Chapter 2
A Formal Model of Maps as a Fundamental Type

The algebra defining the traditional spatial data types, their associated operations, and topological predicates form the the foundation of spatial data processing in spatial processing systems. This algebra also forms the foundation of the Map Framework, as presented in this book.

2.1 Spatial Data Models

We distinguish two *generations* of spatial data types. The types of the first generation have a simple structure, and are known as the *simple spatial types* [2, 7, 9, 11]. The simple spatial types define three structures: *points*, *lines*, and *regions*. A *simple point* describes an element of the Euclidean plane $\mathbb{R}^2$. A *simple line* is a one-dimensional, continuous geometric structure embedded in $\mathbb{R}^2$ with two end points. A *simple region* is a two-dimensional point set in $\mathbb{R}^2$ and is topologically equivalent to a closed disk. Figure 2.1 shows examples of the simple spatial types.

The simple spatial types allow for the representation of zero dimensional, one dimensional, and two dimensional structures in geographic reality, but lack two key properties. First, many objects in geographic reality have multiple *components*, but are intuitively a single object. For example, one may wish to store the shape of the country of Italy in a spatial system; Italy contains islands. A simple region cannot represent Italy's mainland and islands as a single data object. Second, the simple types do not provide type closure guarantees. For example, the intersection of two simple regions may result in multiple disconnected components, as shown in Fig. 2.2; thus, the intersection of two simple regions is not necessarily a simple region. Achieving type closure is a fundamental goal of the Map Framework, precisely to avoid such problems.

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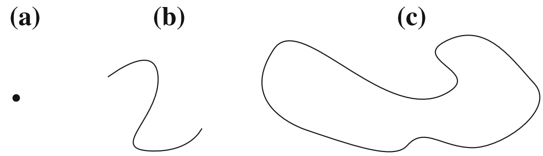

Fig. 2.1 A simple point (**a**), a simple line (**b**), and a simple region (**c**)

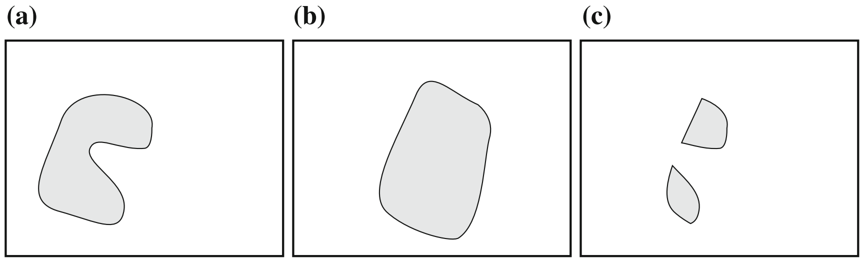

Fig. 2.2 Two simple regions (**a** and **b**), and the result of their intersection (**c**). The result is not a single simple region

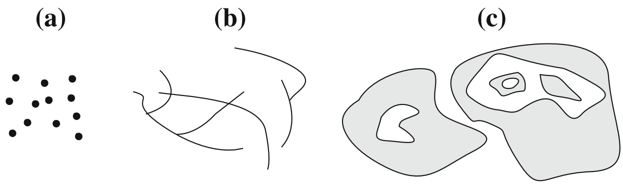

Fig. 2.3 A complex point (**a**), a complex line (**b**), and a complex region (**c**)

Additional requirements of applications as well as needed closure properties of operations led to the second generation of *complex spatial data types* [2, 21, 23] illustrated in Fig. 2.3 ([20] for a survey).

A *complex point* is a finite point collection (e.g., the positions of all lighthouses in Florida). A *complex line* is an arbitrary, finite collection of one-dimensional curves, i.e., a spatially embedded network possibly consisting of several disjoint connected components (e.g., the Nile Delta). A *complex region* permits multiple areal components, called *faces*, and holes in faces (e.g., Italy with its mainland and offshore islands as components and with the Vatican as a hole).

Two additional spatial data types for regions have been proposed as intermediate steps between simple and complex regions. These are, *composite regions* [3] and *simple regions with holes* [2, 6, 23]. A composite region can contain multiple faces, but no holes. In other words, a composite region consists of finitely many simple regions that are either disjoint, or meet at single points. Although holes are not allowed in composite regions, "hole-like" configurations can exist if two components of one region touch at a single point of their boundaries at two different locations (Fig. 2.4).

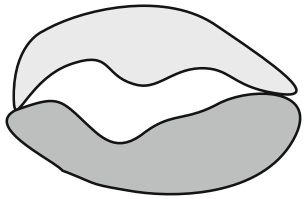

Fig. 2.4 Sample composite region with two components presenting a hole-like structure

A simple region with holes contains only a single face, with finitely many holes. The holes in a simple region with holes are allowed to meet at a point, but cannot form a configuration that causes the interior of the region to be disconnected. In other words, a hole-like structure cannot be formed by the holes in a simple region with holes. Intuitively, a simple region with holes is a complex region that has only a single face.

Each spatial type is defined such that it is made up of three parts: the *interior*, *boundary*, and *exterior*. Given a spatial object A, these components are indicated respectively as A°, ∂A, and A^-. For example, the boundary of a line is its endpoints, and its interior consists of the lines that connect the endpoints. The exterior of a line consists of all points in $\mathbb{R}^2$ that are not part of the interior or boundary. Similarly, the boundary of a region is the line that defines the region's border. The interior consists of all points that lie inside the region, and the exterior consists of all points that are not part of the boundary or interior. These concepts are required for the definition of topological relationships between spatial types (defined in the next section).

2.2 Topological Relationships

In the development of the Map Framework, we rely heavily on concepts of topology and topological relationships between spatial data types. The study of topological relationships between objects in space has been the subject of a vast amount of research [1–4, 6, 8, 9, 14–19, 21]. In the areas of databases and GIS, the motivation for formally defining topological relationships between spatial objects has been driven by a need for querying mechanisms that can filter these objects in spatial selections or spatial joins based on their topological relationships to each other, as well as a need for appropriate mechanisms to aid in spatial data analysis and spatial constraint specification.

Topological relationships indicate qualitative properties of the relative positions of spatial objects that are preserved under continuous transformations such as translation, rotation, and scaling. Quantitative measures such as distance or size measurements are deliberately excluded in favor of modeling notions such as connectivity, adjacency, disjointedness, inclusion, and exclusion. Attempts to model and rigorously define the relationships between certain types of spatial objects have lead to the development of three popular approaches: the *9-intersection model* [9], which is developed based on point set theory and point set topology, the *calculus based method* [2], which is also based on point set topology, and the *RCC model* [19], which utilizes spatial logic. Because the definitions of spatial objects are based on topological principles, and the inability of the calculus based method to identify a complete set of topological relationships, the 9-intersection model is typically used to model topological relationships between spatial objects in the field of spatial information systems.

The 9-intersection model of topological relationships characterizes the topological relationship between two spatial objects by evaluating the non-emptiness of the intersection between all combinations of the interior ($^\circ$), boundary (∂) and exterior ($^-$) of the objects involved. A unique 3×3 matrix, termed the *9-intersection matrix* (9IM), with values filled as illustrated in Fig. 2.5 describes the topological relationships between a pair of objects.

The 9IM is general in the sense that it is applicable to any spatial data types that maintain the topological concepts of interior, exterior, and boundary. Applying the model to different data types results in different sets of applicable topological relationships. For example, Fig. 2.6 depicts the 8 topological relationships between simple regions. There are 33 topological relationships between complex regions [21]. In the construction of a data type for maps, we will primarily refer to the topological relationships *disjoint* and *meet* between pairs of simple regions and pairs of complex regions. The 9IM for those relationships is identical for both simple and complex regions.

Various models of topological predicates based on the 9-intersection model using both *component derivations*, in which relationships are derived based on the interactions of all components of spatial objects, and *composite derivations*, in which relationships model the global interaction of two objects, exist in the literature. Examples

Fig. 2.5 The 9-intersection matrix for spatial objects A and B

$$\begin{pmatrix} A^\circ \cap B^\circ \neq \emptyset & A^\circ \cap \partial B \neq \emptyset & A^\circ \cap B^- \neq \emptyset \\ \partial A \cap B^\circ \neq \emptyset & \partial A \cap \partial B \neq \emptyset & \partial A \cap B^- \neq \emptyset \\ A^- \cap B^\circ \neq \emptyset & A^- \cap \partial B \neq \emptyset & A^- \cap B^- \neq \emptyset \end{pmatrix}$$

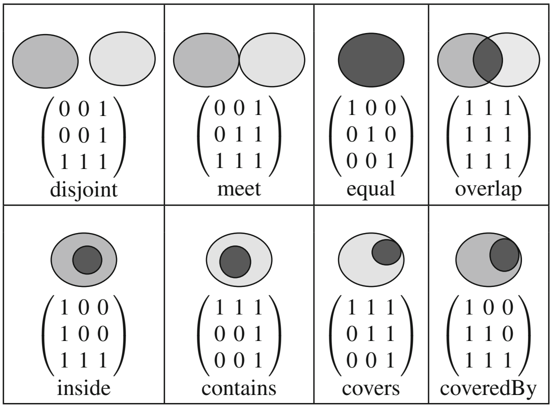

Fig. 2.6 The 8 topological predicates between simple regions. One object is *shaded dark* and the other *light* as in the *disjoint* relationship, whereas the shared areas have the *darkest shade*

of component derivations can be found in [3, 6]. In [6], the authors define topological relationships between regions with holes in which each of the relationships between all faces and holes are calculated. Given two regions, R and S, containing m and n holes respectively, a total of $(n + m + 2)^2$ topological predicates are possible. It is shown that this number can be reduced $mn + m + n + 1$; however, the total number of predicates between two objects depends on the number of holes the objects contain. Similarly, in [3], predicates between complex regions without holes are defined based on the topological relationship of each face within one region with all other faces of the same region, all faces of the other region, and the entire complex regions themselves. Given regions S and R with m and n faces respectively, a matrix is constructed with $(m + n + 2)^2$ entries that represent the topological relationships between S and R and each of their faces.

The most basic example of a composite derivation model (in which the global interaction of two spatial objects is modeled) is the derivation of topological predicates between simple spatial objects in [9]. This model has been used as the basis for modeling topological relationships between object components in the component models discussed above. In [21], the authors apply an extended 9-intersection model to point sets belonging to complex points, lines, and regions. Based on this application, the authors are able to construct a composite derivation model for complex data types and derive a complete and finite set of topological predicates between them, thus resolving the main drawback of the component derivation model.

More recently, it has been observed that composite models of topological relationships between spatial objects are *global*, in that they characterize an entire scene by a single topological relationship that may hide *local* information about the object's relationship [17]. The hiding of local information is expressed in two ways in global topological relationship models: through the *dominance problem*, and the *composition problem*. The dominance problem indicates the property that the global view exhibits *dominance* properties among the topological relationships as defined by the 9-intersection model. For example, while building roads between two adjacent countries, one might be interested to know that there is a disjoint island in one of the countries for which a bridge to the other country is required. The disjointedness in this case is overshadowed or dominated by the existing *meet* (adjacent) situation between the countries' mainlands. The composition problem expresses the property that a global topological predicate may indicate a certain relationship between two objects that does not exist locally. For example, consider two complex regions that have individual faces that satisfy the *inside*, *covers*, and *meet* predicates. Globally, this configuration satisfies the overlap predicate even though no faces overlap locally. These properties have been addressed through the *local topological relationship* models between composite regions [17] and between complex regions [18]. These approaches model a topological relationship between two multicomponent objects based on the topological relationships that exist between the components of the objects. Furthermore, it is shown that these models are more expressive than the global 9IM models in that they can distinguish all the topological scenes that the 9IM models can distinguish, plus many more.

2.3 An Informal Overview of Spatial Partitions

In this paper, we model maps as *spatial partitions*, as discussed in [10, 12–14]. The definition of spatial partitions is rather dense, so we begin by providing an intuitive description of them, and then present the formal definition in later sections.

A spatial partition, in two dimensions, is a subdivision of the plane into pairwise disjoint *regions* such that each region is associated with a *label* or *attribute* having simple or complex structure, and these regions are separated from each other by *boundaries*. The label of a region describes the thematic data associated with the region. All points within the spatial partition that have an identical label are part of the same region. Topological relationships are implicitly modeled among the regions in a spatial partition. For instance, neglecting common boundaries, the regions of a partition are always disjoint; this property causes maps to have a rather simple structure. Note that the *exterior* of a spatial partition (i.e., the unbounded face) is always labeled with the $\perp$ symbol. Figure 2.7a depicts an example spatial partition consisting of two regions.

We stated above that each region in a spatial partition is associated with a single attribute or label. A spatial partition is modeled by mapping Euclidean space to such labels. Labels themselves are modeled as sets of attributes. The regions of the spatial partition are then defined as consisting of all points which contain an identical label. Adjacent regions each have different labels in their interior, but their common boundary is assigned the label containing the labels of both adjacent regions. Figure 2.7b shows an example spatial partition complete with boundary labels.

In [10], operations over spatial partitions are defined based on known map operations in the literature. It is shown that all known operations over spatial partitions can be expressed in terms of three fundamental operations: intersection, relabel, and refine. Furthermore, the type of spatial partitions is shown to be closed under these operations, indicating that the type of spatial partitions is closed under all known operations over them. We will cover operations over spatial partitions in later chapters.

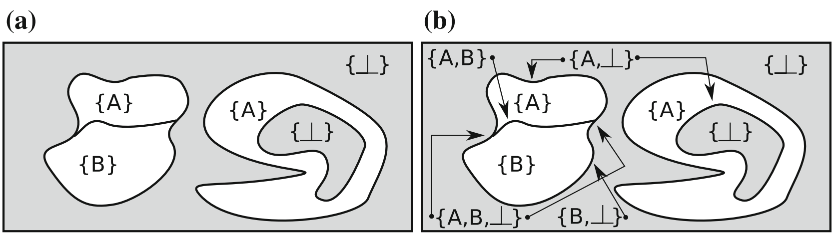

Fig. 2.7 A sample spatial partition with two regions. **a** The spatial partition annotated with region labels. **b** The spatial partition with its region and boundary labels. Note that labels are modeled as sets of attributes in spatial partitions

2.4 Spatial Partitions: A Mathematical Model of Maps

Spatial partitions have been developed in the literature in [10, 12]; those definitions require some modification in order to define both topological predicates over maps and the discrete model of spatial partitions suitable for implementation. We present the modified definition here. We first introduce the mathematical notation and definitions required to formally define spatial partitions. Then, the formal mathematical type definition of spatial partitions is presented.

2.4.1 Notation

The application of a function $f : A \to B$ to a set of values $S \subseteq A$ is defined as:

$$f(S) := \{f(x)|x \in S\} \subseteq B \tag{2.1}$$

In some cases we know that $f(S)$ returns a singleton set, in which case we write $f[S]$ to denote the single element:

$$f(S) = \{y\} \Leftrightarrow f[S] = y \tag{2.2}$$

The inverse function $f^{-1} : B \to 2^A$ of f is defined as:

$$f^{-1}(y) := \{x \in A|f(x) = y\} \tag{2.3}$$

It is important to note that f^{-1} is a total function and that f^{-1} applied to a set yields a set of sets.

We define the range function of a function $f : A \to B$ that returns the set of all elements that f returns for an input set A as:

$$rng(f) := f(A) \tag{2.4}$$

Let (X, U) be a topological space [5] with topology $U \subseteq 2^x$ defined by the basis β (a basis of U is a set such that every element of U is equivalent to a union of elements in the basis). Let $S \in \beta$. The *interior* of S, denoted by S°, is defined as the union of all open sets that are contained in S. The *closure* of S, denoted by $\overline{S}$ is defined as the intersection of all closed sets that contain S. The *exterior* of S is given by $S^- := (X - S)^\circ$, and the *boundary* or *frontier* of S is defined as $\partial S := \overline{S} \cap \overline{X - S}$. An open set is *regular* if $A = \overline{A}^\circ$ [22]. In this paper, we deal with the topological space $\mathbb{R}^2$, i.e., $X = \mathbb{R}^2$.

A *partition* of a set S, requires U to be a *partition topology*. A partition topology, is a topology whose basis β is complete decomposition into non-empty, disjoint subsets $\{b_i|i \in I\}$, called *blocks*, where I is an index set used to name different blocks, where:

$$\begin{aligned}&\text{(i) } \forall i \in I : b_i \neq \emptyset,\\ &\text{(ii) } \bigcup_{i \in I} b_i = \beta, \text{ and}\\ &\text{(iii) } \forall i, j \in I, i \neq j : b_i \cap b_j = \emptyset,\end{aligned} \tag{2.5}$$

We can equivalently represent a partition as a total and surjective function:

$$f : \beta \rightarrow I \tag{2.6}$$

such that the above constraints are satisfied. However, a spatial partition cannot be defined simply as a set-theoretic partition of the plane, that is, as a partition of $\mathbb{R}^2$ or as a function $f : \mathbb{R}^2 \rightarrow I$, for two reasons: first, f cannot be assumed to be total in general, and second, f cannot be uniquely defined on the borders between adjacent subsets of $\mathbb{R}^2$. In the following, we build upon the concept of partitions to create spatial partitions.

2.4.2 *Spatial Partitions*

In this section, we build the definition of spatial partitions based upon the notation developed for partitions, as in [10]. First, we define a *spatial mapping*, then impose constraints on a spatial mapping to create a spatial partition.

A *spatial mapping* of type A is a total function $\pi : \mathbb{R}^2 \rightarrow 2^A$ that maps points in the plane to an element of 2^A where A is a set of *labels*. In this context, a label is a general term and no restrictions are imposed on labels; typically, applications will use labels to represent thematic values or identifiers. The existence of an undefined element $\perp_A$ is required to represent undefined labels (i.e., the exterior of a partition).

Although there are not restrictions on a spatial mapping, the mapping will result in sets of points that map to the same element of 2^A; we denote such sets of points as *components* of the partition imposed by the spatial mapping. Definition 2.1 identifies the different components of a partition imposed by a spatial mapping.

A region in the partition imposed by a spatial mapping (Definition 2.1(i)) is a point set that maps the same label, and that label is an element of the set A. It is important to note that a region in a spatial partition is intuitively the same as a complex region; thus, the region is not necessarily connected (may contain multiple *faces*), and may contain holes. A region in a spatial mapping requires the further constraints imposed by a spatial partition to ensure that regions are mathematically equivalent to complex regions. We use the term *regions* to refer to complex regions in the remainder of this book. The *border* (Definition 2.1(ii)) between two regions will have the labels of both neighboring regions, and thus, will be labeled with an element of 2^A with cardinality greater than 1.

The *interior* of spatial mapping π (Definition 2.1(iii)) is defined as the union of π's regions. Thus, the interior of the spatial mapping is identified by points in the plane that map to a single set that exists in 2^A. The union of all borders in a spatial mapping π form the *boundary* of π (Definition 2.1(iv)). Therefore, the boundary of the of the partition consists of all points that are labeled with non-singleton sets of 2^A. The *exterior* of π (Definition 2.1(v)) is the point set such that each point in that set maps to the label $\perp_A$. The exterior behaves similarly to the exterior of a complex region. The exterior may have multiple connected components. The unique feature about the exterior is that is an *unbounded face*.

Definition 2.1 Let π be a spatial mapping of type A

(i) $\rho(\pi) := \pi^{-1}(rng(\pi) \cap \{X \in 2^A | \, |X| = 1\})$ (*regions*)
(ii) $\omega(\pi) := \pi^{-1}(rng(\pi) \cap \{X \in 2^A | \, |X| > 1\})$ (*borders*)
(iii) $\pi^\circ := \bigcup_{r \in \rho(\pi) | \pi[r] \neq \{\perp_A\}} r$ (*interior*)
(iv) $\partial\pi := \bigcup_{b \in \omega(\pi)} b$ (*boundary*)
(v) $\pi^- := \pi^{-1}(\{\perp_A\})$ (*exterior*)

As an example, let π be the spatial partition in Fig. 2.7 of type $X = \{A, B, \perp\}$. In this case, $rng(\pi) = \{\{A\}, \{B\}, \{\perp\}, \{A, B\}, \{A, \perp\}, \{B, \perp\}, \{A, B, \perp\}\}$. Therefore, the regions of π are the blocks labeled $\{A\}$, $\{B\}$, and $\{\perp\}$ and the boundaries are the blocks labeled $\{A, B\}$, $\{A, \perp\}$, $\{B, \perp\}$, and $\{A, B, \perp\}$.

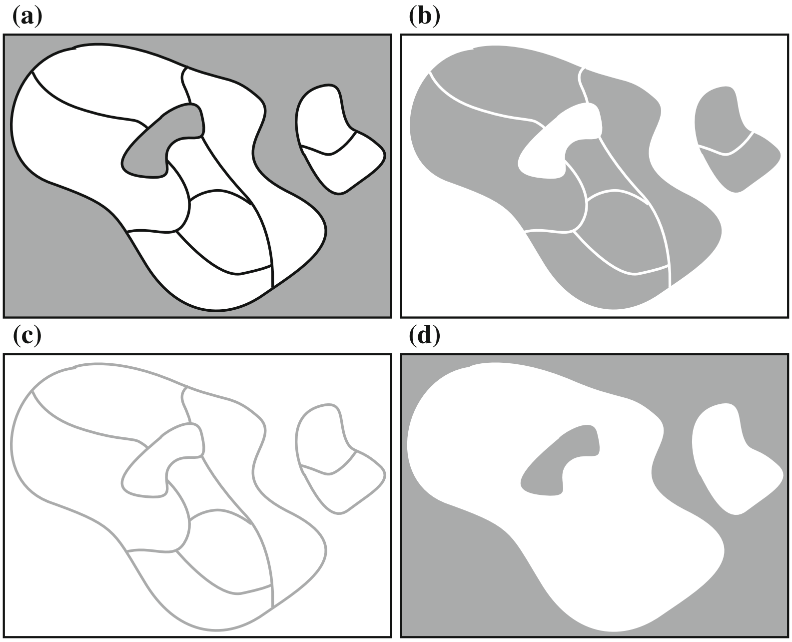

Fig. 2.8 **a** A spatial partition π with two disconnected faces, one containing a hole. **b** The interior (π°). **c** The boundary ($\partial\pi$). **d** The exterior (π^-). Note that the labels have been omitted in order to emphasize the components of the spatial partition

Figure 2.8 provides a pictorial example of the interior, exterior, and boundary of an example spatial mapping (note that the borders and boundary consist of the same points, but the boundary is a single point set whereas the borders are a set of point sets).

A *spatial partition* of type A is then defined as a spatial mapping of type A whose regions are regular open sets [22] and whose borders are labeled with the union of labels of all adjacent regions. From this point forward, we use the term *partition* to refer to a spatial partition.

Definition 2.2 A spatial partition of type A is a spatial mapping π of type A with:
(i) $\forall r \in \rho(\pi) : r = \overline{r}^{\circ}$
(ii) $\forall b \in \omega(\pi) : \pi[b] = \{\pi[[r]] | r \in \rho(\pi) \wedge b \subseteq \partial r\}$

The remaining portion of the definition of spatial partitions requires the use of the *refine operation* over spatial partitions. This operation is formally defined in Chap. 3, so we provide an intuitive definition here.

The refine operation over spatial partitions uniquely identifies the connected components of a partition. Recall that two regions in a partition can share the same label if they are disjoint or meet at discrete points along their boundaries. Given a partition π containing regions with multiple connected components (recall that each connected component of a region has the same label), the operation *refine*(π) returns a partition with identical structure to π, but with each connected component in every region having a unique label. This is achieved by appending an integer to the label of each connected component of all regions in a partition. Figure 2.9 shows an example partition and the same partition after performing a refine operation. To append an integer, we construct a *pair* containing the original label and an integer; this procedure imposes no restrictions on the type of the label.

The boundary of a spatial partition implicitly imposes a graph on the plane. Specifically, the boundaries form an undirected planar graph. The edges of the graph are the points mapped to the boundaries between two regions. The vertices of the graph are the points mapped to boundaries between three or more regions. We identify edges and vertices based on the cardinality of their labels. However, due to degenerate

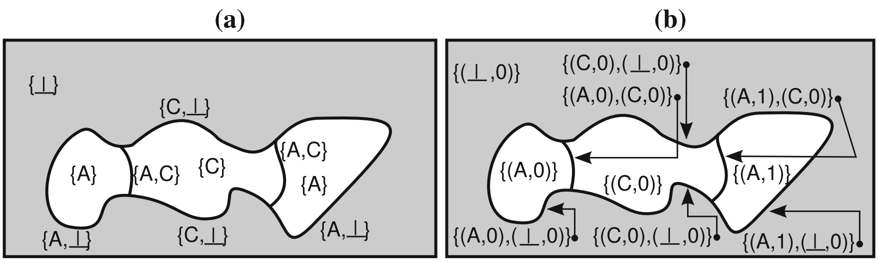

Fig. 2.9 The application of the refine operation to a spatial partition. A spatial partition with two regions and its boundary and region labels (**a**). The result of the refine operation on (**a**). The region labels are $\{A\}$ and $\{C\}$ in (**a**) and $\{(A, 0)\}$, $\{(C, 0)\}$, and $\{(A, 1)\}$ in (**b**). The exterior label is $\perp$ in (**a**) and $(\perp, 0)$ in (**b**). All other labels are boundaries

cases, we must use the refinement of a partition to identify these features. We define the set of edges and vertices imposed on the plane by a spatial partition as follows:

Definition 2.3 A boundary point of a spatial partition π is classified as being a *vertex* or as being part of an *edge* by examining the refinement $\sigma = \mathit{refine}(\pi)$ as follows:
(i) $\varepsilon(\pi) = \{b \in \omega : |\sigma[b]| = 2\}$
(ii) $\nu(\pi) = \{b \in \omega : |\sigma[b]| > 2\}$

2.5 Summary

Spatial partitions provide a data type to represent map constructs and their associated attribute values. The power of the spatial partition data type becomes clear in later sections where we show that nearly all operations over spatial partitions can be expressed as compositions of three fundamental *power operations*. We will prove type closure of those operations under the type of spatial partitions such that any operation defined as a composition of the power operations inherits the type closure properties. Furthermore, complex regions are a special case of spatial partitions which makes spatial partitions a useful tool for studying regions and their operations, in addition to maps and their associated operations.

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