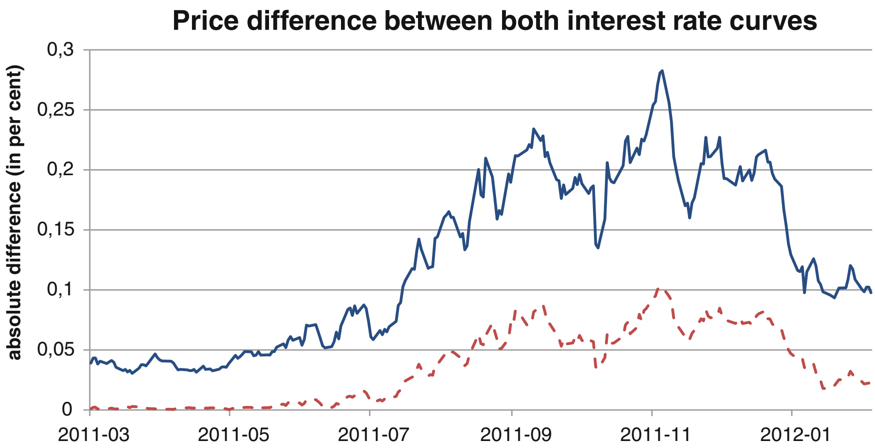

Fig. 2.16 This figure plots the absolute difference between the calculations with both interest curves, where the *blue line* represents the difference for the ten-year maturity and the *red interrupted line* for the five-year maturity

the first few months as we observe in Fig. 2.12. Secondly, the curves converge to each another the longer their runtime. We assume the prices to differ more for the shorter maturities, since the curves are varying the most. Therefore, it is surprising that the difference between the index prices with the single curve and the OIS curve is increasing with maturity. Nevertheless, the prices' difference between the interest rate curves, under the assumption of the same hazard rate structure, only leads to an absolute value of 0.31 % (i.e. 31 bps). With maturities greater than five years, the OIS approach always delivers a lower price than the single curve approach. Figure 2.16 demonstrates the absolute difference between the prices for five and ten year maturities and also shows the increase of the price differences during longer maturities.

If we loosen the assumption that we compare the price difference for different interest curves with the same hazard rate structure, i.e. we compare all eight prices with each other, the price range grows (up to 1.97 i.e. or nearly two percent) and with longer maturities the OIS approach always delivers a lower price than the single curve approach does.

The choice of interest rate curve also affects the implied PD of the referenced underlying if we assume the same CDS spread. As we can see in Fig. 2.17, where we calculated the difference in the implied PD until maturity under the assumption of a linear hazard rate, the PD is always higher under the single curve assumption. Since we can argue that the OIS curve is more likely to be "the interest rate with the lowest risk involved", we can interpret this PD difference as the counterparty risk or, more precisely, the risk that the reference entity as well as the counterpart does default simultaneously. This topic can be widely discussed but it is not the matter for discussion in this dissertation.

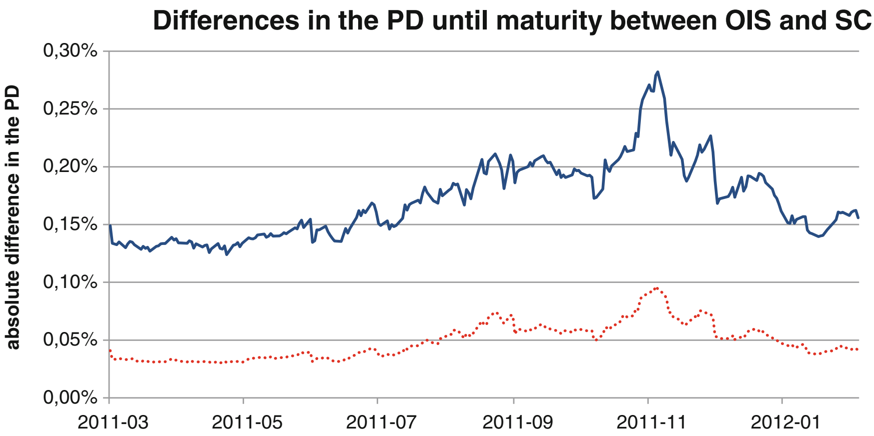

Fig. 2.17 This figure displays the difference in PD until maturity between a determination with the OIS or with the single curve approach, where the *blue line* stands for the ten-year maturity and the *red interrupted* for the five-year contract

Lastly, we analyse the different approaches combined with the topic the proper market quote for the index members according to the price differences between the index and the mean of the index member prices. Before we go into the results in detail, it is worth mentioning that all methods lead to more or less acceptable prices. Nevertheless, some methods lead to an absolute percentage difference of about 2.6 %, i.e. index price being about 90.36 and the replication being about 92.72. As seen before, that means it is not negligible which assumption we consider if we want to derive a CDS price. The following results are all based on the comparison between the index results and the results of replication. The replication values differ more the longer the maturities of the contracts are. The difference $\hat{R}$ ranges from 8 bps on average over all approaches for the three-year contract to 69 bps on average for the ten-year contract. Concerning the optimal market data we would assume, that the constant maturity approach is the worst, since it does not fit the maturity of the index, followed by the interpolated market quote, due to the inaccuracy of the market quotes. We would also assume that the implied hazard rate from the constant maturity quotes applied to the index maturity will lead to the most precise result. However, this presumption only fits either for maturities no longer than three years or if we exclude the linear hazard rate approach from our result set. This is not an absurd conclusion, because the results of the linear hazard rates distort the overall results. The results reveal that the longer the maturity, the better interpolated market quotes, followed by the constant maturity market quotes and finally the implied hazard rate applied to the index maturity. We can observe such results in Table 2.1, where we calculated the $\tilde{R}$ for all different approaches including crossings. With “crossings”, we mean different approaches for the evaluation of the index and index members.

Table 2.1 Here the average values in bps for different maturities are shown. Obviously represent smaller values a better approximation to the pricing index. Each value is based on 242 points of observation

Market data (in bps)	3 Years (n = 242)	5 Years (n = 242)	7 Years (n = 242)	10 Years (n = 242)
Total average	7.90	16.81	37.11	68.91
Const. maturity	14.20	16.80	30.85	66.96
Implied to index mat.	3.99	19.37	47.31	74.33
Interpolated Quotes	5.51	14.28	33.18	65.44

Further, we compare the deviation if the indices and the members are subject to the same hazard rate structure ignoring the market data quote for the members. The data reveal that the approximation with a linear hazard rate structure is without any doubt the worst as we see in Table 2.2. For maturities up to five years the constant hazard rate structure seems to be the optimal choice, for longer maturities the additional market information—the "partial" approaches—deliver more precise results.

At last, we look at different hazard rates structures for the indices and the members, e.g. the constant hazard rate is applied for the index but the members are evaluated with a partial linear hazard rate structure. The results are mapped in Table 2.3.

If we focus on hazard rate structure for the members, we imply that the hazard rate structures being more complex always provide a lower $\tilde{R}$ and therefore a better approximation to the index than the approach, which is simpler. For the approximation of the index, this means it is better to apply all CDS data information available. In our case, this means the "partial" methods dominate the "non-partial" methods. For example, with a partial constant hazard rate structure for the index, the partial linear approximation is better for the members than the linear approximation (28.31–36.51). For the various index hazard rate structures, the effect is not as dominant as before. There still exist cases where a "partial" method dominates the "non-partial" method. For example, if we apply the partial linear hazard rate

Table 2.2 Here, the average values in bps for different hazard rate term structures are shown. Smaller values obviously represent a better approximation to the pricing index. Each value is based on 242 points of observation

Hazard rate index (in bps)	3 Years (n = 242)	5 Years (n = 242)	7 Years (n = 242)	10 Years (n = 242)
Total average	7.90	16.21	35.92	65.95
Constant	5.74	12.87	32.51	67.67
Linear	9.74	21.99	49.75	79.20
Partial constant	8.43	14.04	28.42	60.55
Partial linear	7.70	15.96	32.99	56.38

Table 2.3 The different hazard rate structures for the index and the members are shown with their average difference over all maturities and the maximum difference during our observation period. Obviously smaller values represent a better approximation to the pricing index

Index hazard rate	Members hazard rate	$\tilde{R}$ (n = 888)	Max. value
Constant	Constant	29.70	184.71
Constant	Linear	36.49	184.84
Constant	Partial constant	28.45	187.95
Constant	Partial linear	28.94	184.31
Linear	Constant	36.51	204.73
Linear	Linear	40.17	209.36
Linear	Partial constant	31.71	193.55
Linear	Partial linear	31.51	189.91
Partial constant	Constant	29.63	186.71
Partial constant	Linear	36.51	186.85
Partial constant	Partial constant	27.86	189.96
Partial constant	Partial linear	28.31	186.32
Partial linear	Constant	40.69	214.66
Partial linear	Linear	39.00	244.47
Partial linear	Partial constant	29.22	191.13
Partial linear	Partial linear	28.26	187.48

structures for the members, then the index replication is more precise for the index price evaluated with a partial constant hazard rate structure (28.31) than for the index price evaluated with a constant index hazard rate (28.94). There also exist some counter examples such as the constant hazard rate for the members and the constant respectively partial constant hazard rate structure for the indices. Nevertheless, all different combinations have about the same magnitude of possible maximum differences per day. In other words, the choice of the hazard rate structure for the CDS valuation does not protect for higher difference during the replication of a CDS index. These differences between replication and index can still be caused by their difference in liquidity or simply by the quality of the data.

2.7 Conclusion

The choice of the hazard rate structure can lead to different CDS prices. The consideration of further CDS market data can lead to better approximations and a more realistic mapping of the probability of default. On the one hand, these approaches need more valuation time and market data. The problem of market data with different maturity is that the quality and the liquidity can differ, which always remain a doubt to the calculated results. On the other hand, the results show that the valuation with a constant hazard rate structure—based on a single CDS market quote—is a solid implication for quick information.

According to our research, for CDS contracts with a different maturity than the available constant maturity contracts, the best approximation is the linear interpolation between available CDS market data quotes. If we do not assume a linear hazard rate structure, it is even better to use the implied hazard rate from a market quote with a longer maturity to evaluate the CDS contract. For the index with a runtime of three years, the best approximation is to use the constant three-year maturity CDS market quote for the constituencies.

The selection and determination of an interest rate curve can be important especially for longer maturities. The interest rate curve also determines the probability of default and the counter party risk, which shall be of special interest in the combination of a central counterpart.

We observe that CDS pricing valuation is not as straightforward as one might assume. Each approach has its legitimation and there is no single “correct” way to price a CDS. All impacts are more important and more noticeable for longer maturities. The results show, that the longer the maturity, the more complex is the valuation—complex in the sense of hazard rates (partial constant or even partial linear) as well as market data. Having said this, an even more complex approach such as cubic spline interpolation and a quadratic hazard rate structure could be an interesting topic for future research as well as a the usage of recovery models.

References

S. Amaraoui, S. Hitier, Optimal stochastic recovery for base correlation. BNP Paribas (2008)
M. Bianchetti, Two curves, one price: pricing & hedging interest rate derivatives decoupling forwarding and discounting yield curves. Risk Mag. **10**(8), 70–75 (2010)
F. Black, M.S. Scholes, The pricing of options and corporate liabilities. J. Polit. Econ. (7), 637–654 (1973)
M. Böttger, A. Guthoff, T. Heidorn, Loss given default—Modelle zur schätzung recovery rates. Working paper (2008)
C. Cao, F. Yu, Z. Zhong, The information content of option-implied volatility for credit default swap valuation. J. Finan. Markets **13**(3), 321–343 (2010)
P. Carr, L. Wu, Stock options and credit default swaps: a joint framework for valuation and estimation. J. Finan. Econometrics **8**(4), 409–449 (2010)
M. Krekel, Pricing distressed CDOs with base correlation and stochastic recovery. Working paper (2008)
H. Li, On models of stochastic recovery rate for base correlation. MPRA (2009)
R.C. Merton, On the pricing of corporate debt: the risk structure of interest rates. J. Finance **29**(2), 449–479 (1974)
D. O’Kane, R. McAdie, Trading the basis. Risk Mag. (2001)
D. O’Kane, S. Sen, Up-front credit default swaps. Lehman Brothers fixed income quantitative credit research (2003)
D. O’Kane, S. Turnbull, Valuation of credit default swaps. Lehman Brothers quantitative credit research quarterly (2003)
M.F. Schmidt, Different approaches on CDS valuation—an empirical study. Unpublished working paper (2014)

http://www.springer.com/978-3-319-45969-1

Pricing and Liquidity of Complex and Structured Derivatives
Deviation of a Risk Benchmark Based on Credit and Option Market Data
Schmidt, M.
2016, XVII, 114 p. 32 illus., 16 illus. in color., Softcover
ISBN: 978-3-319-45969-1