2 Basics of fault detection and performance evaluation techniques

Modern large-scale plants are composed of several interconnected process units. Each unit can be approximately modelled using an linear time invariant (LTI) system. Such systems are commonly classified into static and dynamic classes. Mathematical descriptions of them are different, meanwhile, the occurrence of additive and multiplicative faults will impact them in different ways. As the fundamental issues of PM-FD, these two problems will be addressed in the first part of this chapter. Performance evaluation for PM-FD methods is primarily conducted using some standard indices. In the second part, commonly used indices and a new index will be presented.

2.1 Technical description of static processes

In a control system, one of the most important blocks is the sensor, which measures the process value for the control purpose. Due to uncertain environment around the process, the process value returned by the sensor will be affected by various disturbances that will cause the value to deviate from the true value. Furthermore, there may be faults in the process which will disturb the measurement value. For static processes, the observed process value returned by the sensor can be split into three parts: the mean value, changes to the process, and random fluctuations. Therefore, consider a number of sensors, and let the sensor variables be a vector denoted by $\mathbf{y}_{obs} \in \mathbb{R}^m$ and written as [67]

$$\mathbf{y}_{obs} = \mu_y + \underbrace{\mathbf{A}\mathbf{z} + \boldsymbol{\nu}}_{\mathbf{y}} \tag{2.1}$$

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where μ_y is the mean (or expected) process value, $\mathbf{A} \in \mathbb{R}^{m \times m}$ denotes the process parameter matrix, $\mathbf{z} \in \mathbb{R}^m \sim \mathcal{N}_m(0, \mathbf{I}_m)$, $\mathbf{Az}$ represents the relevant process changes, $\boldsymbol{\nu} \in \mathbb{R}^m \sim \mathcal{N}_m(0, \Sigma_\nu)$ denotes the measurement noise, Σ_ν is a diagonal matrix. For such a system, the covariance structure is written as $\Sigma_y = \mathbf{A}\mathbf{A}^T + \Sigma_\nu$ with $\mathbf{y} \sim \mathcal{N}_m(0, \Sigma_y)$.

In general, process faults are classified as either additive or multiplicative faults. An additive fault only changes the mean value of the process and can be modelled as:

$$\mathbf{y}_{obs,f} = \mu_{y,f} + \mathbf{Az} + \boldsymbol{\nu} = \mu_y + \underbrace{\boldsymbol{\Xi} f + \mathbf{Az} + \boldsymbol{\nu}}_{\mathbf{y}_f} \tag{2.2}$$

where $\mathbf{y}_f \sim \mathcal{N}_m(\boldsymbol{\Xi} f, \Sigma_y)$, $\boldsymbol{\Xi} \in \mathbb{R}^m$ is a vector of unit length denoting the fault direction, and $f \geq 0$ denotes the fault magnitude. Additive faults represent changes in a sensor's accuracy. On the other hand, multiplicative faults result from changes in the covariance structure. A typical case is the parameter change occurring in $\mathbf{A}$. For such faults, the measurements are represented as

$$\mathbf{y}_{obs,f} = \mu_y + \underbrace{(\mathbf{A} + \Delta\mathbf{A})\,\mathbf{z} + \boldsymbol{\nu}}_{\mathbf{y}_f} \tag{2.3}$$

where $\mathbf{y}_f \sim \mathcal{N}_m(0, \Sigma_{y,f})$ with $\Sigma_{y,f} = (\mathbf{A} + \Delta\mathbf{A})(\mathbf{A} + \Delta\mathbf{A})^T + \Sigma_\nu$. $\Sigma_{y,f}$ can be simply converted to $\Sigma_{y,f} = \mathbf{M}\Sigma_y\mathbf{M}$ [2, 9, 112, 114], with $\mathbf{M} \in \mathbb{R}^{m \times m}$ and $M_{i,j}$ representing the change of variance and covariance. They are mainly referred to abnormal changes like the degradation of working components in processes. Another common type of multiplicative fault is an independent multiplicative fault that causes changes in the diagonal terms of the covariance matrix, Σ_ν. Such a change represents a change in the precision of the sensor. This fault can be modeled as:

$$\mathbf{y}_{obs,f} = \mu_y + \underbrace{\mathbf{Az} + \boldsymbol{\nu}_f}_{\mathbf{y}_f} \tag{2.4}$$

where $\mathbf{y}_f \sim \mathcal{N}_m(0, \Sigma_f)$, with $\Sigma_{y,f} = \mathbf{A}\mathbf{A}^T + \Sigma_{\nu f}$. Furthermore, $\Sigma_{y,f}$ is likewise represented as $\Sigma_{y,f} = \mathbf{M}\Sigma_y\mathbf{M}$, where $\mathbf{M} = \mathrm{diag}(M_1, \cdots, M_m)$, M_i denotes the variance change in the i^{th} variable.

2.2 Technical description of dynamic processes

LTI systems are widely used to describe a dynamic process using a state-space model. The nominal form of state-space representation of a discrete-time LTI system is

$$\begin{aligned} \mathbf{x}(k+1) &= \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k), \mathbf{x}(0) = \mathbf{x}_0, \\ \mathbf{y}(k) &= \mathbf{C}\mathbf{x}(k) + \mathbf{D}\mathbf{u}(k) \end{aligned} \tag{2.5}$$

where $\mathbf{x} \in \mathbb{R}^n$ is the state vector, $\mathbf{x}_0$ is the initial condition of the system, $\mathbf{u} \in \mathbb{R}^l$ and $\mathbf{y} \in \mathbb{R}^m$ are the input and output vectors. $\mathbf{A} \in \mathbb{R}^{n\times n}$ is the state transition matrix, $\mathbf{B} \in \mathbb{R}^{n\times l}$ is the input matrix, $\mathbf{C} \in \mathbb{R}^{m\times n}$ is the output matrix, and $\mathbf{D} \in \mathbb{R}^{m\times l}$ is the feed-through matrix. Considering the stochastic disturbance, the system is written as

$$\begin{aligned} \mathbf{x}(k+1) &= \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k) + \boldsymbol{\eta}(k) \\ \mathbf{y}(k) &= \mathbf{C}\mathbf{x}(k) + \mathbf{D}\mathbf{u}(k) + \boldsymbol{\nu}(k) \end{aligned} \tag{2.6}$$

where $\boldsymbol{\eta}(k)$ and $\boldsymbol{\nu}(k)$ are Gaussian process, independent of $\mathbf{x}_0$ and $\mathbf{u}(k)$ with

$$\mathrm{E}\begin{bmatrix} \boldsymbol{\eta}(i)\boldsymbol{\eta}^T(j) & \boldsymbol{\eta}(i)\boldsymbol{\nu}^T(j) \\ \boldsymbol{\nu}(i)\boldsymbol{\eta}^T(j) & \boldsymbol{\nu}(i)\boldsymbol{\nu}^T(j) \end{bmatrix} = \begin{bmatrix} \Sigma_\eta & S_{\eta\nu} \\ S_{\nu\eta} & \Sigma_\nu \end{bmatrix}\delta_{i,j}, \delta_{i,j} = \begin{cases} 1, i = j \\ 0, i \neq j \end{cases}$$
$$\mathrm{E}(\boldsymbol{\eta}(i)) = 0, \mathrm{E}(\boldsymbol{\nu}(i)) = 0.$$

The faults occurring in such kind of systems can be modeled in various ways. A commonly used one is to extend Eq. (2.6) to the form of

$$\begin{aligned} \mathbf{x}(k+1) &= \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k) + \mathbf{E}_f\mathbf{f}(k) + \boldsymbol{\eta}(k) \\ \mathbf{y}(k) &= \mathbf{C}\mathbf{x}(k) + \mathbf{D}\mathbf{u}(k) + \mathbf{F}_f\mathbf{f}(k) + \boldsymbol{\nu}(k) \end{aligned} \tag{2.7}$$

where $\mathbf{f}(k) \in \mathbb{R}^{d_f}$ is a unknown vector that denotes all possible faults and will be zero in the fault-free case, $\mathbf{E}_f$ and $\mathbf{F}_f$ are properly dimensioned indicting (1) where a fault occurs; (2) how it impact the system dynamics. According to the location, the faults are divided into three classes [1, 112, 114]:

- Sensor faults, $\mathbf{f}_S$: faults that directly impact the process measurements;
  e.g., let $\mathbf{E}_f = \mathbf{0}$ and $\mathbf{F}_f = \mathbf{I}_l$, $\mathbf{y}_f = \mathbf{y}^* + \mathbf{f}_S$.

- Actuator faults, $\mathbf{f}_A$: faults that could cause changes in actuators; *e.g.*, let $\mathbf{E}_f = \mathbf{B}$ and $\mathbf{F}_f = \mathbf{D}$, $\mathbf{u}_f = \mathbf{u}^* + \mathbf{f}_A$.
- Process faults, $\mathbf{f}_P$: faults that indicate malfunctions within the processes.
  e.g., let $\mathbf{E}_f = \mathbf{E}_p$, $\mathbf{F}_f = \mathbf{F}_p$, $\mathbf{x}_f = \mathbf{x} + \mathbf{f}_P$.

Depending the way how they affect the system dynamics, the faults introduced above are additive faults. They will affect the first-order statistics of output vector, *i.e.* the mean vector of $\mathbf{y}$. It is worth noting that additive faults will not affect the system stability. In practice, there exists another type of fault that is modelled as the change in parameter matrices of Eq. (2.6)

$$\begin{aligned} \mathbf{x}(k+1) &= (\mathbf{A} + \Delta\mathbf{A})\,\mathbf{x}(k) + (\mathbf{B} + \Delta\mathbf{B})\,\mathbf{u}(k) + \boldsymbol{\eta}(k) \\ \mathbf{y}_f(k) &= (\mathbf{C} + \Delta\mathbf{C})\,\mathbf{x}(k) + (\mathbf{D} + \Delta\mathbf{D})\,\mathbf{u}(k) + \boldsymbol{\nu}(k) \end{aligned} \tag{2.8}$$

where $\Delta\mathbf{A}$, $\Delta\mathbf{B}$, $\Delta\mathbf{C}$ and $\Delta\mathbf{D}$ represent the multiplicative fault in system parameters. Compared with the additive fault, this type of fault can influence the second-order statistics of output data. Figure 2.1 shows the two types of faults using a two-variable case, where it can be observed that the additive fault (Eqs. (2.2) and (2.7)) only affects the mean of the data while the multiplicative fault (Eqs. (2.3), (2.4) and (2.8)) does not influence the mean of the measured data, but solely impact the variance.

2.3 FD performance evaluation indices

2.3.1 FDR and FAR

Designing FD methods for monitoring $\mathbf{y}$ consists of (1) defining the detection (test) statistics J with its corresponding threshold J_{th} and (2) comparing the online realization of J, *i.e.* $J(\mathbf{y}(k))$ against J_{th} to make the decision: faulty or fault-free. For example, the extensively used T^2 test statistic is designed as $J_{T^2} = \mathbf{y}^T \Sigma_y^{-1} \mathbf{y}$. The associated J_{th} is commonly of the form: $J_{th,T^2} = \chi^2_{m,\alpha}$. A comprehensive study on J_{T^2} and J_{th,T^2} as well as other statistics will be provided in Chapter 3.

Figure 2.2 shows a typical data display, by which the essential fault detection performance are schematically illustrated. A false alarm occurs when an alarm is announced under normal operating condition. Fault

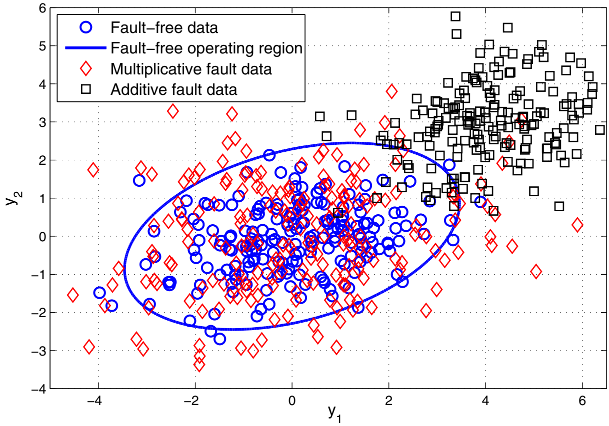

Figure 2.1: Demonstration of additive and multiplicative faults

detection alarm represents an effective alarm issued while there exists a fault [1]. From the probability point of view, FAR and FDR can be correspondingly defined, as they stand for the occurrence probabilities of false alarms and successful fault detection [1]. For the constant fault case, the two definitions are

$$\begin{aligned} \mathrm{FAR} &= \mathrm{prob}\left(J > J_{th} | f = 0\right) \\ \mathrm{FDR} &= \mathrm{prob}\left(J > J_{th} | f = c\left(\neq 0\right)\right) \end{aligned} \tag{2.9}$$

Yin *et al.* have given the following estimates [40]:

$$\begin{aligned} \mathrm{FAR} &= \frac{\text{Number of samples } \left(J > J_{th} \,|\text{fault} - \text{free}\right)}{\text{total fault} - \text{free samples}} \\ \mathrm{FDR} &= \frac{\text{Number of samples } \left(J > J_{th} \,|\text{faulty}\right)}{\text{total faulty samples}} \end{aligned} \tag{2.10}$$

Although widely applied in practice, this method fails when the fault is time varying, for example, in the case of a drift fault, which causes changes in $\mathbf{y}$ slowly. Two FDR-like indices were proposed for quantifying

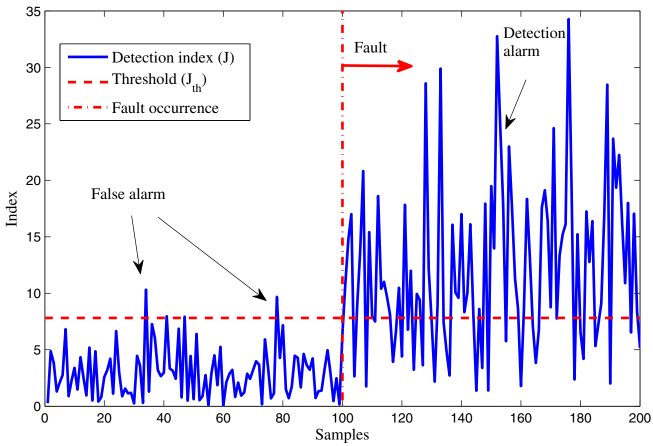

Figure 2.2: Demonstration of false alarm rate and fault detection rate

the probability that $\mathbf{y}(k)$ is faulty [120]:

$$\text{prob}\left(\text{fault}\left|\mathbf{y}\left(k\right)\right.\right) = \text{prob}\left(J\left(k\right) > J\left(\mathbf{y}^*\right)\left|\mathbf{y}^* \in \mathbf{Y}_{tr}\right.\right) \quad (2.11\text{a})$$

$$\text{prob}\left(\text{fault}\left|\mathbf{y}\left(k\right)\right.\right) = \exp\left(-\frac{J_{th}}{\varsigma J\left(k\right)}\right) \quad (2.11\text{b})$$

where $J\left(k\right)$ is shorthand for $J\left(\mathbf{y}(k)\right)$, $k \geq k_f$ with k_f denoting the fault occurring time instance, $\mathbf{Y}_{tr}$ denotes the normal training dataset and $\varsigma > 0$ is a tuning parameter, $\exp(\cdot)$ denotes the exponential function. It can be observed that Eq. (2.11a) does not depend on J_{th} and, thus, the calculated value cannot be used to report the fault. In Eq. (2.11b) $\text{prob}\left(\text{fault}\left|\mathbf{y}\left(k\right)\right.\right)$ approaches 1 only given a significantly large $\varsigma J(k)$, tuning of ς will be a trade-off to fulfil different demands, which can lead to difficulties in implementing this method. From the theoretical viewpoint, it should be assumed that in the case of a constant fault, the FAR at each time k should be constant, namely, $\text{FDR}(k) = c \; \forall k > k_f$. Considering the stochastic nature of $\mathbf{y}$, the methods in Eq. (2.11) cannot ensure a constant probability value for a constant additive fault, and the situation will be worse for a multiplicative fault that can cause significant changes in the variances or covariance of $\mathbf{y}$. Therefore, they are not appropriate for estimating FDR(k). In this chapter, another method that considers the distribution of J is proposed [71, 128],

$$\mathrm{FDR}\left(k\right) = \int\limits_{J_{th}}^{\infty} f_{J,k}\left(x\right)dx = 1 - F_{J,k}\left(J_{th}\right) \tag{2.12}$$

where $f_{J,k}$ and $F_{J,k}$ denote the probability density function (PDF) and the cumulative distribution function (CDF) of J at time k. This method avoids the weakness mentioned in Eq. (2.11), and can be easily realized for constant additive faults when adopting the T^2- and Q-statistics.

2.3.2 Expected detection delay

Although extensively implemented, FDR can only reflect the detectable probability of the PM-FD index for the fault with fixed parameters, but cannot tell whether the fault could be instantaneously detected or not. In addition, when the fault is successfully detected, the time taken to detect the fault is also important. Therefore, in this part, a new index, called EDD, is proposed to deal with this issue. If DD is defined as a random variable that shows the possible time interval between the occurrence of the fault and the successful detection of it.

Let $\mathcal{J}$ denote DD, $\mathcal{J}$ may take the value j, where $j = 0, 1, 2, ...\infty$. The probability that $\mathcal{J}$ takes the value j is based on the fact that $J\left(t_f\right) \leq J_{th}, J\left(k_f + 1\right) \leq J_{th}, ..., J\left(k_f + j\right) > J_{th}$ and is shown as

$$\mathrm{prob}\left(\mathcal{J} = j\right) = \begin{cases} \left(\prod\limits_{k=0}^{j-1}\left(1 - \mathrm{FDR}\left(k_f + k\right)\right)\right)\mathrm{FDR}\left(k_f + j\right), \mathrm{for}\ j \geq 1 \\ \mathrm{FDR}\left(k_f\right), \mathrm{for}\ j = 0 \end{cases} \tag{2.13}$$

where k_f is the fault occurrence time as defined above. EDD takes the expectation of $\mathcal{J}$ based on its distributional information.

Consider the special case where the fault is constant. In this case, $\forall k$, FDR(k) = FDR, where FDR is obtained using Eq. (2.12). Theorem 2.1 shows that for a constant additive fault the expected detection delay provides a valid approach.

Theorem 2.1. *For a constant fault with* FDR(k) $= c < 1\ \forall k$, *then* $\sum\limits_{j=1}^{\infty} \mathrm{prob}\left(\mathcal{J} = j\right) = 1$.

Proof. Based on Eq. (2.13), we can obtain that

$$\begin{aligned}\sum_{\forall j} \operatorname{prob}(\mathcal{J}=j) &= \lim_{n\to\infty} \left(\mathrm{FDR} + (1-\mathrm{FDR})\,\mathrm{FDR} + \cdots + (1-\mathrm{FDR})^n \mathrm{FDR}\right) \\ &= \lim_{n\to\infty} \left(\mathrm{FDR}\frac{1-(1-\mathrm{FDR})^n}{1-(1-\mathrm{FDR})}\right) \\ &= 1\end{aligned} \tag{2.14}$$

□

The above conclusion and Theorem 2.1 demonstrate that the new definition can indicate the PDF of DD for a constant fault. In this case, indeed, $\mathcal{J}$ obeys the geometric distribution [54]. Thus, the expectation of DD is obtained by

$$\mathrm{EDD} \triangleq \mathrm{E}(\mathrm{DD}) = \sum_{j=0}^{\infty} j\operatorname{prob}(\mathcal{J}=j) = \sum_{j=0}^{\infty} j(1-\mathrm{FDR})^j \mathrm{FDR} = \tfrac{1-\mathrm{FDR}}{\mathrm{FDR}} \tag{2.15}$$

From Eq. (2.15), it can be seen that the EDD index indicates the expected time delay given by a method for detecting a fault. As shown in Figure 2.3, if FDR $\to 1$, that leads to EDD $\to 0$ and the detection results are demonstrated in the bottom left figure; else if FDR $\to 0$, EDD approaches infinity, then top left figure of Figure 2.3 shows the detection in this case. Correspondingly, the middle figure gives the intermediate results. Obviously, the result is consistent with the actual situation, and in other words, the new definition of EDD is right for this particular case. This approach allows users to select an appropriate upper limit for the acceptable detection delay. If the computed EDD value is greater than the threshold, then we can say that the method cannot detect the fault. Practically, it is written as $J_{th,\mathrm{EDD}} = (1-\mathrm{FAR})/\mathrm{FAR} \approx (1-\alpha)/\alpha$ with α denoting a pre-specified significance level [71].

As assumed in this work, for constant faults, irrespective of whether the fault be additive or multiplicative, FDR is constant, thus Eq. (2.12) could be directly used. While for the drift fault case, the FDR will be time-varying. Figure 2.4 shows an example using J_{T^2} to detect this type of fault. It can be observed that as $f(k)$ monotonically increases, the calculated FDR(k) likewise increases. Theorem 2.2, which follows, gives the probabilistic property of EDD for this type of fault.

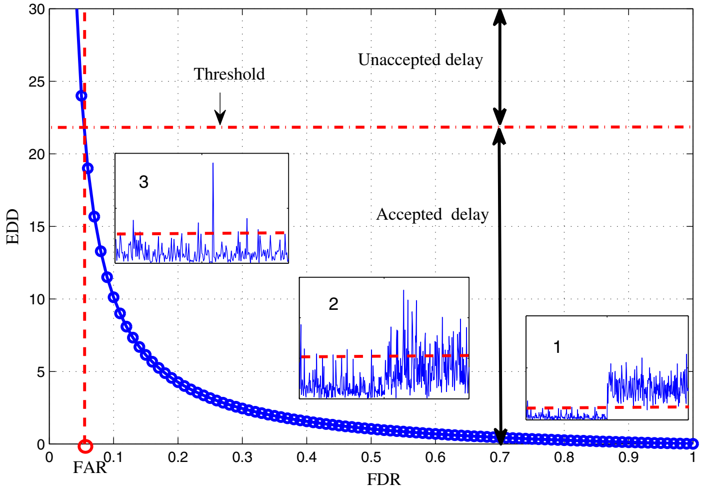

Figure 2.3: Schematic description of detection delay using FAR and FDR

Theorem 2.2. *For a drift fault occurring at time instance k_f, if there exists a time instance k_s that $\forall k \geq k_s$, $\mathrm{FDR}(k) = \mathrm{FDR}(k_s)$, then* $\sum_{j=0}^{\infty} \mathrm{prob}(\mathcal{J} = j) = 1$.

Proof. Let $\mathcal{J} = s+1$ denote the event that the fault can be detected at the k_s+1 time instance. Since FDR(k_s+1)=FDR(k_s), $\mathrm{prob}(\mathcal{J} = s+1)$ is calculated as

$$\begin{aligned} \mathrm{prob}(\mathcal{J} = s+1) &= \tfrac{\mathrm{prob}(\mathcal{J}=s)}{\mathrm{FDR}(k_s)} (1 - \mathrm{FDR}(k_s))\, \mathrm{FDR}(k_s+1) \\ &= \mathrm{prob}(\mathcal{J} = s)(1 - \mathrm{FDR}(k_s)) \end{aligned} \tag{2.16}$$

This leads to $\mathrm{prob}(\mathcal{J} = s+\tau) = \mathrm{prob}(\mathcal{J} = s)(1 - \mathrm{FDR}(k_s))^{\tau}$. Then,

$$\sum_{j=s}^{\infty} \mathrm{prob}(\mathcal{J} = j) = \mathrm{prob}(\mathcal{J} = s) \sum_{\tau=1}^{\infty} (1 - \mathrm{FDR}(k_s))^{\tau} = \frac{\mathrm{prob}(\mathcal{J} = s)}{\mathrm{FDR}(k_s)} \tag{2.17}$$

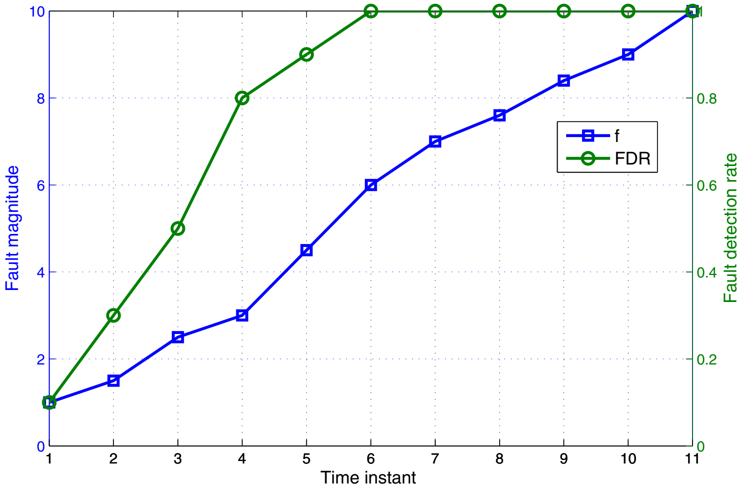

Figure 2.4: An example with FDR for a drift fault

Note that

$$\mathrm{prob}\left(\mathcal{J}=s\right)=\frac{\mathrm{prob}\left(\mathcal{J}=s-1\right)}{\mathrm{FDR}\left(k_s-1\right)}\left(1-\mathrm{FDR}\left(k_s-1\right)\right)\mathrm{FDR}\left(k_s\right) \quad (2.18)$$

Thus,

$$\begin{aligned}\sum_{j=s-1}^{\infty}\mathrm{prob}\left(\mathcal{J}=j\right) &= \sum_{j=s}^{\infty}\mathrm{prob}\left(\mathcal{J}=j\right)+\mathrm{prob}\left(\mathcal{J}=s-1\right)\\ &= \frac{\left(\frac{\mathrm{prob}(\mathcal{J}=s-1)}{\mathrm{FDR}(k_s-1)}(1-\mathrm{FDR}(k_s-1))\mathrm{FDR}(k_s)\right)}{\mathrm{FDR}(k_s)}+\mathrm{prob}\left(\mathcal{J}=s-1\right)\\ &= \frac{\mathrm{prob}(\mathcal{J}=s-1)}{\mathrm{FDR}(k_s-1)}\end{aligned} \quad (2.19)$$

Using Eqs. (2.17) and (2.19) iteratively gives $\sum_{j=0}^{\infty}\mathrm{prob}\left(\mathcal{J}=j\right)=\frac{\mathrm{prob}(\mathcal{J}=0)}{\mathrm{FDR}(k_f)}=\frac{\mathrm{FDR}(k_f)}{\mathrm{FDR}(k_f)}=1.$ □

Thus, like Theorem 2.1, prob($\mathcal{J}=j$) $\forall j$, can be adopted to describe the PDF of DD for this type of fault. Then, Eq. (2.15) is also valid to calculate the EDD. Note in the case that $\mathrm{FDR}\left(k_s\right)=1$ as show in Figure 2.4, despite $\mathrm{FDR}(k_s+i)<1$ with $i>1$, the experiment should be stopped at time k_s. Using Eq. (2.19) and the fact that $\mathrm{prob}(\mathcal{J}=s)=\frac{\mathrm{prob}(\mathcal{J}=s-1)}{\mathrm{FDR}(k_s-1)}(1-\mathrm{FDR}(k_s-1))$, it is obtained that $\sum\limits_{j=0}^{\tau}\mathrm{prob}(\mathcal{J}=s-j)=\frac{\mathrm{prob}(\mathcal{J}=s-\tau)}{\mathrm{FDR}(k_s-\tau)}$. Let $\tau=s$, $\sum\limits_{j=0}^{s}\mathrm{prob}(\mathcal{J}=j)=\frac{\mathrm{prob}(\mathcal{J}=0)}{\mathrm{FDR}(k_f)}=1$. Therefore, for this type of drift fault, EDD can also be usable.

Remark 2.1. *When calculating EDD, for FD methods that there is more than statistic involved, the FDR value can be determined using the maximum of them.*

2.4 Simulation results

A simple numerical model is used to compare the results given by EDD and the numerical approximation approach for J_{T^2}. Assume that $\mathbf{y}$ is normally distributed with zero mean and covariance matrix

$$\Sigma_y=\begin{bmatrix}2 & 0.5 & 0.3\\ 0.5 & 1 & 0.2\\ 0.3 & 0.2 & 0.5\end{bmatrix} \tag{2.20}$$

Three fault scenarios are examined: a constant additive fault, a drift fault, and a multiplicative fault.

In the first case, a constant additive fault occurs in $\mathbf{y}_2$, where $\mathbf{\Xi}=[\begin{array}{ccc}0 & 1 & 0\end{array}]^T$ and f changes from 1 to 8. For the numerical approximation-based method, the simulation is run 100 times. In each run, the detection delay is recorded. The mean value is finally calculated for comparison. EDD is calculated using Eq. (2.15). Figure 2.5 shows the comparison result. Note that, although the two methods deliver comparable results, the numerical approximation approach-based results contain more fluctuations. In the second case, a drift fault occurs in $\mathbf{y}_3$. $\mathbf{\Xi}=[\begin{array}{ccc}0 & 0 & 1\end{array}]$, and $f(k)=\rho k$ with ρ varying from 0.1 to 0.8. The comparison results are presented in Figure 2.6, where the two methods match well. EDD is calculated in each run when $\mathrm{FDR}(k)$ equals 1. In the third case, a multiplicative fault occurs in the system. $\mathbf{M}=\mathrm{diag}(M_1,1,M_3)$, with M_1 and M_3 varying from 1 to 50. The results are shown in Figure 2.7. We can see that, for both test statistics,

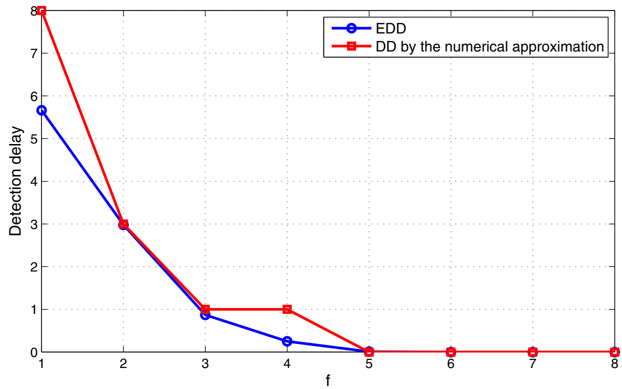

Figure 2.5: Comparison between EDD and the detection delay by numerical approximation for a constant additive fault.

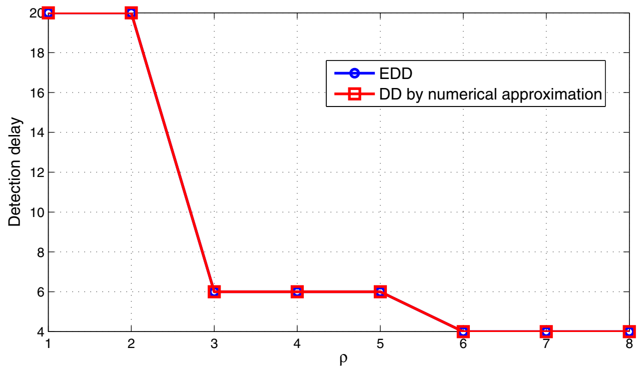

Figure 2.6: Comparison between EDD and the detection delay by numerical approximation for a drift fault.

although the two methods agree with each other, like in the additive fault case, the numerical approximation method contains more fluctuations than the EDD index.

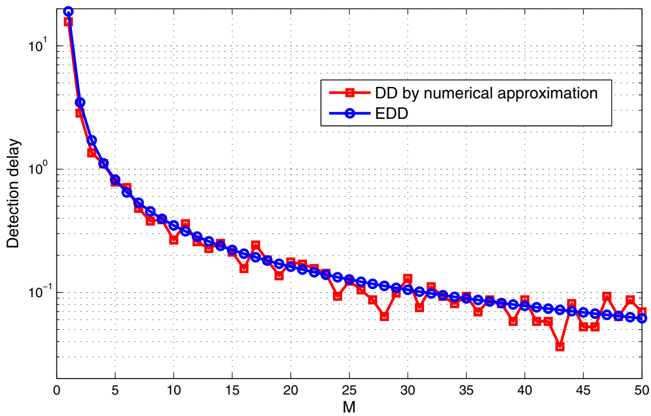

Figure 2.7: Comparison between EDD and the detection delay by numerical approximation for a multiplicative fault.

2.5 Conclusions

This chapter has addressed the fundamental issues for FD of static and dynamic processes. Two types of faults, additive and multiplicative faults have been studied in the way of how they impact the two kinds of processes. To detect such faults, the concept of fault detection statistics was introduced in the statistical framework. In evaluating the performance of FD statistic, the widely accepted performance indices like FDR, FAR have been investigated. In addition, an EDD index was proposed to assess the performance of FD methods for detecting constant additive and multiplicative faults as well as drift faults in stochastic processes. The relationship between EDD and FDR has been also investigated under certain condition. A simple numerical case has been utilized to show the performance of EDD. Compared to numerical approximation-based methods, EDD shows more accurate results.

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