

Preface

Beginnings are always troublesome.—George Eliot

This thesis was written during my last year at the Leung Center for Cosmology and Particle Astrophysics (LeCosPA), National Taiwan University, Taipei. I always knew I wanted to write a thesis about black holes, since I have always been interested in them. The problem was finding a topic that would be sufficiently interesting *as a thesis*, instead of merely as a journal publication. Perhaps I am somewhat old-fashioned in this regard, but I am quite adamant that a thesis should be a scholastic work that not only consists of new results, but is also a reasonably self-contained, good introduction to the field, with digested insights of the author over his or her many years of study. This is what I have striven to do, although perhaps far from perfect, despite further improvements incorporated for the published version with Springer (which include the addition of a new chapter and appendices).

The subject matter for my thesis was finally settled when I attended the Strings 2012 conference in München, Germany. “Strings” is a yearly conference for the string theory community, and I thought it might be a good idea to attend one of these just for the experience, even though I am not a string theorist. Raphael Bousso was asked to give a special talk¹ during the conference to explain firewalls—which was very recently proposed back then—to the perplexed participants. I was rather intrigued, but was only finally convinced by Brett McInnes, my Masters thesis supervisor, that this topic would be suitable as a Ph.D. thesis.

I hope that I am able to convey the mysteries and the beauty of black holes to the reader, and that he or she will be at least sufficiently intrigued to continue reading the subsequent pages. I also hope that other researchers who would like to get into this field of research will benefit from this thesis-turned-monograph, and its many references at the back.

¹The talk is available on the conference website: <http://www.theorie.physik.uni-muenchen.de/videos/strings2012/bousso/index.html>.

As a Ph.D. thesis, most of the content assumes some proficiency with graduate level physics and mathematics. *I will assume the reader to have a good background in basic quantum mechanics, differential geometry, differential equations, and topology.* By basic I mean, specifically, knowledge of

- (1) **Quantum Mechanics** At the level of a typical first undergraduate course; but actually not much is required beyond knowing basic concepts such as the wave function, state vectors $|\psi\rangle$, quantum operators, unitarity, and the principle of superposition.
- (2) **Differential Geometry** Enough to understand what a Riemann curvature tensor, a Ricci tensor, and a scalar curvature are. This means a typical first course in Riemannian geometry, which is taught in most universities at the graduate level. A prior knowledge of differential geometry of curves and surfaces embedded in \mathbb{R}^3 is helpful but not required.
- (3) **Differential Equations** Very basic knowledge about differential equations—most equations we will be solving are just linear ordinary differential equations. Some knowledge of partial differential equations is required to *appreciate* the Einstein Field Equations, but we will not really solve the field equations, so this requirement can be relaxed.
- (4) **Topology** A typical first course of topology is sufficient. In particular, one should understand basic concepts such as compactness and orientability. At one point the concept of covering space will be needed, but this can be safely skipped on first reading.

Some knowledge of general relativity and quantum field theory would be helpful, but is not necessary to understand this thesis, provided the readers are willing to take the results for granted without proofs. Even the aforementioned assumed background items (1)–(4) are not really necessary if, e.g., one is willing to take terms like “curvature” at superficial level without getting into its technical definitions. Relevant chapters and appendices had been added to provide some background to the readers. These are, unfortunately, necessarily brief, and may not help someone with zero knowledge in the subjects. They would, however, be hopefully sufficient to allow physicists in other fields (and students with sufficient mathematical maturity) to appreciate much of this thesis.

The first chapter of this work is meant to be an introduction to the thesis, and is at least partially aimed at a wider audience that may not necessarily have had physics training beyond that of their high school education. The second chapter, which is newly added for this Springer Theses publication, is a quick summary of general relativity. The anti-de Sitter (AdS) spacetime is introduced in some detail. This will be useful for physicists who are not experts in the field. In addition, it also contains some—perhaps biased—opinions of the author about what general relativity is about, as well as subtleties of the theory that are not more widely recognized or appreciated. Some discussions are more philosophical than what one may find in a typical physics text, but after all, the “P” in a Ph.D. does stand for

philosophiae, and it would be fitting to include some philosophical thoughts in a Ph.D. thesis.

In Chap. 3, we discuss the much celebrated positive mass theorem in mathematical relativity, stability of gravitational configurations, and phase transitions between them. In particular, the famous Hawking-Page phase transition is explained in detail. Some parts of this chapter require a good background knowledge of real analysis, but these parts can be skipped without affecting the understanding of the rest of the thesis. This chapter provides the background for some arguments we will use in the later chapters. However, by itself, Chap. 3 is also a nice glimpse into mathematical relativity—a huge effort from mathematicians to give general relativity the rigorous treatment it deserves.

The main parts of the thesis are in Chaps. 4 and 5. We investigate the Harlow-Hayden conjecture (that it takes a vastly longer time to decode Hawking radiation than the lifetime of a black hole) in the context of charged black holes with flat event horizons in AdS spacetime. This is motivated by the fact that in the application of the anti-de Sitter/conformal field theory (AdS/CFT) correspondence, such black holes are dual to a field theory that behaves very much like a Quark-Gluon Plasma (QGP), and are thus arguably the most well-understood “quantum gravity system”, especially where charged black holes are concerned.

It is essential to study charged black holes because even neutral black holes inevitably pick up electrical charges as they evaporate (as long as the theory admits charged particles). By modeling Hawking evaporation using an extension of the Hiscock and Weems analysis, we show that charged, flat black holes inevitably evolve toward the extremal limit, and are destroyed either by brane-pair production induced by the Seiberg-Witten instability, or by a phase transition into a type of soliton. The lifetime of such black holes is thus cut short, as Harlow and Hayden require, in order to evade the firewall argument.

Lastly in Chap. 6, we also investigate the possibility that black holes can store a huge amount of information behind their horizon. Since black holes are formed from gravitational collapse, it would be interesting to see if *non-black hole* configurations can have arbitrarily large volumes bounded by finite horizon areas. Such a “monster,” if it exists, could be the stage that leads to a black hole with arbitrarily large statistical entropies, far beyond the bound set by the Bekenstein-Hawking entropy of the black hole with the same mass. Again, by investigating the issue in AdS spacetimes, we found that monsters most probably do not exist in quantum gravity. This suggests—although it does not prove—that black holes formed from collapse do not have arbitrarily large statistical entropies. However, this does not mean that black holes cannot have large interiors.

The thesis concludes with an epilogue that discusses the current state of the firewall controversy, and what else can be done to further understand this topic. Several useful appendices are provided at the end.

Readers are warned that, in a work of this length, it is unlikely to be free of mistakes despite rounds of proofreading.

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the Firewall Controversy

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