

Contents

1	Introduction	1
1.1	What Is this Book About: Consequence Relations and Other Logical Issues	1
1.2	Finiteness Versus Infiniteness	3
1.2.1	ω -rule	4
1.2.2	Infinitary Languages	5
1.2.3	Hyperfinite Numbers and Infinitesimals	5
1.2.4	Admissible Sets	7
1.2.5	Ranges of Probability Functions	9
1.3	Modal Logics	9
1.4	Kolmogorov's Axiomatization of Probability and Probability Logics	11
1.5	An Overview of the Book	12
	References	14
2	History	19
2.1	Pre-leibnitzians	20
2.2	Leibnitz	21
2.3	Jacob Bernoulli	24
2.4	Probability and Logic in the Eighteenth Century	29
2.4.1	Abraham de Moivre	29
2.4.2	Thomas Bayes	31
2.4.3	Johann Heinrich Lambert	33
2.5	Laplace and Development of Probability and Logic in the Nineteenth Century	35
2.5.1	Laplace	35
2.5.2	Bernard Bolzano, Augustus de Morgan, Antoine Cournot	37
2.5.3	George Boole	40
2.5.4	J. Venn, H. MacColl, C. Peirce, P. Poretsky	43

2.6	Rethinking the Foundations of Logic and Probability in the Twentieth Century	47
2.6.1	Logical Interpretation of Probability	48
2.6.2	Subjective Approach to Probability	51
2.6.3	Objective Probabilities as Relative Frequencies in Infinite Sequences	54
2.6.4	Measure-Theoretic Approach to Probability	56
2.6.5	Other Ideas	60
2.7	1960s: And Finally, Logic	62
2.7.1	Probabilities in First-Order Settings	63
2.7.2	Probability Quantifiers	64
2.7.3	Probabilities in Modal Settings	66
2.7.4	Probabilistic Logical Entailment	68
	References	69
3	LPP₂, a Propositional Probability Logic Without Iterations of Probability Operators	77
3.1	Syntax and Semantics	78
3.1.1	Syntax	78
3.1.2	Semantics	79
3.1.3	Atoms	81
3.2	Complete Axiomatization	82
3.3	Non-compactness	84
3.4	Soundness and Completeness	86
3.4.1	Soundness	86
3.4.2	Completeness	87
3.4.3	The Role of the Infinitary Rule	94
3.4.4	Completeness for Other Classes of Models	95
3.5	Decidability and Complexity	97
3.6	A Heuristic Approach to the $LPP_{2, \text{Meas}}$ -Satisfiability Problem PSAT	98
3.6.1	Other Heuristics for PSAT and Similar Problems	107
	References	107
4	Probability Logics with Iterations of Probability Operators	109
4.1	Introduction	110
4.2	Syntax and Semantics of $LFOP_1$	110
4.2.1	Syntax	110
4.2.2	Semantics	111
4.3	Axiom System Ax_{LFOP_1}	113
4.4	Soundness and Completeness	114
4.4.1	Semantical Consequences	116
4.4.2	Completeness for Other Classes of Measurable First-Order and Propositional Models	117

4.5	Modal Logics Versus Probability Logics	118
4.6	(Un)decidability	121
4.6.1	The First-Order Case.	121
4.6.2	The Propositional Case	122
4.7	A Discrete Linear-Time Probabilistic Logic	125
4.7.1	Semantics	126
4.7.2	Axiomatization	127
	References.	130
5	Extensions of the Probability Logics LPP_2 and $LFOP_1$	133
5.1	Generalization of the Completeness-Proof Technique	134
5.2	Logic $LPP_2^{Fr(n)}$	135
5.3	Logic $LPP_2^{A, \omega_1, Fin}$	137
5.4	Probability Operators of the Form Q_F	141
5.4.1	Complete Axiomatization	141
5.4.2	Decidability.	142
5.4.3	The Lower and the Upper Hierarchy.	143
5.4.4	Representability.	144
5.4.5	The Upper Hierarchy	147
5.4.6	The Lower Hierarchy	149
5.5	Qualitative Probabilities	151
5.6	An Intuitionistic Probability Logic	152
5.6.1	Semantics	153
5.6.2	Axiomatization, Completeness, Decidability	154
5.7	Logics with Conditional Probability Operators	156
5.7.1	A Logic $LPCP_2^{[0,1]_{Q(e)}, \approx}$ with Approximate Conditional Probabilities	156
5.7.2	Axiomatization	157
5.8	Polynomial Weight Formulas	158
5.9	Logics with Unordered or Partially Ordered Ranges.	159
5.9.1	A Logic for Reasoning About p -adic Valued Probabilities	160
5.10	Other Extensions	162
	References.	162
6	Some Applications of Probability Logics	165
6.1	Nonmonotonic Reasoning and Probability Logics	165
6.1.1	System P and Rational Monotonicity	165
6.1.2	Modeling Defaults in $LPCP_2^{[0,1]_{Q(e)}, \approx}$	166
6.1.3	Approximate Defaults and $LPCP_2^{[0,1]_{Q(e)}, \approx}$	173
6.2	Logic for Reasoning About Evidence.	175
6.2.1	Evidence	175
6.2.2	Axiomatizing Evidence	178

6.3	Formalization of Human Thinking Processes in $L_{\mathbb{Q}_p}$	180
6.4	Other Applications	183
	References.	184
7	Related Work	187
7.1	Papers on Completeness of Probability Logics	187
7.2	Papers on (Infinitary) Modal Logics	194
7.3	Papers on Temporal Probability Logics	194
7.4	Papers on Applications of Probability Logics	195
7.5	Books About Probability Logics.	196
	References.	197
	Erratum to: Extensions of the Probability Logics LPP₂ and LFOP₁ . . .	E1
	Appendix A: General Notions	201
	Index	209

Probability Logics

Probability-Based Formalization of Uncertain Reasoning

Ognjanović, Z.; Rašković, M.; Marković, Z.

2016, XI, 215 p. 5 illus., Hardcover

ISBN: 978-3-319-47011-5