Preface

The theory of time scales was introduced by Stefan Hilger in his Ph.D. thesis [31] in 1988 (supervised by Bernd Aulbach) in order to unify continuous and discrete analysis and to extend the continuous and discrete theories to cases “in between.” Since then, research in this area of mathematics has exceeded by far a thousand publications, and numerous applications to literally all branches of science such as statistics, biology, economics, finance, engineering, physics, and operations research have been given. For an introduction to *single-variable* time scales calculus and its applications, we refer the reader to the monograph [21] by Bohner and Peterson.

In this book, we offer the reader an overview of recent developments of *multivariable* time scales calculus. The book is primarily intended for senior undergraduate students and beginning graduate students of engineering and science courses. Students in mathematical and physical sciences will find many sections of direct relevance. This book contains nine chapters, and each chapter consists of results with their proofs, numerous examples, and exercises with solutions. Each chapter concludes with a section featuring advanced practical problems with solutions followed by a section on notes and references, explaining its context within existing literature. Altogether, the book contains 123 definitions, 230 theorems including corollaries, lemmas, and propositions, 275 examples, and 239 exercises including advanced practical problems.

The first three chapters deal with single-variable time scales calculus. Many of the presented results including their proofs are extracted from [21]. Chapter 1 introduces the most fundamental concepts related to time scales, namely the forward and backward jump operators and the graininess. In addition, the induction principle on time scales is given. Chapter 2 deals with differential calculus for single-variable functions on time scales. The basic definition of delta differentiation is due to Hilger. Many examples on differentiation in various time scales are included, as well as the Leibniz formula for the nth derivative of a product of two functions. Mean value results are presented that will be used later on in the book in the multivariable case. Several versions of the chain rule are included. Sufficient conditions for a local maximum and a minimum are given. Moreover, sufficient conditions for convexity and concavity of single-variable functions are presented. A sufficient condition for complete delta differentiability of single-variable functions is given, and the geometric sense of differentiability is discussed and illustrated. In Chapter 3, the main concepts for regulated, rd-continuous, and pre-differentiable functions are introduced. The indefinite integral and the Riemann delta integral are defined and many of their properties are deduced. Hilger's complex plane is introduced. Some elementary functions such as the exponential functional, hyperbolic functions, and trigonometric functions are defined and their properties are given. Moreover, Taylor's formula and L'Hôpital's rule are presented. Improper integrals of the first and the second kind are introduced and studied.

The next two chapters discuss sequences and series of functions as well as parameter-dependent integrals. The results of these chapters are adopted from [36, 40]. In Chapter 4, the Dini theorem, the Cauchy criterion for uniform convergence of a function series, the Weierstraß M-test for uniform convergence of a function series, the Abel test, and the Dirichlet test are presented and numerous examples are given. Chapter 5 introduces and studies both normal parameter-dependent integrals and improper parameter-dependent integrals of the first kind.

The final four chapters deal with multivariable time scales calculus. The presented results including their proofs are n-dimensional analogues of the two-dimensional results given in [8, 9, 14, 17]. Chapter 6 is devoted to partial differentiation on time scales. Definitions for partial derivatives and completely delta differentiable functions are given. Some sufficient conditions for differentiability are presented. The chain rule and some of the properties of implicit functions are given. The directional derivative is introduced. In Chapter 7, multiple Riemann integrals over rectangles and over more general sets are introduced. Many of their properties are given. Mean value results are presented. Chapter 8 defines the length of time scale curves. Line integrals of the first kind and of the second kind are introduced. Moreover, Green's formula is derived. Chapter 9 deals with surface integrals. Many of their properties are given.

The aim of this book is to present a clear and well-organized treatment of the concept behind the development of mathematics as well as solution techniques. The text material of this book is presented in a readable and mathematically solid format. Many practical problems are given, displaying the power of multivariable dynamic calculus on time scales.

Many of the results presented in this book are based on the work by one of the authors (Martin Bohner) and Professor Gusein Shirin Guseinov, who unexpectedly passed away on March 20, 2015. Both authors have attended the memorial conference for Professor Guseinov at Atılım University in Ankara, Turkey, July 11–13, 2016, and they have decided there to dedicate this book to the memory of Professor Guseinov.

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