Generalized Minimum Distance Decoder in a DHA FH OFDMA Employing Concatenated Coding

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Abstract. In what follows a coded DHA FH OFDMA employing robust reception and concatenated coding construction is considered. The problem of employing Generalized Minimum Distance (GMD) decoder as an outer code decoder is considered. The effectiveness of the proposed decision is verified by means of simulation. In particular, performance of the communication system under intensive mixed interference is considered.

Keywords: Multiple access · Coded DHA FH OFDMA · Robust reception · Generalized Minimum Distance decoder · Iterative decoding

1 Introduction

Interference mitigation is one of the key issues in modern telecommunication systems design (see e.g.). This is mainly due to the fact that interference can be caused by different factors: authorized users' activity in a multiple access system (multi-user interference, MUI), signals transmitted by the users of other telecommunication systems operating within the same frequency bands or intentional jamming. If the interference is severe traditional reception techniques turn out to be ineffective due to low reliability of the computed decision statistics. Recently several coded modulation schemes [1–5] employing robust reception techniques were proposed to solve the problem. Due to their immunity to different types of interference they can be thought of as promising candidates for certain communication systems (e.g. M2M communications). In what follows a coded DHA FH OFDMA employing robust reception and concatenated coding construction is considered. In particular, decoding of the outer code by means of the Generalized Minimum Distance (GMD) decoder will be considered.

This paper is organized as follows. In Sect. 2 a short description of a coded DHA FH OFDMA system is given. In Sect. 3 a description of two different types

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of noncoherent detectors will be given. In Sect. 4 concatenated construction in use will be described and the criterion for reliability values estimation for inner codes decoding will be proposed. In Sect. 5 the simulation scenario under consideration will be discussed. The effectiveness of the proposed approach will be verified by means of simulation in Sects. 6 and 7 respectively. Finally in Sect. 8 the obtained results will be summarized.

2 A DHA FH OFDMA System: Transmission and Reception

Let us consider a multiple access system in which K active users transmit information via a channel split into Q identical nonoverlapping subchannels by means of OFDM. In what follows it will be assumed that information that is to be transmitted is encoded into a codeword of a q-ary (n, k, d) block code $(q < Q)$. This code will be further on referred to as "inner code in time domain" (in what follows a specific case will be considered. In particular we shall consider the case in which the inner code in time domain is a short Maximum Distance Separable (MDS) code obtained by puncturing form a longer Reed-Solomon (RS) code). Whenever a user is to transmit a $q-$ary symbol it places 1 in the position of the vector $\bar{a}_g$ corresponding to the symbol in question within the scope of the mapping in use (in what follows it will be assumed that all the positions of the vector are enumerated from 1 to Q, moreover for the sake of simplicity and without loss of generality we shall assume that the 1st subchannel corresponds to 0, the 2nd subchannel corresponds to 1 and so on). Thus, each $q - ary$ symbol to be transmitted is mapped in to a weight 1 binary vector which will be referred to as "inner code in frequency domain". Then a random permutation of the aforesaid vector is performed and the resulting vector $\pi_g(\bar{a}_g)$ is used to form an OFDM symbol (permutations are selected equiprobably from the set of all possible permutations and the choice is performed whenever a symbol is to be transmitted). The transmission technique in question can be interpreted in the following way: assume that whenever a certain user is to transmit a symbol it randomly chooses q out of Q available subchannels (subcarriers). Since the list of the subchannels that can be used to transmit a signal (or a hopset) is allocated to each user in a dynamic fashion the technique under consideration is referred to as Dynamic Hopset Allocation Frequency Hopping OFDMA (DHA FH OFDMA). An example of a transmitter employing the strategy considered above is shown in Fig. 1 (symbols corresponding to the signals transmitted by the user under consideration are shown in italics). Therefore in order to transmit a codeword a user is to transmit n OFDM symbols. A sequence of OFDM symbols corresponding to a certain codeword that has been sent by a certain user will be referred to as a frame. Note that frames transmitted by different users need not be block synchronized, i.e. if within the time interval a certain user transmits a frame that corresponds to a codeword, symbols transmitted by another user within the same time period do not necessarily all comprise one codeword. Moreover, it will be assumed that transmissions from different

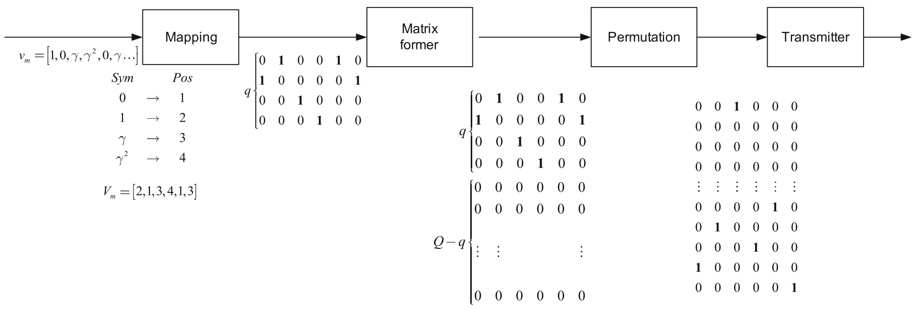

Fig. 1. Transmitter structure (for the case $q = 4$)

users are uncoordinated, i.e. none of the users has information about the others. In what follows we shall assume that all users transmit information in OFDM frames and the transmission is quasisynchronous. In terms of the model under consideration this assumption means that transmissions from different users are symbol synchronized.

Within the scope of a certain codeword reception the receiver is to receive n OFDM symbols corresponding to the codeword in question. Note that the receiver is assumed to be synchronized with transmitters of all users. Therefore all the permutations performed within the scope of transmission of the codeword in question are known to the user. The receiver measures energies at the outputs of all subchannels (let us designate the vector of the measurements as b_g where g is the number of the OFDM symbol) and applies inverse permutation to each vector b_g corresponding to the respective OFDM symbol thus reconstructing the initial order of elements and obtaining vector $\tilde{b}_g = \pi_g^{-1}(b_g)$. Let us consider a matrix X that consists of vectors $\tilde{b}_g = \pi_g^{-1}(b_g)$ that correspond to the codeword of inner code. Let us consider the submatrix $\Re = [\bar{\Re}_1, \bar{\Re}_2, \ldots, \bar{\Re}_n]$ (here $\Re$ is submatrix corresponding to the q first rows of the matrix X and each vector $\bar{\Re}_s$ is the height q column vector corresponding to the $s - th$ symbol of the codeword). Please note that $\Re$ provides all the information necessary to decode the codeword of the inner code.

3 Detection

The detection procedure that will be considered in this section can be decomposed in two successive procedures: reliability values computation and decoding. First and foremost let us consider the former procedure.

Let us assume that within the scope of the detection technique in use a reliability value is computed for each element of the matrix $\Re$. Let us designate the matrix of reliability values corresponding to the matrix $\Re$ with $Y = [\bar{Y}_1, \bar{Y}_2, \ldots, \bar{Y}_n]$. The main idea of the detection techniques under consideration is to use the reliability values stored in the matrix Y to compute decision statistics (reliability values) for codewords of the inner code in time domain.

Thus for the sake of brevity the matrix Y will be further on referred to as "decision matrix". The detector considered in [1] computes decision statistics called ranks. The easiest way to introduce ranks is to consider the indicator function of the following form:

$$I(x^*, x) = \begin{cases} 1 & x \le x^* \\ 0 & x > x^* \end{cases} \tag{1}$$

Rank of the element of the matrix $\Re(i,j)$ is given by:

$$\rho(i,j) = \sum_{k \neq i} \sum_{m \neq j} I\left(\Re(i,j), \Re(k,m)\right). \tag{2}$$

In other words a certain element $\Re(i,j)$ has rank ϱ if there are ϱ elements in the matrix having value lower than that of the element $\Re(i,j)$.

In RANK decoder introduced in [1] the decision matrix is the rank matrix considered above. The structure of the communication system employing the α detector is depicted in Fig. 2 (symbols corresponding to the signals transmitted by the user under consideration are shown in bold).

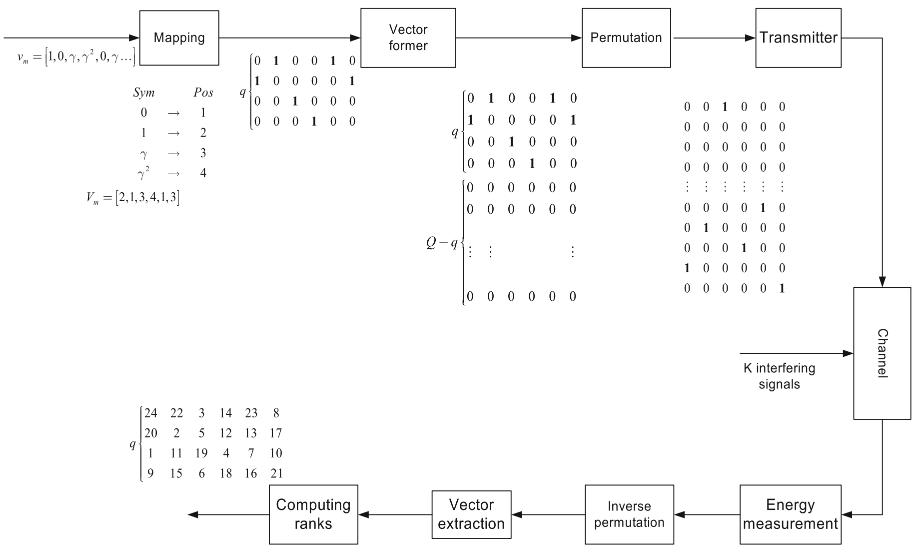

Fig. 2. A communication system employing RANK detector (for the case $q = 4$)

The RANK detector considered above requires rank matrix calculation. The latter operation requires sort of the elements of the $q \times n$ size matrix (where n is the codeword length of the inner code in time domain). Moreover in the case under consideration the decoding delay is lower bounded by the frame duration. In [5] a modified reception technique that provides lower complexity and delay has been proposed (in what follows it will be referred to as α detector). Let us assume that each column of the matrix $\Re$ is sorted in the descending order. Let us designate the α-th element of the vector v_j obtained by sorting the $j-th$ column of the matrix $\Re$ in the descending order by $v_j(\alpha)$. Let us consider the matrix D^α:

$$D^\alpha(i,j) = \begin{cases} 1 \; \Re(i,j) \geq v_j(\alpha) \\ 0 \; \Re(i,j) < v_j(\alpha). \end{cases} \tag{3}$$

Each column of the matrix D^α contains exactly α nonzero entries. The nonzero entries in a certain column correspond to the elements of the respective column of the matrix $\Re$ having values greater or equal than α-th q quantile of this column. The matrix D^α is the decision matrix for the detector under consideration.

In Fig. 3 the structure of the communication system employing α detector is depicted (symbols corresponding to the signals transmitted by the user under consideration are shown in bold, symbols corresponding to the nonzero elements of the decision matrix are shown in bold italics).

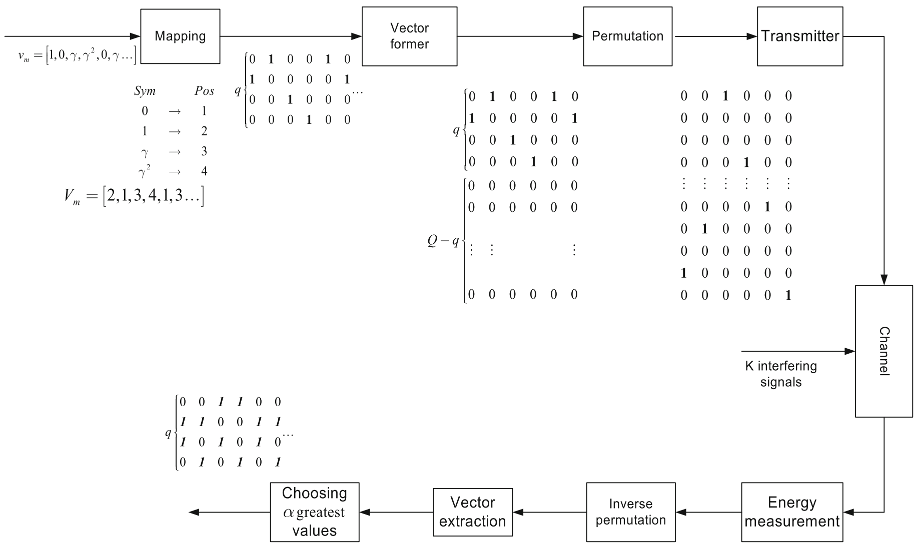

Fig. 3. A communication system employing α detector (for the case $q=4$, $\alpha=2$)

Let us assume that the user under consideration transmits a codeword v_m and within the scope of the mapping in use this vector is mapped into a matrix $X^m = \left[\bar{X}_1, \bar{X}_2, \ldots, \bar{X}_n\right]$. Let us denote the matrix

$$M = X^m * Y \tag{4}$$

where Y is the decision matrix (i.e. $Y = \rho$ for the RANK detector and $Y = D^{\alpha}$ for the α detector) and $*$ stands for Hadamard product. The decision statistic for the mth codeword is then given by:

$$S_m = \sum_{i=1}^{n} \sum_{j=1}^{q} M(i,j) \tag{5}$$

The decoding rule boils down to choosing codeword number $m^* = \arg\max_m (S_m)$ if m^* is unique (i.e. $\forall\, m = 1 : \Omega, m \neq m^* \; m^* > m$, where $\Omega = |C|$), otherwise a denial decision is taken.

4 Concatenated Code and Generalized Minimum Distance Decoder

Even though it has been shown in [1–3,5] that the detectors described in the preceding section can provide relatively low probabilities of erroneous decoding and denial even under drastic interference the probabilities in question are not sufficiently low to meet the QoS requirements of modern communication systems. Thus some kind of concatenated coding is to be used. In what follows we shall consider the simplest case: the information to be transmitted is encoded with an outer (N, K, D) error correcting code over $GF(q^k)$ each symbol of this code being treated as k tuple over $GF(q)$ which in turn is encoded with an (n, k, d). Within the scope of the conventional decoding algorithm decision taken by the inner code decoder (in the case under consideration an inner decoder is replaced by a detector) are transferred directly to the decoder of the outer code i.e. in case of successful decoding (correct or erroneous) the symbol of the outer code corresponding to the respective codeword of the inner code is chosen, whereas denial decision results in erasure of the respective symbol of the outer code. Thus the decoder of the outer code is to correct both errors and erasures. If the number of errors and erasures is beyond the error correction capability of the outer code in use decoding can result in denial decision.

In this paper another approach that uses a classical Generalized Minimum Distance (GMD) decoder [6] is proposed. A GMD decoder makes use of symbol reliability values and combines it with the hard decision decoding. The codeword is decoded using a conventional decoding algorithm. If decoding fails (i.e. results in denial decision) two symbols with lowest reliability values are erased, the obtained vector being than decoded. The process is to be repeated iteratively until either the number of erasures to be corrected is beyond the error correction probability of the code in use or decoding is successful.

Within the scope of the reception strategy considered hereinabove reliability values for the symbols of outer code are not available. This is mainly due to the fact that channel state information cannot be evaluated with acceptable precision and thus the distributions of decision statistics cannot be computed. Thus the idea behind the proposed approach is to use information obtained from inner code decoder to evaluate the reliability of the decision taken by the inner code decoder. Let us consider a vector of decision statistics $\bar{S}^t = [S_1^t, S_2^t \dots, S_M^t]$ corresponding to the t-th symbol of the outer code. Let us consider the vector $\bar{\Omega}^t = [\omega_1^t, \omega_2^t, \dots, \omega_M^t]$ obtained by sorting the vector $\bar{S}^t$ in the descending order. Let us denote the reliability value λ^t for t-th symbol of the outer code as

$$\lambda^t = \begin{cases} \omega_1^t - \omega_2^t & \omega_1^t - \omega_2^t > 0 \\ n & \omega_1^t - \omega_2^t = 0 \end{cases} \tag{6}$$

The reliability value is computed for each matrix Y^t corresponding to the t-th symbol of the outer codeword. Thus the GMD decoding algorithm obtains the vector $\bar{r}$ of decisions obtained from inner decoder and the vector $\Lambda = [\lambda^1, \lambda^2, \dots, \lambda^N]$ of reliability values (the last equality in Eq. 6 is to ensure that the symbols for which the erasure decision has been taken will not be erased once again by the GMD algorithm).

5 Simulation

To verify the effectiveness of the proposed solution simulation will be used. An OFDM system with Q subcarriers has been considered. Please note that no power control has been considered within the system under consideration i.e. the signals from the interfering users (at the receiver end) have powers κ greater than that of the signal from the user under consideration (in what follows we shall assume that $Q = 4096$ and $\kappa = 10^4$). The number of signals transmitted by interfering users is equal to K. It has been assumed that the received signal is affected by the broadband noise jamming. The power at the receiver end is described in terms of the signal to interference ratio SNR per bit (please note that the power of the broadband noise is determined in the entire band whereas the power of the signal is determined in the instantaneous band.

Since decoding of inner code boils down to exhaustive search the size of codebook should be moderate. Since the rate of inner code predetermines the overall concatenated code rate the length is to be relatively small. Thus in what follows we shall consider short block code as inner code in time domain. Hereinafter (n, k) MDS code obtained by puncturing $(q-1, k, q-k)$ RS code will be used as inner code. We shall also choose (N, K) MDS code obtained by puncturing RS code over $GF(q^k)$. In particular, we shall assume inner codes with $k = 2$ over $GF(16)$ and $240, 160$ punctured RS code as outer code.

6 Simulation Results: GMD Decoding

In Figs. 4 and 5 the dependencies of the probability of undetected error (i.e. the probability of the fact that decoding will result in denial or erroneous decoding) for RANK detector, different rates of inner code and $K = 300$ are shown.

As can be seen from the obtained curves the proposed solution can decrease the probability of undetected error up to one order magnitude. The following curves (Figs. 6 and 7) show that the same effect can be archived for RANK detector for other values of K.

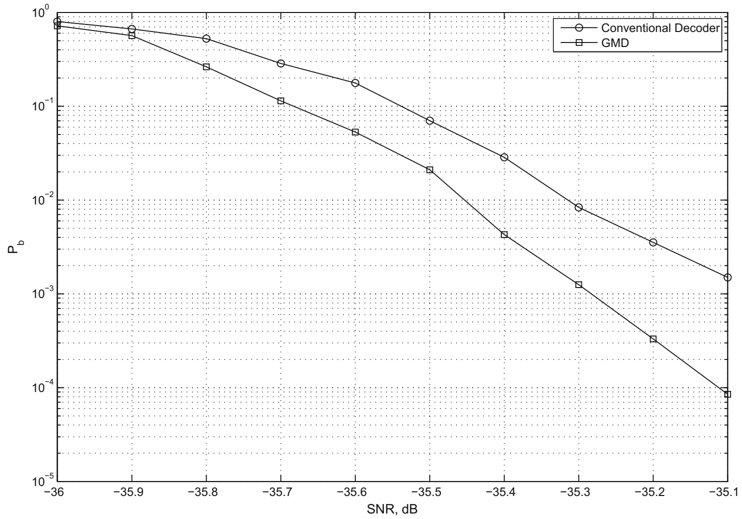

Fig. 4. Dependencies of the probability of undetected error on the value of SNR for RANK detector (number of interfering signals K = 300, inner code rate R = 1/3)

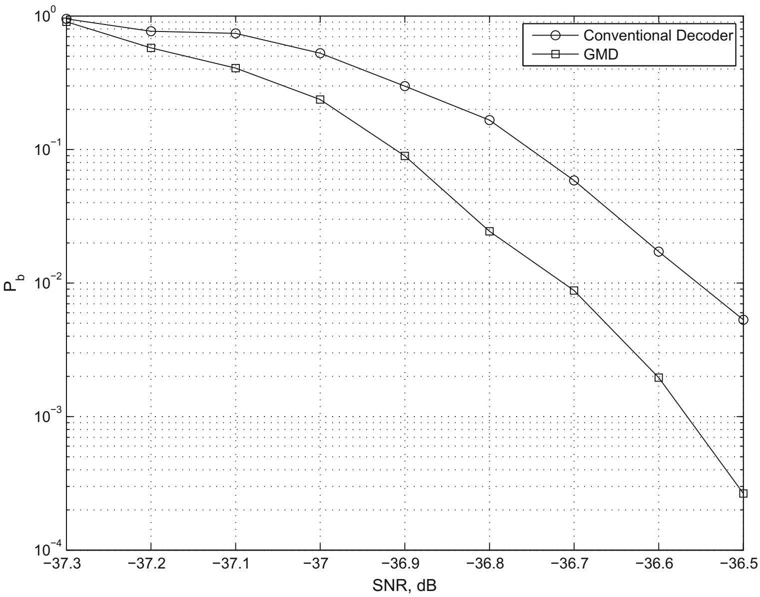

Fig. 5. Dependencies of the probability of undetected error on the value of SNR for RANK detector (number of interfering signals K = 300, inner code rate R = 1/4)

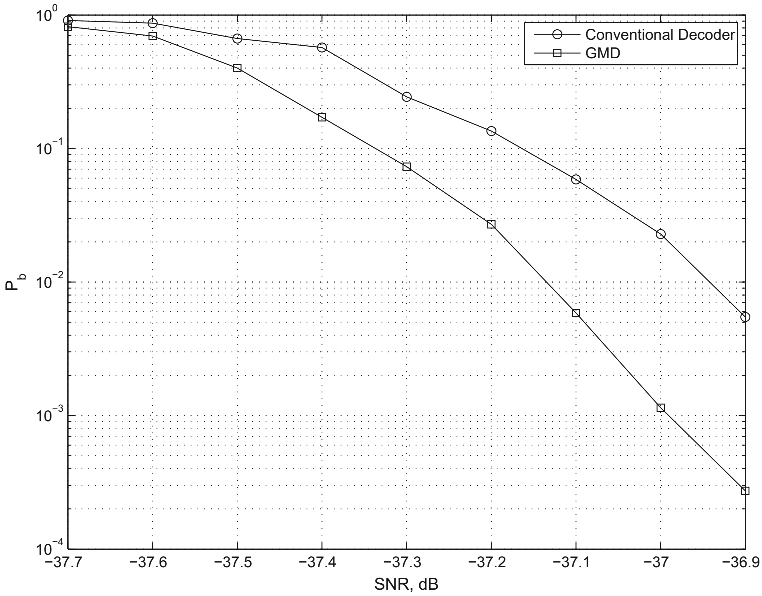

Fig. 6. Dependencies of the probability of undetected error on the value of SNR for RANK detector (number of interfering signals K = 200, inner code rate R = 1/4)

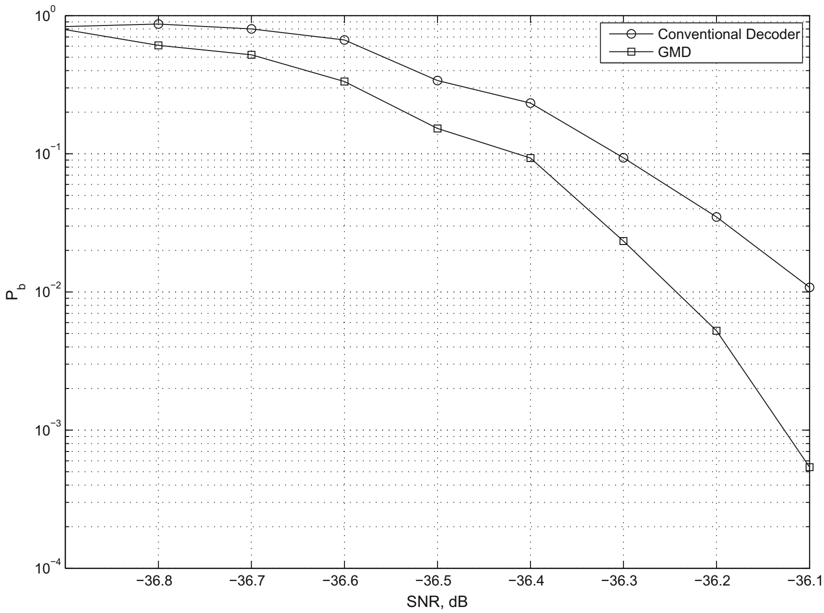

Fig. 7. Dependencies of the probability of undetected error on the value of SNR for RANK detector (number of interfering signals K = 400, inner code rate R = 1/4)

Since the outer code has sufficient error-correction capability the probability of denial is sufficiently higher than the probability of erroneous decoding. Thus it seems reasonable to consider the probability of denial separately. The respective dependencies are depicted in Fig. 8.

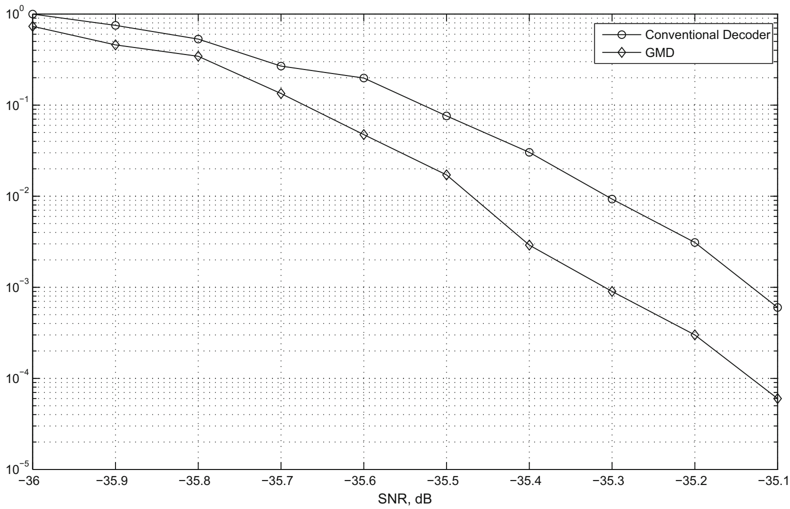

Fig. 8. Dependencies of the probability of denial on the value of SNR for RANK detector (number of interfering signals K = 300, inner code rate R = 1/5)

Let us now consider how the application of the purposed approach to GMD decoding influences the performance of the system employing α detector as inner decoder. As an example let us consider the performance of such system for inner code with rate $R = 1/6$ and various values of K (see Figs. 9 and 10).

Similarly to the case of the RANK detector the application of GMD results in substantial performance gain. The obtained curves demonstrate that the proposed solution can provide low probability of undetected error (per codeword) even under drastic interference. Finally, let us consider the effect of applying GMD on the denial probability of the outer code decoding see Fig. 11.

Again, similarly to the case of RANK detector the application of GMD in the case were α detector results in probability of denial decision decrease up to one order of magnitude.

7 Simulation Results: Restricted GMD Decoding

Hereinabove it has been demonstrated that by applying GMD as an outer code decoder in the way proposed in the previous sections the performance of the system can be significantly improved (as compared to that ensured by the conventional algebraic decoder correcting errors and erasure). However, this gain is

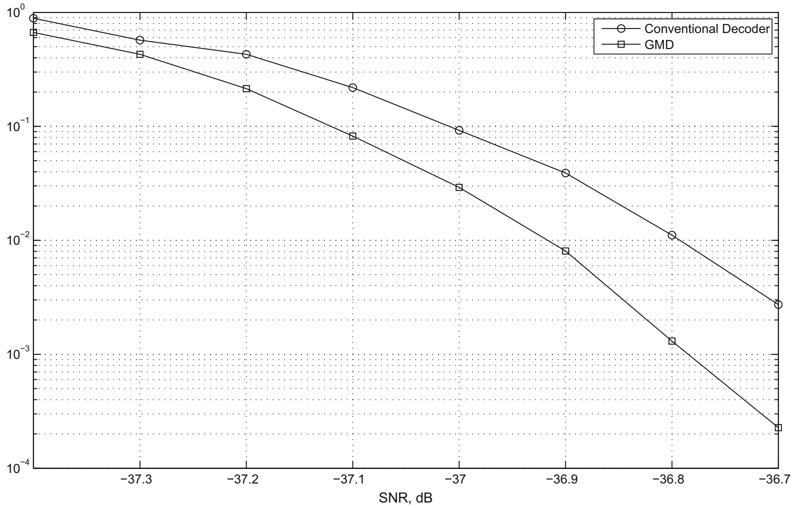

Fig. 9. Dependencies of the probability of undetected error on the value of SNR for α detector (number of interfering signals $\mathrm{K} = 400$, inner code rate $\mathrm{R} = 1/6$)

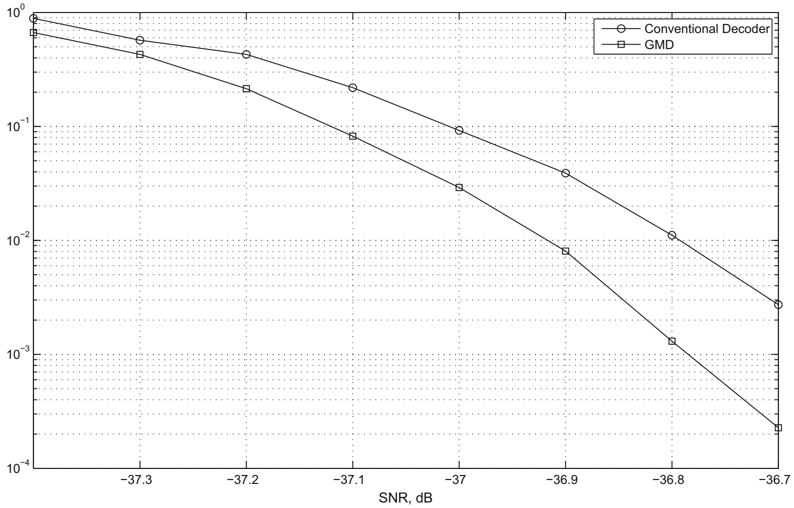

Fig. 10. Dependencies of the probability of undetected error on the value of SNR for α detector (number of interfering signals $\mathrm{K} = 600$, inner code rate $\mathrm{R} = 1/6$)

obtained at the expense of increased complexity and delay (due to the iterative nature of the decoding algorithm). Thus it seems interesting to investigate the case when the decoder is allowed to make only moderate number of iterations (i.e. not only is the number of symbols of the outer code to be erased is limited by the error correction capability of the code in use but rather is restricted in

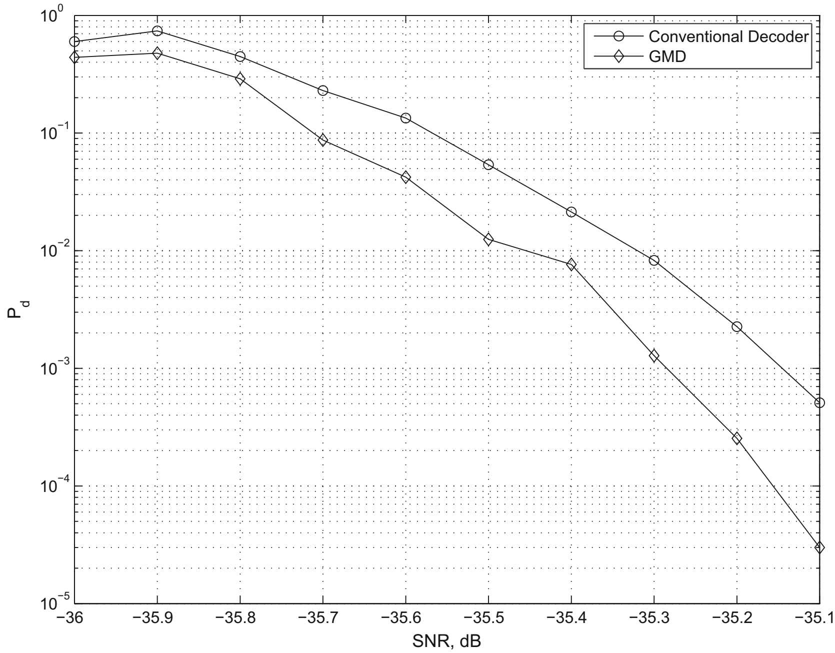

Fig. 11. Dependencies of the probability of denial on the value of SNR for α detector (number of interfering signals $\mathrm{K} = 500$, inner code rate $\mathrm{R} = 1/5$)

advance). In Fig. 12 the dependencies of the probability of undetected error on the value of SNR for the case when the number of iterations is upper bounded for the case $\mathrm{K} = 300$ are depicted (the curves for the conventional decoder and for the unrestricted GMD decoder are depicted for comparison).

It can be noted that although the GMD decoder with limited number of iterations cannot provide the reliability that can be obtained with the unrestricted GMD decoder it is still possible to decrease the probability of undetected error substantially while with fixed complexity and delay. For instance for SNR values less than −35.15 dB GMD detector with 25 iterations can decrease the probability of undetected error (per codeword) by an order of magnitude or more. Let us now consider the denial probability. In Fig. 13 the dependencies of the denial probability on the value of SNR for different values of the maximum number of iterations allowed for the case $\mathrm{K} = 300$ are depicted (the curves for the conventional decoder and for the unrestricted GMD decoder are depicted for comparison). As can be seen Figs. 13 and 12 almost coincide i.e. the probability of the fact that an uncorrectable error pattern will result in erroneous decoding is negligible as compared to the probability of the fact that an uncorrectable error pattern will result in denial.

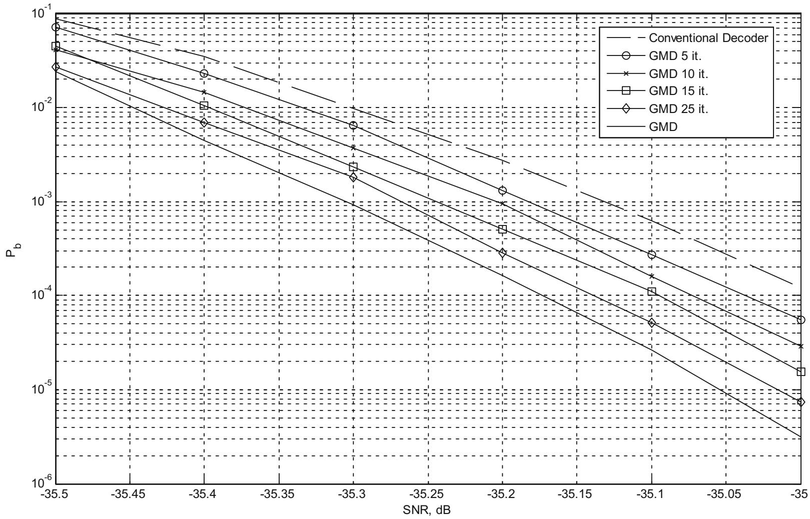

Fig. 12. Dependencies of the probability of undetected error (per codeword) on the value of SNR for RANK detector and various types of outer decoder (number of interfering signals K = 300, inner code rate R = 1/3)

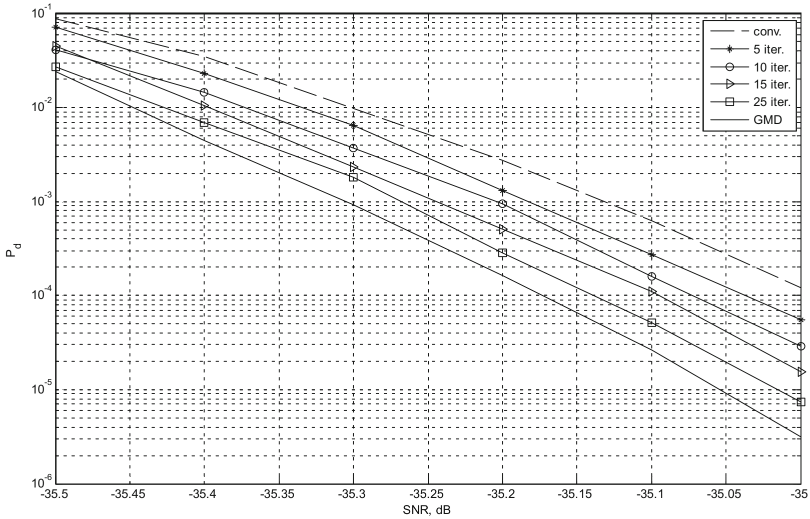

Fig. 13. Dependencies of the denial probability (per codeword) on the value of SNR for RANK detector and various types of outer decoder (number of interfering signals K = 300, inner code rate R = 1/3)

Let us now consider the case K = 200. In Fig. 14 the dependencies of the probability of undetected error (per codeword) on the value of SNR for this case and various values of maximal number of iterations are shown (the curves

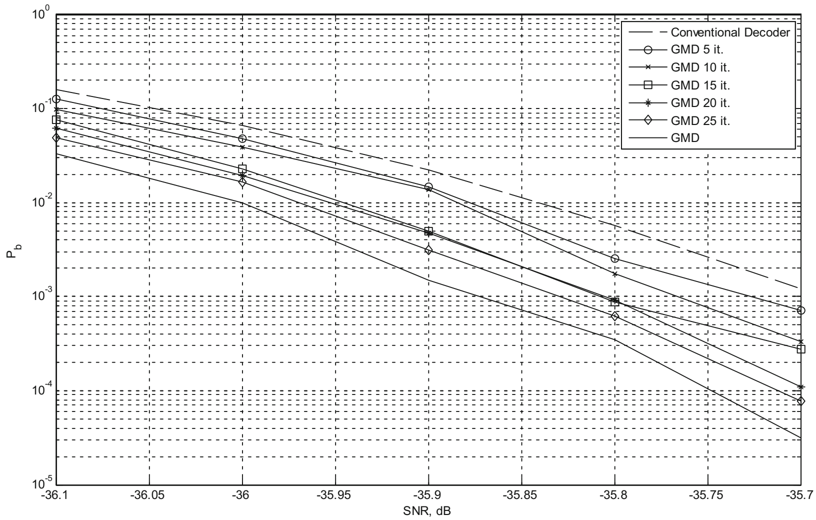

Fig. 14. Dependencies of the probability of undetected error (per codeword) on the value of SNR for RANK detector and various types of outer decoder (number of interfering signals K = 200, inner code rate R = 1/3)

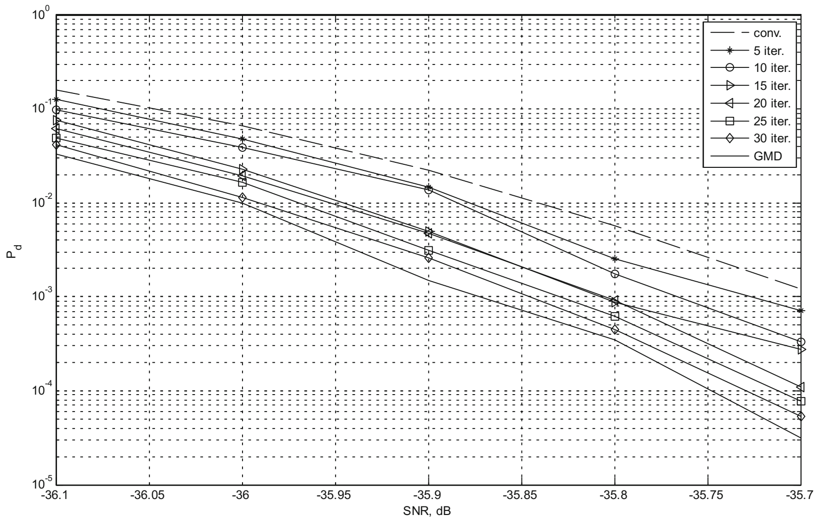

Fig. 15. Dependencies of the denial probability (per codeword) on the value of SNR for RANK detector and various types of outer decoder (number of interfering signals K = 200, inner code rate R = 1/3)

for the conventional decoder and for the unrestricted GMD decoder are also shown in Fig. 14 for comparison). The curves shown on Fig. 14 confirm that the probability of undetected error can be substantially reduced by applying

GMD decoder even if maximum number of iterations allowed is fixed i.e. the situation is very similar to that considered previously. Finally let us consider the dependencies of the denial probability on the value of SNR for different values of the maximum number of iterations allowed for the case $K = 200$. The respective curves are shown in Fig. 15.

Again we can claim that the curves for the denial probability and probability of undetected error almost coincide.

8 Conclusion and Future Work

In this paper a coded DHA FH OFDMA employing concatenated coding and a GMD as an outer code decoder have been considered. It has been demonstrated that the proposed approach can result in substantial reliability increase. It has been demonstrated that even if the number of iteration is upperbounded the GMD-type decoder proposed hereinabove can provide substantial reliability increase. Specific parameters choice is to be based on the tradeoff between the desired QoS and complexity and delay requirements. The choice in question essentially depends on the specific application and scenario. The applicability of the proposed approach to the specific communication scenarios is a subject of future work.

References

1. Kondrashov, K., Afanassiev, V.: Ordered statistics decoding for semi-orthogonal linear block codes over random non-Gaussian channels. In: Proceedings of the Thirteenth International Workshop on Algebraic and Combinatorial Coding Theory, Pomorie, Bulgaria, 15–21 June 2012, pp. 192–196 (2012)
2. Kreshchuk, A., Potapov, V.: New coded modulation for the frequency hoping OFDMA system. In: Proceedings of the Thirteenth International Workshop on Algebraic and Combinatorial Coding Theory, Pomorie, Bulgaria, 15–21 June 2012, pp. 192–196 (2012)
3. Osipov, D.: Inner convolutional codes and ordered statistics decoding in a multiple access system enabling wireless coexistence. In: Jonsson, M., Vinel, A., Bellalta, B., Marina, N., Dimitrova, D., Fiems, D. (eds.) MACOM 2013. LNCS, vol. 8310, pp. 33–38. Springer, Heidelberg (2013). doi:10.1007/978-3-319-03871-1_4
4. Osipov, D.: On jamming-proof signal-code constructions for a multiple access system. In: Proceedings of the XIV International Symposium on Problems of Redundancy in Information and Control Systems, 1–5 June 2014, pp. 74–77. IEEE Press (2014)
5. Osipov, D.: Reduced-complexity robust detector in a DHA FH OFDMA system under mixed interference. In: Jonsson, M., Vinel, A., Bellalta, B., Belyaev, E. (eds.) MACOM 2014. LNCS, vol. 8715, pp. 29–34. Springer, Heidelberg (2014). doi:10.1007/978-3-319-10262-7_3
6. Forney, G.D.: Generalized minimum distance decoding. IEEE Trans. Inf. Theory **IT–12**, 125–131 (1966)

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