2 Theory of the Construction of Hedonic Price Indices

In order to fully understand what the strengths and weaknesses of the hedonic method are and how it provides a more robust methodology for the development of office rent indices, an in-depth discussion of its theoretical underpinnings is necessary. *Triplett (2004)* points out that each hedonic price index study has two empirical parts. The researcher has to first estimate the hedonic function and then decide how to use the hedonic function to calculate the price index.[84] *Wallace (1996)* likewise states that "[…] it is useful to view the theoretical implications by distinguishing between the estimation stage concerning the appropriate specification of the hedonic price function […] and the composition stage in which the estimated hedonics are used to obtain the price indexes."[85] This chapter is organized in a corresponding manner. First, the underlying theory of the hedonic function is explained, and second, a price index[86] is developed from the hedonic function. Thereafter, some practical issues that will be addressed later in this thesis are highlighted.

2.1 Hedonic Function (Estimation Stage)

The hedonic function relates the prices of heterogeneous goods with the quantities of characteristics contained in them:

$$P = h(Z)$$

Equation 2-1

P is an *n*-element vector of prices and (Z) is a *k x n* matrix of characteristics. Hence, there exist *n* varieties of the product and *k* characteristics. This is the basic hedonic function and accounts for all heterogeneous goods.[87] In the context of this thesis, the product is a lease contract for an underlying office rental unit.[88] The term 'hedonic' is used because it is determined by the different qualities of the heterogeneous good and

[84] Cf. Triplett, J. E. (2004), p. 136
[85] Wallace, N. (1996), p. 38-39
[86] The purpose of this thesis is to develop hedonic office rent indices. In chapter 3 it will be shown, that rent is the price paid by tenants in order to obtain a certain amount of space over a specific time. As in most academic literature the term price is used to explain the hedonic theory, the same terminology will be used.
[87] Cf. Brachinger, H. W. (2003), p. 2-3
[88] An office rental unit comprises the area unit that is usable by the tenant and lettable by the landlord. Cf. Flüshöh, C. /Stottrop, D. (2005), p. 75

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by the ‘pleasure’ these would bring to the purchaser. In economic terms, ‘pleasure’ should be translated as ‘utility.’[89]

The economic interpretation of hedonic functions lies in the hedonic hypothesis. The hedonic hypothesis states that the heterogeneous goods are aggregations of their characteristics. The economic behaviour does not relate simply to the goods but also to the characteristics. In addition, the hedonic hypothesis postulates that the transaction of a heterogeneous good is a tied-in sale of a bundle of characteristics. In other words, the price of a good is the aggregation of the prices and quantities of its characteristics. *Triplett (2004)* illustrates this notion with his example of a grocery cart. If one imagines that a heterogeneous good is a filled shopping cart, then the groceries in the cart are the characteristics of the heterogeneous good. The consumer does not buy the cart but rather the groceries in the cart. When the groceries are purchased at the checkout line of a store, the aggregation of prices paid for groceries multiplied by the quantities purchased yields the total expenditure on groceries.[90] In the office property market, a tenant pays for the flow of services derived from the characteristics of the office rental unit per period of time.

The hedonic hypothesis implies that the characteristics of heterogeneous goods are the true variables in utility functions. Thus, the consumer utility function can be written as

$$Q = Q(Z, x)$$

Equation 2-2

where Q is utility, x is a vector of other, homogeneous consumption goods, and, for expositional simplicity, only one heterogeneous good in the system is specified with characteristics, (Z).[91]

Equation 2-2 also can be interpreted as a production function, where Q is the output, which has some heterogeneous inputs, with characteristics, (Z), and other, homogeneous inputs, x. If a production function is intended, then the hedonic function has to be rewritten as

[89] Cf. Triplett, J. E. (2004), p. 223, Day, B. (2001), p. 25
[90] Cf. Triplett, J. E. (2004), p. 223-224
[91] Cf. Ibid., p. 224-225, Jonker, N. (2001), p. 5-7

$$t\,(Z, K, L, M) = 0$$

Equation 2-3

Because the production of heterogeneous goods can be seen as transformation, the characteristics *Z* are produced with the standard inputs of capital *K*, labour *L*, and materials *M*. As a matter of fact, the inputs can be heterogeneous.[92]

So far, it is understood that hedonic functions can be interpreted either from the demand, or user, side (demander for office space) or from the supply, or producer, side (landlords). The following two sections illuminate both sides and explain why the approach of one side will be followed in this thesis.

2.1.1 Demand or User Side

In general, the user's choice of characteristics is determined in a two-stage budgeting process. A company looking for new or additional office space first decides on the budget for office rent expenses. After the level of office rent expenses is determined, the company decides how this budget will be allocated among the characteristics.[93] In other words, as in consumer theory, utility is maximized subject to a budget constraint.[94]

$$\begin{aligned} &MaxQ(x, Z, ; \alpha) \\ &s.t. y = x + P(Z) \end{aligned}$$

Equation 2-4

where y is the total budget of the company, *Q* is the utility, *x* is the composite good representing all other goods outside the office-property market, *Z* is a vector of office characteristics of the office rental unit $(Z = z_1, ... z_n)$, and *α* is a vector of socioeconomic characteristics.[95] The latter is necessary because office users can no longer be considered one monolithic group.[96] Companies choose levels of *x* and *Z* to maximize utility. The following equation is the first order condition of equation 2-4 and states that the

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[92] Cf. Triplett, J. E. (2004), p. 224-225

[93] Cf. Ibid., p. 226-228

[94] Thereby, the hedonic price functions are always nonlinear, since office space demanders are unable to repackage the offered office rental unit. In other words, office users cannot break up the office rental unit into its constituent characteristics and enjoy the benefit of each characteristic separate from the whole. The possibility of repackaging is referred to as arbitrage opportunity. If an arbitrage is not possible, the budget constraint is nonlinear. Cf. Day, B. (2001), p. 26

[95] Cf. Clauw, F. (2005), p. 5, Rosen, S. (1974), p. 38-39

[96] Cf. Dowall, D. E. (1988), p. 35

ratio of marginal utilities equals the marginal implicit price of the characteristic. In other words, the ratio of marginal utilities is the marginal rate of substitution between z_i and x, which describes the preparedness of a company to give up a certain quantity of one good to obtain an extra unit of another good, ceteris paribus. In addition, the term $h_i(z, y-P(Z);\alpha)$ can be seen as the marginal rate of substitution function and as the hedonic demand function.[97]

When the marginal utility of money is constant, y-P(Z) will disappear in the marginal rate of substitution functions, and they will revert to traditional demand functions. When the marginal utility of money is not constant, y-P(Z) will appear in the demand functions.[98]

$$\frac{Q_{z_i}}{Q_x} = \frac{\partial Q / \partial z_i}{\partial Q / \partial x} = \frac{\partial P}{\partial z_i} = P_i = h_i(Z, y - P(Z);\alpha)$$

Equation 2-5

In this context, the term ‘utility,’ rather than ‘profit,’ is used because office rental units not only represent a cost factor to the company but also provide benefits.

Rosen (1974) formulates a user’s action by defining a bid function $\theta(Z,q,y;\alpha)$ to stress the spatial context of the problem. In the context of the office property market, this function indicates the willingness to pay rent for an office rental unit with characteristics Z, given a variable for budget y and a level of utility. It is defined by $Q(y-\theta,z_{1\ldots}z_n;\alpha)=q$. The bid function can be demonstrated by bid curves, which are the inverted indifference curves. For a certain characteristic z_1 the optimal bundle $(\hat{x},\hat{z}_1)$ will be found on the tangency point between the highest indifference curve and the nonlinear budget constraint.[99] Similarly, a company maximizing its utility will choose the bundle of characteristics on the bid curve with the highest level of utility whilst still being able to fulfill the market conditions given by the hedonic price function.[100]

The slope of the hedonic price function and the slope of the bid function are the same. If a company wants an extra unit of characteristic z_1, its willingness to pay for that unit

[97] Cf. Bartik, T. J. (1987), p. 82, Clauw, F. (2005), p. 5, Rosen, S. (1974), p. 38-40, Day, B. (2001), p. 29-37

[98] Cf. Clauw, F. (2005), p. 5

[99] See Figure 5

[100] Cf. Clauw, F. (2005), p. 6, Rosen, S. (1974), p. 38

equals the market price of that extra unit. In addition, the chosen bundle of characteristics must be purchased by the market price as defined by the hedonic price function. This forms an upper envelope of different optimising bid functions.[101]

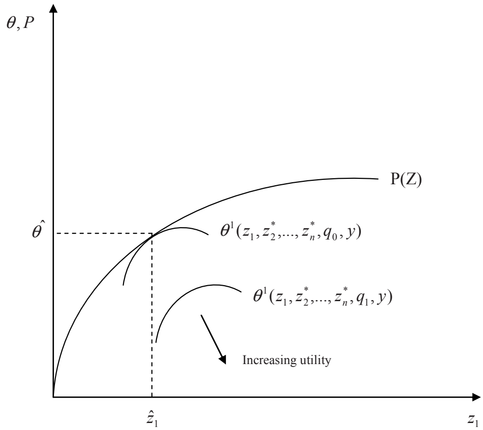

Figure 5: User Choice of Characteristic[102]

2.1.2 Supply, Producer, or Landlord Side

The landlords represent the supply side. They maximize their profits as follows:

$$Max \prod = M.P(Z) - C(M, Z; \beta)$$

Equation 2-6

where $C(M, Z; \beta)$ represents the total cost derived from minimizing factor costs to a joint production function constraint relating *M*, *Z*, and factors of production. *M(Z)* is the number of office units produced by a firm of designs offering specification *Z*. The shift parameter β reflects underlying parameters in the cost minimization problem, namely, factors prices and production function parameters. *C* is assumed to be convex with *C (0,Z)*, and the first derivatives to, respectively, *M* and z_i strictly positive. Furthermore, the production process is characterized by constant returns to scale.[103]

[101] Cf. Clauw, F. (2005), p. 6, Rosen, S. (1974), p. 40, Day, B. (2001), p. 38-45
[102] Clauw, F. (2005), p. 5
[103] Cf. Triplett, J. E. (2004), p. 229-330, Clauw, F. (2005), p. 7, Rosen, S. (1974), p. 41-43

The first order conditions of the optimal choice of *M* and *Z* require

$$P_i(Z) = \frac{\partial P}{\partial z_i} = \frac{\partial C}{\partial z_i} / M$$
$$P(Z) = C_M(M, z_1, ..., z_n)$$

Equation 2-7

In other words, this implies that, in the optimal design, the marginal price for each characteristic equals the marginal cost per unit of increasing the amount of that characteristic. Furthermore, quantities are produced up to the point where unit revenue *P(Z)* equals marginal production costs, evaluated at the optimal bundle of characteristics.[104]

In his seminal paper, *Rosen (1974)* formulates an offer function $\phi(Z, \Pi; \beta)$. This function shows the price at which the producer will offer the office unit to obtain a particular profit level π, given a particular value of *Z*. The offer function can be solved for by using the following first order conditions:[105]

$$\Pi = M\phi - C(M, z_1, ..., z_n)$$
$$C_M(M, z_1, ..., z_n) = \phi$$

Equation 2-8

The offer function can be depicted by offer curves. A certain optimal bundle $(\hat{\phi}, \hat{z}_1)$ will be found on the tangency point between the offer curves with the highest profit level whilst still being able to fulfill the market conditions given by the hedonic price function.[106] Differentiating both equations given the marginal reservation supply price for attribute *i*, increasing in z_i:[107]

$$\frac{\partial \phi}{\partial z_i} = \frac{C_{z_i}}{M} > 0$$
$$and \quad \frac{\partial \phi}{\partial \pi} = \frac{1}{M} > 0$$

Equation 2-9

[104] Cf. Clauw, F. (2005), p. 7
[105] Cf. Rosen, S. (1974), p. 42
[106] See Figure 6
[107] Cf. Clauw, F. (2005), p. 7-8

In general, there is a distribution of β across all potential suppliers. The producer equilibrium is then characterized by a family of offer functions that envelop the market hedonic price functions. The empirical content of β is anything that shifts cost conditions among firms.[108]

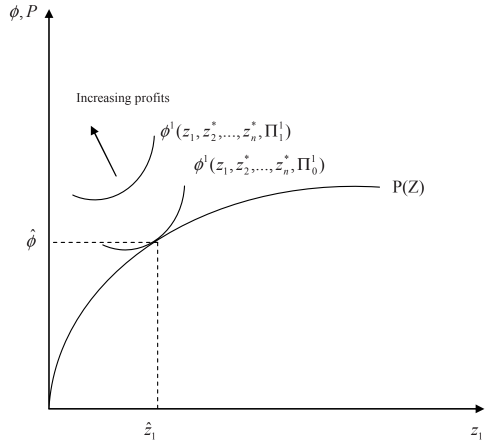

Figure 6: Landlord Choice of Characteristic[109]

2.1.3 Market Equilibrium

Different office users have different sets of bid curves, and different landlords have different sets of offer curves. Office users maximize their utility and look for the lowest possible bid, and landlords attempt to maximize their profits and look for the highest possible offer. Therefore, different choice points exist along the hedonic price function. Moreover, each point along the hedonic price function corresponds to tangency between the bid function of an office user and the offer function of a landlord.[110]

On the one hand, the requirement that the amount the consumer is willing to pay must be equal to the price (P(Z)) makes the optimal point between the bid function and price function. On the other hand, the requirement that the amount the landlord is willing to offer must be equal to the price (P(Z)) makes the optimal point to be the tangency point between the offer function and the price function. Therefore, it can be said that the ob-

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[108] Cf. Rosen, S. (1974), p. 43
[109] Clauw, F. (2005), p. 8
[110] Cf. Rosen, S. (1974), p. 44, Clauw, F. (2005), p. 8-9, Day, B. (2001), p. 46-48

servations that constitute P(Z) represent a joint envelope of a family of bid functions and a family of offer functions. In addition, it can be concluded that the hedonic model assumes a market equilibrium throughout the property market.[111]

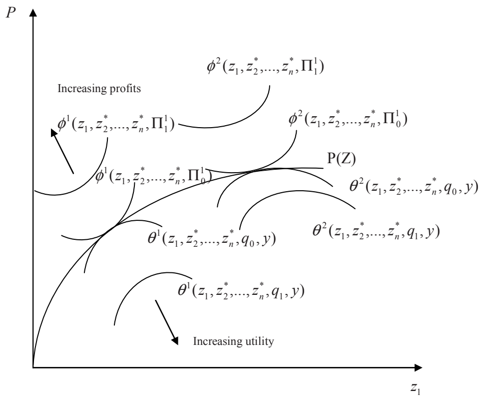

Figure 7: Market Equilibrium[112]

From the above discussion it can be deducted that one can model the hedonic price function by modeling either the supply side or the demand side. In this thesis, a demand side approach is followed. Because the stock of existing office rental units dominates the office property market in most areas and the supply of office rental units is fixed in the short run, it is assumed that office rent is demand determined.[113]

Some researchers go beyond the estimation of the hedonic function and attempt to identify supply and demand parameters for individual characteristics. This undertaking, which is often referred to as 'the second stage' hedonic price model, constitutes a separate field of research, as different methodological problems have to be tackled.[114]

2.1.4 Interpretation of Regression Coefficients in Hedonic Functions

The hedonic price function is determined using the common multiple regression technique. Thereby, the dependent variable price is regressed against the characteristics that are thought to be of highest value to the user. The regression coefficients measure the

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111 Cf. Clauw, F. (2005), p. 8-9, Rosen, S. (1974), p. 44-45, Day, B. (2001), p. 46-48

112 Clauw, F. (2005), p. 9

113 Cf. Geltner, D. M., et al. (2007), p. 6-7, Ball, M., et al. (1998), p. 22-24, Kurzrock, B.-M. (2007), p. 88, Jedem, U. (2006), p. 32, Day, B. (2001), p. 47

114 Cf. Brasington, D. M. /Hite, D. (2005), p. 4-5, Malpezzi, S. (2002), p. 59

implicit prices for characteristics. From a user's perspective, the implicit prices ought to have some relation to what the users are paying for units of the characteristic. In hedonic studies, it is common to inspect estimated hedonic functions for the plausibility of the estimated coefficients.[115] This undertaking again implies that the appropriate set of characteristics has been included in the hedonic function. In other words, the search for the characteristics for which the user is willing to pay is of utmost importance. If the wrong variables have been selected, then the hedonic function and, as a consequence, the hedonic price index are wrong and useless. *Triplett (2004)* underscores this point: "[…] the first principle for conducting a hedonic study is: Know your product."[116]

Besides the in-depth search for the correct set of characteristics, which will be undertaken separately,[117] the interpretation of the regression coefficient depends on the choice of the functional form of the hedonic function. In hedonic price studies, the following four functional forms are widely used:[118]

The simplest functional form is the ordinary linear approach given by

$$p = \beta + \sum_{i=1}^{K} \beta_i z_i + u$$

Equation 2-10

with hedonic prices

$$\frac{\partial p}{\partial z_i} = \beta_i$$

Equation 2-11

The regression coefficient β_i (i=1,…,K) indicates the marginal change of the price with respect to a change of the i-th characteristic z_i of the good. The term u represents a disturbance or error in the regression model.

[115] Cf. Triplett, J. E. (2004), p. 137-138
[116] Ibid., p. 138
[117] See Chapter 4
[118] Cf. Maurer, R., et al. (2001), p. 4-6, Li, W., et al. (2006), p. 13-14, Brachinger, H. W. (2003), p. 3-4

Another functional form is the semilog form given by

$$\ln p = \ln \beta_o + \sum_{i=1}^{K} \beta_i z_i + u$$

Equation 2-12

with the hedonic prices

$$\frac{\partial p}{\partial z_i} = \beta_i p$$

Equation 2-13

Here the regression coefficients can be interpreted as growth rates. The coefficient β_i (i=1,…,K) indicates the rate at which the price increases at a certain level, given the characteristic z.

In addition, a double log form can be chosen as the functional form given by

$$\ln p = \ln \beta_o + \sum_{i=1}^{K} \beta_i \ln z_i + u$$

Equation 2-14

with the hedonic prices

$$\frac{\partial p}{\partial z_i} = \frac{\beta_i}{z_i} p$$

Equation 2-15

The regression coefficients can be interpreted as partial elasticities. The coefficient β_i (i=1,…, K) indicates how much in percent terms the price *p* increases at a certain level if the i-th characteristic z_i changes by one percent.

Few hedonic studies recommend the Box-Cox form given by

$$p^{\lambda} = \beta_0 + \sum_{i=1}^{K} \beta_i z_i + u$$

Equation 2-16

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