# 2 State of the Art

Towards a Gesture Perspective on Epistemic Processes

The starting point for this study stems from an interest in how gestures contribute to learning processes in social interaction. By investigating this issue, I aim to provide a theoretical basis from which can be hypothesized on the potential of gestures as a didactical means. This chapter seeks to embed this into recent and current research, singling out how the study aims to take part in investigating the epistemic role of gestures. However, the study of gestures in mathematics teaching and learning is a still young research field. Having just started to gain attention during the last two decades, this interest can be considered being initiated by the rise of the *embodied cognition theory* that embraces the body's role in shaping the mind (Varela, Thompson, & Rosch, 1991).

## 2.1 A Psychologist's Approach to Gestures in the Learning of Mathematics

Based on this and on the close relationship between gesturing, thinking and speaking as proposed by McNeill (1992; 2005) and Kendon (1988; 2004), researchers especially from the field of psychology started to investigate gestures in relation to students' mathematical thinking. Their studies shed light on how students' use of gestures reveals mathematical conceptualization and learning strategies (Alibali & Goldin-Meadow, 1993; Alibali, 1999; Goldin-Meadow, Nusbaum, Garber, & Breckinridge Church, 1993) and on how it influences their mathematical skills (Alibali & Goldin-Meadow, 1993; Alibali & diRusso, 1999; Goldin-Meadow, 2003; Broaders, Cook, Mitchell, & Goldin-Meadow, 2007; Goldin-Meadow, Cook, & Mitchell, 2009; Goldin-Meadow, 2010). The insights gained from these studies can be considered as groundbreaking from a psychological perspective to investigate students' mathematical knowledge, providing a diagnostic potential the teacher can react to[6].

[6] See also chapter 3.2.2 for their results on the concept of gesture-speech mismatches and its role in teaching and learning as described in these studies.



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Nevertheless, it does not cover two main issues that are sought to be addressed in this study:

First, learning mathematics is seen as a social process that takes place in social interaction, as for example in the mathematics classroom. The aforementioned experimental studies did not consider the acquisition of knowledge in collaboration with peers and the teacher but observed gestures related to the students' current state of mathematical conceptualization. Second, the mathematics within these studies concerns elementary topics such as 'learning to count' (e.g. Alibali & diRusso, 1999) and mathematical equivalence problems.[7] The studies do not make any statements about students' gestures related to mathematical ideas on a higher abstract level, comprising generalization and reasoning.

## 2.2 Research in Mathematics Education on the Role of Gestures in Learning Processes

Recent research on gestures in mathematics education points out the crucial role gestures fill in communication in mathematical talk (Edwards, 2009) and in collaborative learning processes (Arzarello, Robutti, Paola, & Sabena, 2009; Bjuland, Cestari, & Borgersen, 2008; Reynolds & Reeve, 2002; Yoon, Thomas, & Dreyfus, 2011), giving a special remark about students' reasoning and argumentation (Bjuland et al., 2008; Chen & Herbst, 2013; Marghetis, Edwards, & Núñez, 2014; Rasmussen, Stephan, & Allen, 2004). Furthermore, studies also highlight the potential of gestures for exploration and experimentation while working out mathematical ideas (Chen & Herbst, 2013; Sabena, 2007; Yoon et al., 2011), and for coordinating the work with given inscriptions, for example through focusing by pointing on different representations of the same situations, and by sliding between them (Bjuland et al. 2008).

Reynolds and Reeve (2002) conducted a problem solving situation to find out how students use gestures to communicate their interpretations of unknown graphs. The students worked on different tasks, each concerning the graphical representation of speed as a rate. In both cases, graphs were given to be investigated in dyadic collaboration: First, by assigning specific situations to given graphs; second, by inventing a description of a bus trip that fits a speed-time graph provided by the task. Reynolds and Reeves observed that gestures are used to direct the others' attention to certain aspects of the graphs, and also as a means to express an understanding of a concept that has not been fully elaborated at that point in time (Reynolds & Reeve, 2002, p. 457). They show how missing or insecure mathematical terms can be overcome by providing additional reference to a

[7] That is, problems that concern filling in a missing number in equations of the kind 2+7+8 = _+8 (Alibali & Goldin-Meadow, 1993; Goldin-Meadow, 2003, pp. 41-47).

mathematical entity. Gesture thus can support mathematical communication by providing a joint visual reference for the participants.

This has also been identified as benefiting reasoning in collaborative problem solving in a study conducted by Bjuland, Cestari, and Borgersen (2008). Concluding from results of their empirical study, they determined that gestures and discourse "constitute mathematical reasoning" (ibid., p. 289). In this study, they asked 12-year-old students to transfer a real world representation of a situation to its corresponding representation in a coordinate system: Four people were displayed, all differing in age and height. The coordinate system displayed the age on the y-axis and the height on the x-axis. The students were asked to assign one person from the real world representation to each point within the coordinate system. In order to convince their classmates, the students reasoned their hypotheses first and foremost helping themselves by pointing gestures to complement their verbal utterances. In this regard, gesture is considered a *necessary* complementation to verbal discourse. Furthermore, this study shows how gesture helps coordinating and relating the different representations between which information needs to be transferred.

Chen and Herbst (2013) set a special focus on students' use of diagrams in conjecturing and reasoning processes in geometry class. According to them, the amount of information given in a diagram influences the configuration of gestures: They argue that "when limited information is given in a diagram, students make use of gestural and verbal expressions to compensate for those limitations as they engage in making conjectures" (ibid., p. 285). This has been concluded from observations made about students' work on similar diagrams in two different geometry classrooms. They compared students' use of the diagram in an 'intact lesson' in which a complete, "intact" diagram was handed in, with the use of the diagram in an 'intervention lesson' in another class. In the intact lesson, marks and labels in the given diagram provided a lot of information, whereas the students in the intervention lesson worked with a similar diagram that contained less information. According to Chen and Herbst, gestures have been used differently: While the students in the intact lesson exclusively used pointing gestures to refer to facts given in the diagram (ibid., p. 291), the students in the intervention lesson also made use of iconic gestures to refer to the mathematical object. They "were extensively employed to represent the object that had not been represented in the diagrams, and so to represent the students' conception of figures" (ibid., p. 293).

These three examples make apparent that gesture, speech, and inscription crucially work together when knowledge is constructed in social interaction. To investigate this interplay more profoundly, Arzarello developed a model to analyze the relationships between semiotic resources of different kinds as they are used in learning processes (Arzarello, Ferrara, Robutti, Paola, & Sabena, 2005; Arzarello, 2006, pp. 280-288). He adopts a multimodal approach to learning and proposes to always observe signs within the intertwined relationships among gesture, speech, and inscription, constituting *semiotic bundles*.[8] These semiotic bundles are dynamic structures, the relations between them changing over time. Because of that, both a synchronic analysis of mutually used signs and a diachronic analysis of the use of signs across the process are considered for investigating students' semiotic bundles in learning processes. One diachronic relationship is given by the so-called *genetic conversion*, which is identified when one semiotic set is converted into another one (Arzarello, 2006, p. 281). This is given, for example, when the reference made by a gesture is fixed so that the inscription arises from a former gesture and by this, enlarges the semiotic bundle.[9] Concluding from a study carried out in a 11th grade calculus class, Arzarello et al. (Arzarello, Robutti, Paola, & Sabena, 2009) emphasized that "the synchronic and diachronic analyses show the complex intertwining of gestures, speech, and inscriptions in learning mathematics. These ingredients jointly support the thinking processes of students in a unitary way" (ibid., pp. 106-107). Arzarello and his colleagues observed that students create personal signs to express mathematical meaning. These can become endowed with meaning within social interaction by being used repeatedly in similar contexts. Those signs are called *basic signs* (Arzarello & Paola, 2007) and their use can be initiated and supported by the teacher, playing a *semiotic game* with the students. This allows the teacher "[to tune] with the students' semiotic resources and [to use] them to guide the evolution of mathematical meaning" (Arzarello, Bikner-Ahsbahs, & Sabena, 2009, p. 1547). Arzarello, Robutti, Paola, and Sabena further note that "in order that such opportunities can become concretely realized, the teacher must be aware of the role that multimodality and semiotic games can play in teaching" (Arzarello, Robutti, Paola, & Sabena, 2009, p. 107).

## 2.3 The Epistemic Role of Gestures Against The Background of Different Theoretical Frameworks

The abovementioned applications of the semiotic bundle analyses have been carried out against the background of a semiotic-cultural approach (Radford, 2003). That is, they take into account the students' semiotic-cultural background, referring to Radford's process of objectification, described etymologically as "a process aimed at bringing something in front of someone's attention or view" (Radford, 2002, p. 14). For example, gestures referring to a graphical representation may function as a means to make something visible or indicate something that has not been apparent before. Representations are one specific form of what Radford calls *semiotic means of objectification*, generally denoted as any "objects, tools, linguistic devices, and signs that individuals intentionally use in

[8] Also other semiotic sets, for example artefacts used by the students may be part of a semiotic bundle.
[9] The model of the semiotic bundle will be described more in detail in chapter 4.3.

social meaning-making processes to achieve a stable form of awareness, to make apparent their intentions, and to carry out their actions to attain the goal of their activities" (Radford, 2003, p. 41). Furthermore, he sees great potential in gathering insights into the process of the social construction of knowledge by investigating learners' use of semiotic means of objectification. Radford has been one of the first to appreciate the role of different signs as they play together to constitute mathematical meaning, and to establish a theory about it. Against the background of this theory, Sabena (2007) investigated the role "gestures play in the semiotic bundle intervening in processes of knowledge objectification in Calculus" (Sabena, 2007, p. 257). Among others, she found that a gesture may contribute to a blended character of the semiotic bundle, i.e. to the possibility of merging two related meanings in a gesture by using it in two different, but related contexts. In Sabena's example, a gesture referred to as the Δ-gesture[10] is used in a numerical and in a geometrical context: Using this gesture in a geometrical environment but using language related to Numerics, the two meanings are blended within the gesture (ibid., pp. 259-260). Sabena also suggests that the less contextualized the character of the mathematical activity becomes, the more conventionalized are the gestures used in the discourse as carriers of a certain meaning. In other words, they get a symbolic character developing somehow in classroom interaction (ibid., pp. 261-262). Objectification, and with that "learning, […] can be theorized as those processes through which students gradually become acquainted with historically constituted cultural meaning and forms of reasoning and action" (Radford, 2010, p. 3). This approach to learning considers the teacher as participating and mediating a mathematical culture within the mathematics classroom. While this becomes an important component in the actual application of theories for teaching practice, it leaves aside how, and what kind of knowledge is constructed *by the students*. However, getting a better understanding of students' epistemic processes is considered important in order to become aware of conditions that foster or hinder them. Since gestures constitute a part of the social interaction in which the epistemic processes take place, my study addresses the conditions shaped by gestures.

Just recently, a case study of networking two approaches demonstrated that the epistemic bundle model can be adopted to provide deeper insights into epistemic processes: Two research groups, each focusing on different aspects of students' learning processes, collaborated to add a semiotic perspective to analyzing an epistemic process (Dreyfus, Sabena, Kidron, & Arzarello, 2014). With this, they sought to answer *whether* gestures contribute to the *individual* construction of mathematical knowledge and of course, also the ways in which they contribute to it. For this purpose, both groups decided about a

[10] The shape of the gesture can be described as follows: The thumb and index finger are held as if they were holding something, like an imaginary line segment, between them. The fingers indicate a starting point and end point of this segment. Depending on the mathematical context, i.e. depending on whether the context is geometrical or numerical, narrowing the fingers means a line segment getting shorter or a number that is decreasing.

piece of data together: Dreyfus and Kidron (in the following referred to as the 'epistemic group') searched for episodes that bear a "potential for the emergence of constructing knowledge" (ibid., p. 131), based on an a-priori-analysis of the task. The 'semiotic group' (Arzarello and Sabena), however, preferred to analyze episodes in which gestures were used in order to communicate. The amount of data interesting for both groups resulted to be very small (ibid., p. 129). Therefore, their initial plan to compare two parallel analyses of the same episode was changed in favor of integrating gesture analysis to the epistemic analysis of an episode in which the process of knowledge construction has been found to be hardly accessible from the verbal utterances alone. This way they aimed to enrich the theoretical and methodological framework of the epistemic group. The semiotic perspective on the epistemic process allowed the epistemic group to attain better comprehension of the students' construction of knowledge by considering the gestures as an individual and a social means of expression to "better understand the interplay between the social and the cognitive dimensions" (ibid., p. 149). In fact, they found that gestures contribute to the process of constructing knowledge in different ways, for example to "illustrate or clarify to [the students] themselves the mathematical objects and their properties rather than to communicate to one another" (ibid., p. 146). Dreyfus et al. call gesture that fulfil such a function *epistemic gestures* (ibid. p. 146). Given another example, they identified a gesture that recurred during the epistemic process. The semiotic group interprets these to recall meaning as embodied in a reasoning situation, while the epistemic group considers it as an additional sign for consolidating constructed knowledge, confirming what they have already reconstructed from "the progressively more elaborated language" (ibid., p. 136).

While at first sight this seems to deal with exactly the same issue as does the study presented here, crucial differences are evident when taking a second look: The epistemic group focuses on *individual* processes of constructing knowledge. That is, the reconstructions of the epistemic processes concern the individual constructs of the students as they develop in social interaction through communication as cognition-driven. It does not take into account the *social* constructs developing in social interaction. Furthermore, a case study cannot provide in-depth insights into such a phenomenon as epistemic gestures, but can indicate their existence and significance.

However, this case of networking shows that the integration of gestures in the analysis of epistemic processes can supplement an approach that had been mostly restricted to the interpretation of verbal means of expression to this point.

## 2.4 The Epistemic Role of Gestures in This Project

The current study understands knowledge as constructed as shared among the participants. To analyze this, the epistemic action model developed by Bikner-Ahsbahs will be used (Bikner-Ahsbahs, 2005; 2006, see also chapter 4.1). Bikner-Ahsbahs obeserved how interest-dense situations (Bikner-Ahsbahs, 2004; Bikner-Ahsbahs, 2005, pp. 119-149) emerge in the mathematical classroom and describes characteristics of such situations in which mathematical knowledge is constructed (Bikner-Ahsbahs, 2005). As a consequence, her findings lead to conclusions about conditions that foster and hinder the emergence of interest-dense situations and with that, of the construction of mathematical knowledge. Her epistemic action model embraces the three epistemic actions of **g**athering mathematical entities, **c**onnecting them and seeing mathematical **st**ructures, such as patterns or generalities (GCSt). Knowledge in this regard is understood as shared structural relation generated through connecting and structure-seeing in social interactions (Bikner-Ahsbahs, 2005, pp. 200-204, see also chapter 4.1). The model has been developed based on reconstructions of the social interaction tracing the development of knowledge in a *semiotic sequence analysis* (Bikner-Ahsbahs, 2005; 2006; see also section 5.2.3.1) . However, the reconstruction was framed by speech act analysis and non-verbal signs have been taken into account only when needed for the interpretation of the speech act. Observations of social interactions in the mathematics classroom and also the research presented in this chapter show that non-verbal signs cannot be seen as merely supporting verbal language, but also as a particular part of social interactions in which epistemic processes take place. Hence, excluding them from analysis might lead to overlooking important aspects. This study will focus on gestures as specific kinds of signs to get insights about how their use can foster the construction of mathematical knowledge, but also how it may hinder it.

The review of literature leads to focussing on two aspects of gestures in mathematical epistemic processes:

1. **The communicational aspect**: As a social means of expression, gestures support the verbal utterance and facilitate the expression of non-elaborated ideas so that they are offered to the shared pool of ideas. Gestures may help to compensate for lacking terms (Reynolds & Reeve, 2002), to simplify the verbal expression by using deictics (Bjuland et al., 2008), or to test ideas or approaches without going too much into detail (Sabena, 2007, pp. 260-261). All of these communicative functions are suggested to lighten the cognitive load[11] by allowing verbal utterances to be imprecise up to a certain point while still giving reference to the core of the idea. This unburdening of speech when words are hard to find can also

[11] In cognitive science, the cognitive load described the cognitive effort that has to be overcome in order to make progress in learning. Sweller, as one of the first ones to highlight this topic, investigated students‘ problem solving strategies and how they are influenced by the instructional design (Sweller, 1988). He established Cognitive Load Theory to investigate the factors that influence this effort, integrating the concept of the schema thinking pattern to release this effort: "[…] in contrast to a huge long-term memory, working memory is very limited. Working memory can store and process no more than a few discrete items at a given time" (Sweller, 1994, p. 299). The research just cited suggests that gesture can reduce the number of items and processes stored in order to communicate information by means of words.

be regarded as benefitial from the theory of interest-dense situations: According to Bikner-Ahsbahs (2004), one of the characteristics of interest-dense episodes lies in the progressive dynamics of the epistemic process[12] in which knowledge is socially negotiated. This can be disturbed "by the unwillingness to accept indistinct meanings" (Bikner-Ahsbahs, 2004, para. 6). Her findings reveal that a hindering condition for the emergence of interest-dense situations can be caused by the teacher when "forcing the students to use precise words before the interaction process may continue. This preciseness of words disturbs the flow of ideas and prevents the students from presenting their own ideas" (Bikner-Ahsbahs, 2004, para. 6). The possibility to express ideas in a premature stadium is thus essential for social learning processes and may be benefited by using a communicative function of gestures. In this regard, gestures may ground the shared understanding on visual means of expression rather than on verbal ones.

2. **The representational aspect:** Gestures can themselves provide visual representations of a mathematical object that are not or cannot be represented in inscription. Studies of more advanced learners and of teachers give evidence that the conceptualization of more sophisticated mathematical ideas may be revealed by gestures in the gesture space in front of a person (Edwards, 2009; Marghetis et al., 2014; Sabena, 2007). This can also be endowed with mathematical meaning to be used as shared in social interaction, as described by Yoon, Thomas and Dreyfus (2011). It allows to collaboratively elaborate more general ideas by detaching the reference from the concrete inscription and provides the potential for using the gesture space as an experimental space for trying out and representing ideas (ibid., p. 390). The difference to the communicational function as mentioned above is seen here in the potential to represent something to work on and not to compensate speech, as mentioned above. This has been seen also in the study of Chen and Herbst (2013), where gestures have been used to complement inscription in order to carry out or test hypotheses: "[Iconic] gestures represented virtual mathematical objects […] or mathematical relationships […]. Students also gestured to animate the diagrams" (ibid., p. 302). Gestural representation can thus be considered to potentially bring forward the epistemic process by representing the unrepresented.

Nevertheless, also possible pitfalls are to be mentioned: Yoon et al. (2011) point out the aspect of inaccuracy of gestures that "may lead to significant miscommunications and misconceptions" (ibid, p. 390). They suggest that there need to be moments of synchronizing details, for example by producing inscriptions. Furthermore, and probably of more importance, the meaning of gesture is not given per se. It needs to be elaborated by the students together with the knowledge of the object it refers to

[12] Literally, she refers to this as '*progressive Erkenntnisdynamik*' (Bikner-Ahsbahs, Kidron, & Dreyfus, 2011) in German and "dynamic of the epistemic process" in English (Bikner-Ahsbahs, 2004).

in order to express *shared* meaning in social interaction and to be potentially have a beneficial effect for all the participants.

This study aims at investigating how both, the communicational as well as the representational aspect of gestures may play a part in fostering and in hindering social processes of constructing mathematical knowledge. An analysis of the epistemic process carried out by epistemic actions according to the GCSt-model shall thus be combined with an analysis of gestures within the semiotic bundle, shedding light on gestures' contribution to social epistemic processes.

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