Preface

Stability theory was first established by Aleksandr Lyapunov in 1892 (see [70]). Due to the wide application of stability theory, many mathematicians are devoted to this area. For an unstable differential system, how to stabilize it, with the aid of a feedback control, becomes an important subject in control theory of differential equations. This is termed feedback stabilization. Studies on this subject started with finite dimensional systems in the 1950s and extended to infinite dimensional systems in the 1960s. This subject contains two important themes: criteria to judge whether a controlled system is feedback stabilizable and design of feedback laws to stabilize systems.

In most past publications on criteria of stabilization, control systems have been linear and time invariant. There are quite limited studies on criteria of the periodic stabilization for linear and time-periodic controlled systems. The reason for studying the latter can be explained as follows: Mature theories have been established on the stability and stabilization for time-invariant linear ODEs. Regarding stability, it is a well-known result that the equation: $\dot{y}(t) = Ay(t), t \geq 0$, with $A \in \mathbb{R}^{n \times n}$, is exponentially stable if and only if the spectrum of A is in the half plane $\mathbb{C}^- \triangleq \{z \in \mathbb{C} : Re(z) < 0\}$(see, for instance, [2]). The most important result regarding stabilization is Kalman's criterion: A pair of matrices $[A, B]$ in $\mathbb{R}^{n \times n} \times \mathbb{R}^{n \times m}$ is stabilizable (i.e., there exists a matrix K in $\mathbb{R}^{m \times n}$ so that the spectrum of $(A + BK)$ is in $\mathbb{C}^-$) if and only if the rank of $(\lambda I - A, B)$ equals to n for each $\lambda \in \mathbb{C} \backslash \mathbb{C}^-$ (see, for instance, [24, 86]). With respect to the stability of linear time-periodic ODEs, one of the most important results is that the periodic equation: $\dot{y}(t) = A(t)y(t), t \geq 0$, with $A(\cdot)$ T-periodic in $L^\infty(\mathbb{R}^+; \mathbb{R}^{n \times n})$, is exponentially stable if and only if the spectrum of $\Phi_A(T)$ belongs to the open unit ball in $\mathbb{C}$, where $\Phi_A(\cdot)$ is the fundamental solution associated with $A(\cdot)$ (see, for instance, [38, 40]). In view of Kalman's criterion on the stabilization of time-invariant pairs and the above-mentioned criterion on the stability of periodic equations, it is natural to ask for criteria on the periodic stabilization for periodic pairs.

When a pair $[A, B] \in \mathbb{R}^{n\times n} \times \mathbb{R}^{n\times m}$ is stabilizable, any matrix $K \in \mathbb{R}^{m\times n}$, with the spectrum of $(A + BK)$ in $\mathbb{C}^-$, is called a feedback stabilization law for the pair $[A, B]$. It is important to find ways to construct feedback laws. The usual structure of feedback laws is connected with either the LQ theory, as well as Riccati equations, or Lyapunov functions (see, for instance, either Chap. 9, [59] or Chap. 5, [86]). Correspondingly, how to construct periodic feedback stabilization laws for a given stabilizable T-periodic pair should also be important.

According to our understanding, for each unstable T-periodic $A(\cdot) \in L^\infty(\mathbb{R}^+; \mathbb{R}^{n\times n})$, the procedure to stabilize periodically the system: $\dot{y}(t) = A(t)y(t), t \geq 0$, is as follows: First, one builds up a T-periodic $B(\cdot) \in L^\infty(\mathbb{R}^+; \mathbb{R}^{n\times m})$ so that $[A(\cdot), B(\cdot)]$ is T-periodically stablizable; Second, one designs a T-periodic $K(\cdot) \in L^\infty(\mathbb{R}^+; \mathbb{R}^{m\times n})$ so that $A(\cdot) + B(\cdot)K(\cdot)$ is exponentially stable. We call the aforementioned $B(\cdot)$ a control machine and the corresponding $K(\cdot)$ a feedback law. It should be interesting to study the question of how to design a *simple* T-periodic $B(\cdot)$ for a given T-periodic $A(\cdot)$ so that $[A(\cdot), B(\cdot)]$ is T-periodically stabilizable. Of course, we can define what *simple* means according to our needs.

The aim of this monograph is to present recent advances regarding periodic stabilization for some linear and time-periodic evolution equations which contain both finite and infinite dimensional systems. These advances may lead us to a comprehensive understanding of the subject of periodic stabilization. The monograph summarizes our ideas, results, and methods with respect to the subject during recent years. Insofar as possible, we have tried to make the material self-contained. There is much literature on the stabilization of differential equations, and we are unable to give a complete list of references. Consequently, it is possible that some important works in the field will have been overlooked.

The monograph is organized as follows: Chapter 1 presents some preliminaries on linear periodic evolution equations, in particular, the connection between the LQ theory and periodic stabilization. Chapter 2 studies the periodic stabilization for some infinite dimensional linear periodic evolution equations. Three criteria on the periodic stabilization for a linear periodic evolution equation are provided. One is a geometric condition which is related to the attainable subspaces, while the other two are analytic conditions which are connected with some unique continuation properties of dual equations. Some applications of these criteria are also given in this chapter. Chapter 3 provides two criteria on the periodic stabilization of periodic linear ODEs. One is an algebraic condition which is an extension of Kalman's crite- rion to the periodic case, while the other is a geometric condition which is connected with the null-controllable subspace. Chapter 4 shows how to find a *simple* control machine $B(\cdot) \in L^\infty(\mathbb{R}^+; \mathbb{R}^{n\times m})$, for a given unstable periodic $A(\cdot) \in L^\infty(\mathbb{R}^+; \mathbb{R}^{n\times n})$, so that $[A(\cdot), B(\cdot)]$ is T-periodically stablizable. This is an application of the geometric criterion in Chapter 3.

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