

Preface

Overview

The advent of the digital information and communication era has resulted in image processing and computer vision playing more important roles in our society today. These roles include creating, delivering, processing, visualizing, and making decisions from information in an efficient and visually pleasing manner.

The term *digital image processing* or simply *image processing* refers to comprehensively processing picture data by a digital computer. The term *computer vision* refers to computing properties of the three-dimensional world from one or more digital images.

The theoretical bases of image processing and computer vision include mathematics, statistics, signal processing, and communications theory. In a wide variety of theories for image processing and computer vision, optimization plays a major role. Although various optimization techniques are used at different levels for those problems, there has not been a sufficient amount of effort to summarize and explain optimization techniques as applied to image processing and computer vision.

The objective of this book is to present practical optimization techniques used in image processing and computer vision problems. A generally ill-posed problem is introduced and it is used to show how this type of problem is related to typical image processing and computer vision problems.

Unconstrained optimization gives the best solution based on numerical minimization of a single, scalar-valued *objective function* or *cost function*. Unconstrained optimization problems have been intensively studied, and many algorithms and tools have been developed to solve them. Most practical optimization problems, however, arise with a set of constraints. Typical examples of constraints include (a) prespecified pixel intensity range, (b) smoothness or correlation with neighboring information, (c) existence on a certain contour of lines or curves, and (d) given statistical or spectral characteristics of the solution.

Regularized optimization is a special method used to solve a class of constrained optimization problems. The term *regularization* refers to the transformation of an objective function with constraints into a different objective function, automatically reflecting constraints in the unconstrained minimization process. Because of its simplicity and efficiency, regularized optimization has many application areas, such as image restoration, image reconstruction, and optical flow estimation.

Optimization-Problem Statement

The fundamental problem of optimization is to obtain the best possible decision in any given set of circumstances. Because of its nature, problems in all areas of mathematics, applied science, engineering, economics, medicine, and statistics can be posed in terms of optimization.

The general mathematical formulation of an optimization problem may be expressed as

$$\min_{x \in R^N} f(x) \quad \text{subject to } x \in C, \quad (1.1)$$

where $f(x)$ represents the objective function, C the constraint set in which the solution will reside, and R^N the N -dimensional real space.

Various optimization problems can be classified based on different aspects. At first, classification based on the properties of the objective function $f(x)$ is:

1. Function of a single variable or multiple variables,
2. Quadratic function or not, and
3. Sparse or dense function.

A different classification is also possible based on the properties of the constraint C as:

1. Whether or not there are constraints,
2. Defined by equation or inequality of constraint functions, i.e., $C = \{x | c(x) = 0\}$ or $C = \{x | c(x) \geq 0\}$, where $c(x)$ is termed the constraint function, and
3. The constraint function is either linear or nonlinear.

If, for example, we want to reduce random noise from a digital image, a simple way is to minimize the extremely high-frequency component while keeping all pixel intensity values inside the range $[0, 255]$. In this case, the objective function of multiple variables represents the high-frequency component. The range of pixel intensity values plays a role in inequality constraints.

Optimization for Image Processing

Various optimization techniques for image processing can be summarized as, but are not limited to, the following:

1. Image quantization: The optimum mean square (*Lloyd-Max*) quantizer design.
2. Stochastic image models: Parameter estimation for auto-regressive (AR), moving average (MA), or auto-regressive-moving average (ARMA) model.
3. Image filtering: Optimal filter design and Wiener filtering.
4. Image restoration: Wiener restoration filter, constrained least squares (CLS) filter, and regularized iterative method.
5. Image reconstruction: Convolution/filtered back-projection algorithms.
6. Image coding: Energy compaction theory and optimum code-book design.

Optimization for Computer Vision

Some examples of optimization techniques for computer vision are summarized as:

1. Feature detection: Optimal edge enhancer, ellipse fitting, and deformable contours detection.
2. Stereopsis: Correspondence/reconstruction procedures, three-dimensional image reconstruction.
3. Motion: Motion estimation, optical flow estimation.
4. Shape from single image cue: Shape from shading, shape from texture, and shape from motion.
5. Recognition
6. Pattern matching

Organization of This Book

This book has five self-contained parts. In Part I, Chap. 1 introduces the scope and general overview of the material. Chapter 1 gives an introduction into ill-posed problems. This chapter also discusses practical reasons why many image processing and computer vision problems are formulated as ill-posed problems. Chapter 1 also presents typical examples of ill-posed problems in image processing and computer vision areas. Chapter 2 discusses different techniques to select regularization parameter.

Part II summarizes the general optimization theory that can be used to develop a new problem formulation. Practical problems are solved using the optimization formulation. Chapter 3 presents a general form of optimization problems and summarizes frequently used terminology and mathematical background. Chapters 4 and 5 describe in-depth formulation and solution methods for unconstrained and

constrained optimization problems, respectively. Constrained optimization problems are more suitable for modeling real-world problems. This is true because the desired solution of the problem usually has its own constraints.

In Part III, we discuss regularized optimization, or simply regularization, that can be considered a special form of a general constrained optimization. In Chap. 6, frequency-domain regularization is discussed. Chapters 7 and 8 describe iterative type implementations of regularization and fusion-based implementation of regularization.

Part IV provides practical examples for various optimization technique applications. Chapters 9, 10, 11, and 12 give some important applications of two-dimensional image processing and three-dimensional computer vision.

Appendices summarize commonly used mathematical background.

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