

Foreword

“anyway the exponential function would ,wouldn't go up again' because one but it would flatten further that would than haven u-h ,as its asymptote” is Tim’s commentary as he watches the computer screen in front of him. On the computer screen we find a coordinate system. Starting from the first quadrant, a parabola is swept out, crossing the y-axis in the curve’s lowest point. However, Tim does not directly refer to the parabola but to the graph of an exponential function, that is not visible on the screen. In his commentary, Tim wants to say that the visual piece of the curve on the screen cannot represent an exponential function, “... because one but it would flatten further that would than haven u-h ,as its asymptote”. The last part of his utterance is accompanied by a gesture: directly in front of the computer screen, Tim’s finger draws the graph of an exponential function in the air, passing from the right top downwards, a bit up again before going to the left and approximating the x-axis.

Such situations often happen in mathematical learning situations. Even when unobserved, students may use gestures to approach mathematical ideas or clarify aspects, specifically when solving problems. This does not only happen with graphical or geometrical topics. Learners may represent all kinds of things by gestures, for instance if they indicate the substitution of an algebraic expression by a “grasping hand” as is described in this book. Such representational gestures are relevant tools for constructing mathematical knowledge, as has been known for quite some time. However, what has not been known is how gestures enable students to advance their epistemic processes. In her thesis, Christina Krause investigates this research problem. Her main approach is to merge two models, an epistemic action model, representing the three epistemic actions of gathering and connecting mathematical meaning as a basis for structure seeing, and the semiotic bundle model, connecting semiotic means, such as gestures, language and representations. Empirically based, she identifies ‘specifying gestures’, specifying the *Where*, the *What* and the *How* of mathematical ideas and also *Relationships* between mathematical aspects. In the example above, Tim not only specifies what he refers to verbally, the exponential function, but also its graphical style, hence the *How* and, in terms of direction, also the *Where*. Through comparison, Tim also constitutes a relationship between two graphical representations: one represented on the screen, the other by tracing the style of its shape with a gesture in front of the screen in the air. In the investigated data more than 90% of the gestures referring to mathematical objects were specifying gestures, and on average 40% of them specified several things at the same time.

For her empirical investigation, Christina Krause uses given data about pairs of students solving mathematical problems. These data, already transcribed, were re-worked in detail and enriched by pictures of gestures and inscriptions. In her fine-grained analysis,

she focused on so called “epistemic-dense episodes”, the most relevant parts for structure seeing. For that she clarified the epistemic role of gestures: two planes of analyses are distinguished, one in which gestures represent mathematical aspects related to speech and inscriptions, the second in which gestures relate to epistemic actions. For the latter, Christina Krause is able to reconstruct two different types of epistemic functions affecting the epistemic processes: *performing functions* and *forming function*. This frame of concepts linked to results from gesture studies allows her to reconstruct processes of coming to know mathematics as they take place in epistemic-dense situations. The results show that gestures may prepare, shape, substitute or directly advance epistemic actions. In the example above, Tim’s gesture sources parts of the argumentation out from speech to visualize them by gesturing. By this, gesturing simplifies verbal argumentation.

Gestures do not only advance epistemic insights, they may even assist to create them and to elaborate them. For example, as the abovementioned “grasping hand” was repeated, the idea of substitution afforded by a task on continued fractions evolved towards new insights. Hence, gestures may indicate on which idea a student is working, thus providing diagnostic potential for teachers’ noticing. For this fresh area of research, Christina Krause has made a significant contribution by showing how gestures are used spontaneously to complement speech and advance mathematical insights.

She succeeds not only in presenting her research results in an exciting manner, her book is also shaped in an attractive way. This shaping is not just an end in itself: it makes her fine-grained gesture analyses accessible and transparent to the reader. Her analyses do not only affect the reader by consistent argumentation but they often happen to be much more directly convincing, e.g. when one realizes that while reading the meaning of the “grasping hand” is grasped by gestural imitation.

At first sight the data base seems rather small, but the added value of this research lies in the theoretical and methodological deepness having led to a framework for studying a new topic in a fresh field of research. Only now it will be possible to explore the epistemic function of gestures systematically with qualitative as well as quantitative methods. Therefore, we wish that the national and international community of researchers interested in gesture research and beyond will adopt the results of this dissertation and will be inspired by this book and its exciting contribution to our understanding of mathematics education practices and research.

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