Chapter 2
Anti-disturbance Control for Nonlinear Systems with Mismatched Disturbances Based on Disturbance Observer

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Abstract An anti-disturbance control scheme based on disturbance observer is proposed for a class of nonlinear system with mismatched disturbances. With such control approach, the disturbances can be rejected and the semi-global uniformly ultimate bounded (SGUUB) stability of the closed-loop system can be achieved. Finally, simulations for a numerical example are given to demonstrate the feasibility and effectiveness of the proposed scheme.

Keywords Anti-disturbance control · Disturbance observer · Mismatched disturbances

2.1 Introduction

Disturbance-observer-based control (DOBC) strategy appeared in the late 1980s [1] and has been applied in many control fields [2]. In recent years, a composite DOBC and other control approaches has been proposed, such as H_∞ control [3], sliding mode control [4],adaptive control [5], fuzzy control [6]. Back-stepping method is one of the most important design techniques in the nonlinear control area [7]. Recently, back-stepping method has been integrated with other control approaches, such as adaptive control [8], sliding mode control [9], H_∞ control [10], fuzzy control [11].

However, there are few reports about composite DOBC scheme and back-stepping method. In [12], a control scheme combining disturbance observer technique and back-stepping method was proposed for a class of nonlinear system with disturbances and nonlinear functions. Considering that it is linear for the main part of the system in [12], we will extend to a class of more general nonlinear systems with mismatched disturbances.

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The main contributions of this paper are summarized as follows:

(1) The disturbances with partially-known information represented by exogenous systems are mismatched disturbances, which appear in different channels as the control inputs.
(2) A composite anti-disturbance control strategy with nonlinear disturbance observer and back-stepping method is proposed to apply in a class of more general nonlinear systems with mismatched disturbances.

2.2 Formulation of the Problem

The following nonlinear system with mismatched disturbances is described as

$$\begin{aligned}
\dot{x}_1 &= f_1(x_1) + g_1(x_1)x_2 + h_1(x_1)d_1(t),\\
\dot{x}_2 &= f_2(x_1,x_2) + g_2(x_1,x_2)x_3 + h_2(x_1,x_2)d_2(t),\\
&\vdots\\
\dot{x}_n &= f_n(x_1,\dots,x_n) + g_n(x_1,\dots,x_n)u + h_n(x_1,\dots,x_n)d_n(t),\\
y &= s(x),
\end{aligned} \tag{2.1}$$

where $x = [x_1, x_2, \dots, x_n]^T \in R^n$, $u \in R$, y are the system states, the control input and the system output, respectively. f_i, g_i, h_i, s are smooth functions and are differentiable with $f_i(0) = 0$ and $g_i(\cdot) \neq 0$ for $i = 1, 2, \dots, n$. $d_i(t) \in R$ represents the external disturbance.

Assumption 1 The disturbance $d_i(t)$ can be described by the following exogenous system:

$$\begin{aligned}
\dot{w}_i(t) &= A_i w_i(t),\\
d_i(t) &= C_i w_i(t),
\end{aligned} \tag{2.2}$$

where $A_i \in R^{m\times m}$ and $C_i \in R^{1\times m}$ are proper matrices. Generally, the exogenous system (2.2) is deemed to be neutral stable.

2.3 Nonlinear Disturbance Observer

A new nonlinear disturbance observer is introduced as follows:

$$\begin{aligned}
\dot{q}_1 = {}& [A_1 - l_1(x_1)h_1(x_1)C_1]q_1 + A_1 p_1(x_1) - l_1(x_1)[h_1(x_1)C_1 p_1(x_1) + f_1(x_1)\\
&+ g_1(x_1)x_2],\\
\dot{q}_2 = {}& [A_2 - l_2(x_2)h_2(x_1,x_2)C_2]q_2 + A_2 p_2(x_2) - l_2(x_2)[h_2(x_1,x_2)C_2 p_2(x_2)
\end{aligned}$$

$$
\begin{aligned}
&+f_2(x_1,x_2)+g_2(x_1,x_2)x_3],\\
&\quad\vdots\\
\dot{q}_n &= [A_n - l_n(x_n)h_n(x_1,\ldots,x_n)C_n]q_n + A_n p_n(x_n)\\
&\quad - l_n(x_n)[h_n(x_1,\ldots,x_n)C_n p_n(x_n) + f_n(x_1,\ldots,x_n) + g_n(x_1,\ldots,x_n)u],\\
\hat{w}_i &= q_i + p_i(x_i),\\
\hat{d}_i &= C_i \hat{w}_i,
\end{aligned}
\tag{2.3}
$$

where $\hat{w}_i$ is the estimation of w_i, $q_i \in R^{m\times 1}$ is the auxiliary vector and $p_i(x_i) \in R^{m\times 1}$ is an nonlinear function to be designed. The nonlinear observer gain $l_i(x_i)$ is determined by $l_i(x_i) = \frac{\partial p_i(x_i)}{\partial x_i}$. The estimation error is denoted as $e_i = w_i - \hat{w}_i$. Then the estimation errors dynamics are presented as

$$\dot{e} = [A - l(x)h(x)C]e, \tag{2.4}$$

where $e = [e_1, e_2, \ldots, e_n]^T$, $A = diag\{A_1, A_2, \ldots, A_n\}$, $l(x) = diag\{l_1(x_1), l_2(x_2), \ldots, l_n(x_n)\}$, $h(x) = diag\{h_1(x_1), h_2(x_1,x_2), \ldots, h_n(x_1,\ldots,x_n)\}, C = diag\{C_1, C_2, \ldots, C_n\}$.

Assumption 2 Suppose that the relative degree from the disturbance d_i to the output $r_i \geq 1$ such that $m_i(x) = L_{H_i} L_F^{r_i-1} s(x) \neq 0$, where L denotes Lie derivatives, $m_i(x)$ is bounded with respect to x in its operation region.

Select $l_i(x_i) = K_i \frac{\partial L_F^{r_i-1} s(x)}{\partial x_i}$, then $l_i(x_i)h_i(x_1,\ldots,x_i) = K_i m_i(x)$. If there exists a constant m_{0i} and a bounded nonlinear function $m_{ii}(x)$ satisfying $m_i(x) = m_{0i} + m_{ii}(x)$, where $m_{ii}^2(x) \leq \bar{m}_{ii}^2$, $\bar{m}_{ii}$ is known constant, then the disturbance estimation errors system (2.4) can be rewritten as

$$\dot{e} = [\bar{A} - KM(x)C]e, \tag{2.5}$$

where $\bar{A} = A - KMC$, $K = diag\{K_1, K_2, \ldots, K_n\}$, $M = diag\{m_{01}, m_{02}, \ldots, m_{0n}\}$, $M(x) = diag\{m_{11}(x), m_{22}(x), \ldots, m_{nn}(x)\}$. Defining matrix $\bar{M} = diag\{\bar{m}_{11}^{-1}, \bar{m}_{22}^{-1}, \ldots, \bar{m}_{nn}^{-1}\}$, based on Assumptions 1 and 2, the following result can be obtained.

Theorem 1 *For given matrix M, $\bar{M}$, if there exist $P > 0$ and Q satisfying*

$$\begin{bmatrix} A^T P + PA - C^T M^T Q^T - QMC + C^T C & Q \\ Q^T & -\bar{M}^2 \end{bmatrix} < 0, \tag{2.6}$$

where $K = P^{-1}Q$, then based on the nonlinear disturbance observer (2.3), the disturbance estimation errors system (2.5) is asymptotically stable.

2.4 Back-Stepping Controller Design and Stability Analysis

Based on the back-stepping method, the anti-disturbance controller u is constructed as

$$\begin{aligned} u &= \alpha_n(x_1, \ldots, x_n, \hat{d}_1, \ldots, \hat{d}_n) \\ &= \frac{1}{g_n}\{-z_{n-1}g_{n-1} - f_n - h_n\hat{d}_n + \sum_{j=1}^{n-1}[\frac{\partial \alpha_{n-1}}{\partial x_j}(f_j + g_j x_{j+1} + h_j\hat{d}_j) \\ &\quad + \frac{\partial \alpha_{n-1}}{\partial \hat{d}_j} C_j A_j \hat{w}_j] - k_n z_n + r_n\}, \end{aligned} \tag{2.7}$$

where $k_n > 0$ is an adjustable controller parameter, and $r_n = -\sum_{j=1}^{2n-1} \delta_{nj} z_n$, $\delta_{nj} > 0$, $j = 1, 2, \ldots, 2n-1$ are adjustable parameters. Thus,

$$\begin{aligned} \dot{V}_n &= \sum_{j=1}^{n}(-k_j z_j^2 + z_j h_j C_j e_j + z_j r_j) - \sum_{t=2}^{n}[z_t \sum_{j=1}^{t-1}(\frac{\partial \alpha_{t-1}}{\partial x_j} h_j C_j e_j \\ &\quad + \frac{\partial \alpha_{t-1}}{\partial \hat{d}_j} C_j l_j h_j C_j e_j)] \\ &\le -\sum_{j=1}^{n} k_j z_j^2 + \varepsilon_n, \end{aligned} \tag{2.8}$$

where $\varepsilon_n = \frac{1}{\delta_{n1}} h_n^2 C_n e_n e_n^T C_n^T + \frac{1}{\delta_{n2}}(\frac{\partial \alpha_{n-1}}{\partial x_1} h_1)^2 C_1 e_1 e_1^T C_1^T + \cdots + \frac{1}{\delta_{n(n+1)}}(\frac{\partial \alpha_{n-1}}{\partial \hat{d}_1} h_1)^2 C_1 l_1 C_1 e_1 e_1^T C_1^T l_1^T C_1^T + \cdots + \varepsilon_{n-1}$.

The inequality (2.8) can be further rewritten as

$$\dot{V}_n \le -\lambda V_n + \varepsilon_n, \tag{2.9}$$

where $\lambda = \min\{k_1, k_2, k_3, \ldots, k_n\}$. The inequality (2.9) is equivalent to

$$0 < V_n \le \frac{\varepsilon_n}{\lambda} + [V_n(0) - \frac{\varepsilon_n}{\lambda}]e^{-\lambda t}. \tag{2.10}$$

From (2.10) and in the same proof as [9, 11], it can be shown that the closed-loop system is semi-global uniformly ultimate bounded (SGUUB). According to the above design and analysis, the stability of the closed-loop system is summarized in the following theorem:

Theorem 2 *Consider nonlinear system (2.1) with mismatched disturbances under Assumptions 1 and 2. By designing the nonlinear disturbance observer (2.3) and the anti-disturbance controller (2.7), the closed-loop system is semi-global uniformly ultimate bounded (SGUUB).*

2.5 Simulation Example

The mathematical model of a second-order nonlinear system with mismatched disturbances is described as follows:

$$\begin{aligned}
\dot{x}_1 &= x_1^3 + 2x_1 + x_2 + (1 + x_1^2)d_1(t), \\
\dot{x}_2 &= -x_1 + x_2 + (2 + 3x_1^2 + x_2^2)u + (1 + x_1^2 + x_2^2)d_2(t), \\
y &= x_1 + x_2,
\end{aligned} \tag{2.11}$$

where $x = [x_1, x_2]^T, u \in R$ are the system states and input, respectively. $d_i(t), i = 1, 2$ is assumed to be an unknown harmonic disturbance described by (2.2) with

$$A_1 = \begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix}, \; A_2 = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}, \; C_1 = \begin{bmatrix} 2 & 0 \end{bmatrix}, \; C_2 = \begin{bmatrix} 3 & 0 \end{bmatrix}.$$

The relative degree from the disturbance $d_i, i = 1, 2$ to the output is calculated as $(r_1, r_2) = (2, 2)$. And $m_1(x) = 3x_1^4 + 4x_1^2 + 1,\; m_2(x) = x_1^2 + x_2^2 + 1$. We can choose $m_{01} = 1,\; m_{02} = 1,\; \bar{m}_{11} = 0.5,\; \bar{m}_{22} = 0.5$. According to Theorem 1, it can be obtained that

$$K = \begin{bmatrix} 0.5324 & 0 \\ -0.0384 & 0 \\ 0 & 0.5277 \\ 0 & -0.0973 \end{bmatrix}.$$

The controller is obtained by the proposed method in Sect. 2.4,

$$\begin{aligned}
u = \frac{1}{2 + 3x_1^2 + x_2^2}\{ &-x_2 - (1 + x_1^2 + x_2^2)\hat{d}_2 + \frac{\partial \alpha_1(x_1, \hat{d}_1)}{\partial x_1}[x_1^3 + 2x_1 + x_2 \\
&+ (1 + x_1^2)\hat{d}_1] + \frac{\partial \alpha_1(x_1, \hat{d}_1)}{\partial \hat{d}_1} C_1 A_1 \hat{w}_1 - k_2 z_2 + r_2\},
\end{aligned} \tag{2.12}$$

with $\frac{\partial \alpha_1(x_1, \hat{d}_1)}{\partial x_1} = -3x_1^2 - 2x_1\hat{d}_1 - (k_1 + \delta_{11} + 2),\; \frac{\partial \alpha_1(x_1, \hat{d}_1)}{\partial \hat{d}_1} = -x_1^2 - 1,\; z_2 = x_2 + x_1^3 + (1 + x_1^2)\hat{d}_1 + (k_1 + \delta_{11} + 2)x_1,\; r_2 = -(\delta_{21} + \delta_{22} + \delta_{23})z_2$, where $k_1,\; k_2$ are positive numbers, $\delta_{11} > 0,\; \delta_{2j} > 0, j = 1, 2, 3$ are adjustable parameters. In simulation, the initial value of the states are set to be $x_1(0) = -1,\; x_2(0) = 4$ and the design parameters are chosen as $k_1 = 0.1,\; k_2 = 0.1,\; \delta_{11} = 1,\; \delta_{21} = 1,\; \delta_{22} = 1,\; \delta_{23} = 1$.

The simulation results are shown in Figs. 2.1, 2.2 and 2.3. Figure 2.1 demonstrates the system performance using the proposed control scheme. Figure 2.2 illustrates the estimation error for system disturbances with the anti-disturbance approach. Figure 2.3 shows the responses of the control input signal using the proposed method.

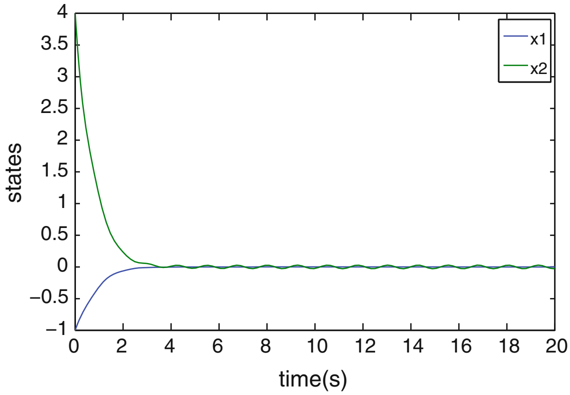

Fig. 2.1 System performance for disturbances

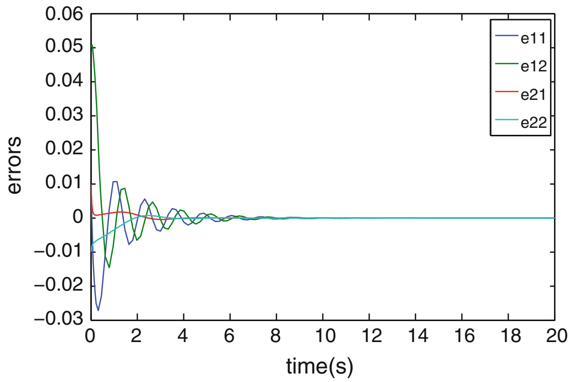

Fig. 2.2 Estimation errors for disturbances

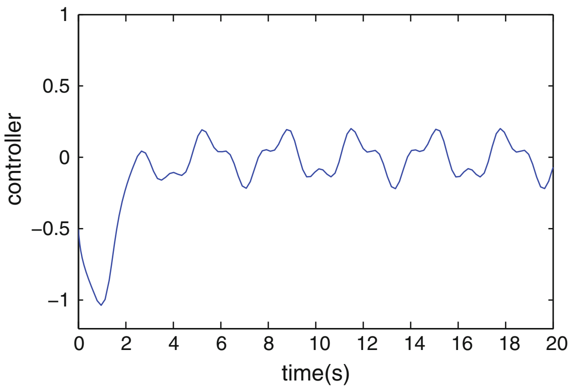

Fig. 2.3 The trajectory of control input

2.6 Conclusions

It is shown that the proposed anti-disturbance control scheme based on nonlinear disturbance observer is an effective control approach for a class of general nonlinear system subject to mismatched disturbances. However, in practical engineering, many complex systems can be described as mathematical models with multiple disturbances. If the disturbances in the nonlinear system (2.1) are multiple disturbances with unknown parameters, the situations will be more sophisticated. Consequently, new anti-disturbance control approaches are required to be considered and further research is needed in the future.

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