

Chapter 2

Measurement Uncertainty and Requirements of Production System. Selected Issues of Measurement Uncertainty Theory

Abstract This chapter describes the theoretical bases together with the author's concept of identification of reproducibility error of the measuring point (REMP), as the basis of the matrix method (MM) used for CMM error identification and the assessment of their accuracy. Also the importance of coordinate metrology for quality management and the evaluation of measurement uncertainty as a key task in deciding product geometry compliance with its specifications (GPS, geometrical product specification) are pointed out. There is also presented one of the most accurate coordinate measuring machines in the laboratory, with the air-conditioning system ensuring thermal stability in the range of ± 0.05 °C. This chapter presents the theory of measurement uncertainty, the vector concept of describing coordinate measurement accuracy, and the REMP. It discusses the results of the author's original work concerning the determination of coordinate system accuracy at the measuring point and presents it as the basis for the new concept of CMS accuracy assessment. It also examines the issue of standards construction and the methodology of their application, which includes the possibility of identifying the accuracy at the measuring point. The discussion on coordinate measurement uncertainty in accordance with international standards and the author's research results, including the method using a calibrated object or standard, the concept of a multiposition method with the use of a noncalibrated object or simulative, analytical and expert methods is also presented.

2.1 Coordinate Measurement During Production Process

The acceptance of system solutions in the field of quality management and their later certification for compliance with the ISO 9000 series of standards means that many companies have to face concepts such as uncertainty, calibration, and metrological traceability. The implementation of these concepts into metrological practice has to be provided by a huge number of metrological standards and recommendations. The most important issue is the evaluation of measurement

uncertainty. It should be treated as a key task in deciding about the compliance of the product geometry with its specifications (GPS, geometrical product specification) [36]. We get used to the fact that in advanced technology industries, especially in the aerospace, space, and arms industries, the emphasis is put on the evaluation of measurement uncertainties, but recently the problem is also becoming crucial for other industries. It is particularly important for the automotive industry and its suppliers, producers of household appliances, medical and telecommunication devices, or mechatronic systems. In the case of small and medium-sized enterprises (SMEs) it can be noted that the use of accuracy assessment not only for customers, but also for internal processes, is not only useful, but necessary. It is mainly related to narrow tolerance ranges of feedback on tools and measuring systems development (particularly the coordinate ones), which in turn is reflected in product reliability and safety. A similar problem arises in the research and calibration laboratories, certified for compliance with PN-EN ISO/IEC 17025 [88]. But of course in this case the requirements of measurement accuracy are usually determined by higher precision requirements. By performing dimensionally shaped evaluations for each produced object, its shape is compared with the designed one. In traditional technical documentation the geometric object is presented as a view or cross-section.

Figure 2.1 includes formal information such as size, shape, location, and condition of the surface, together with the relevant tolerances. Tolerances describe the geometrically limited space, which sets the limits for the real geometric shape changes. The measurement instruments used in the metrology of geometric quantities were constructed to perform direct measurement of one characteristic. Other necessary dimensions were calculated by simple operations based on the results of direct measurements. It did not allow carrying out comprehensive measurement of spatial objects produced for the rapidly developing industry of the early 1970s. Common use of numerically controlled machine tools brings in new design possibilities, but also new measurement problems, especially in the case of outlines with variable curvature (gears, cams, etc.). In addition there was a whole group of indirect measurements (e.g., distance between axes of holes) for dimensionally expanded objects such as a corpus. The consequence of this was the introduction of a large number of specialized measurement instruments for the separate measurement of each dimension. It was an obstacle for the processes of control automation and thus for effectiveness growth. The control took too much time in comparison to the processing time and generated difficulties in the integration of the material and information flow.

The need to ensure production quality necessitated the search for new efficient measurement methods. The problem was solved by the coordinate measuring technique (CMT), originally based on measuring machines (CMM) that made representations of measured objects. Figure 2.1 presents one of the most accurate coordinate measuring machines in a modernized laboratory with an air-conditioning system maintaining temperature stability within ± 0.05 °C.

The basic concepts used here are position and direction vector. Position vectors represent the location of measuring points, and direction vectors the angular

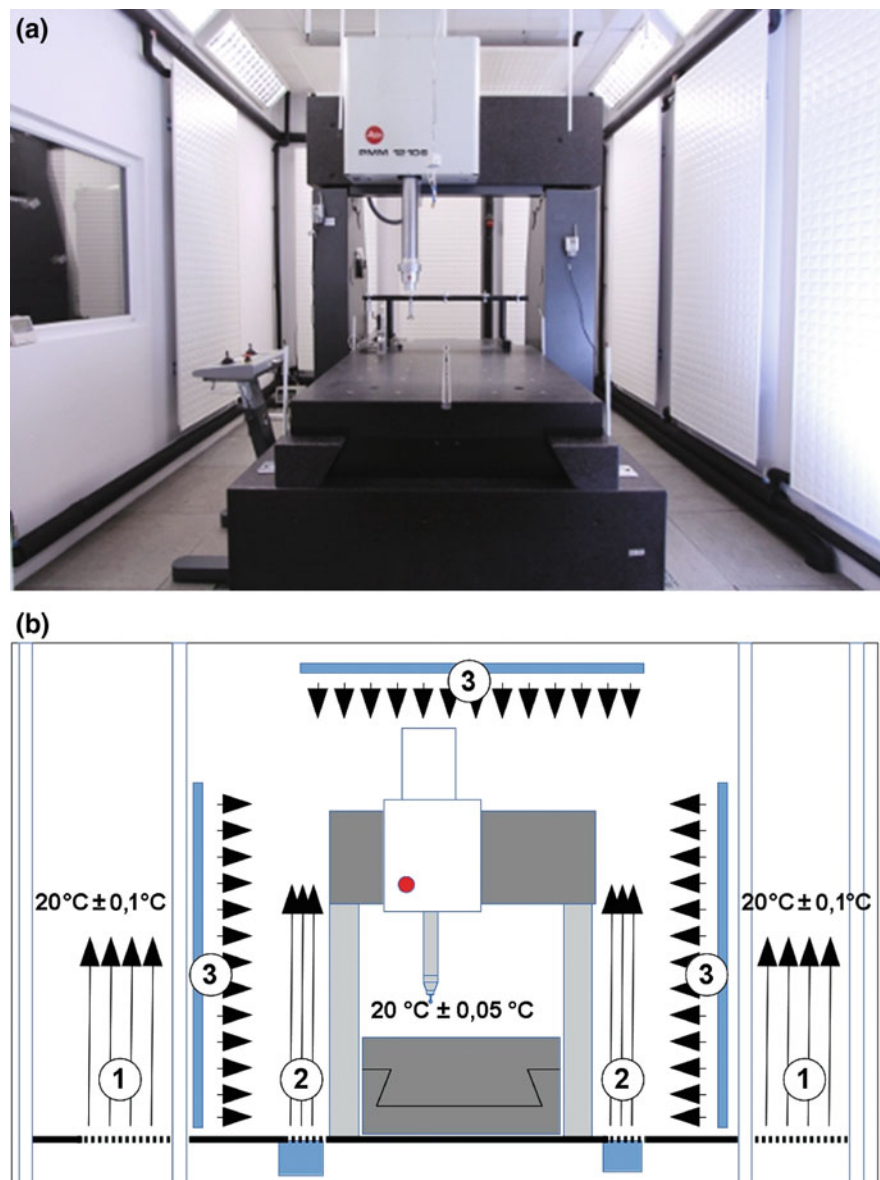


Fig. 2.1 **a** Multicoordinate measuring machine—PMM12106 Leitz—Laboratory of Coordinate Metrology at Cracow University of Technology; **b** air-conditioning system with thermal stability 0.05 °C. 1 isolation zone, 2 air curtain, 3 water panels

location in the object's space created from these points, such as straight lines, planes, and the like. Dimensional characteristics are represented by scalar quantity (length, radius of the circle or cylinder, cone angle). The representation of the

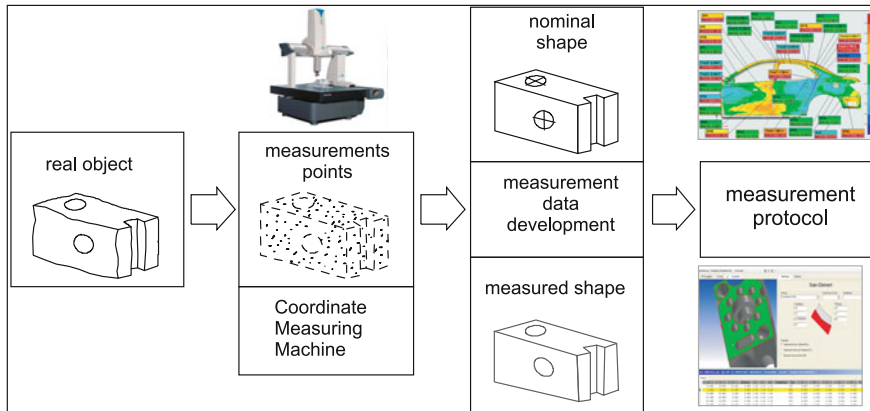


Fig. 2.2 Use of the coordinate measuring technique to geometric quantity measurements with the schematically indicated current method of measurement protocol preparation [10, 106]

measured object, however, involves the problem of determining its reference surface, having the shape of the assumed nominal surface.

The application of coordinate technique in the measurement process is shown in Fig. 2.2. As shown in the figure, in the coordinate measurement technique identified points are situated on a shell, and their coordinates are used exactly to calculate the ideal surface. Its location depends on the coordinates of individual points, and for its determination it requires, from the geometric point of view, a specified number of the points, whereas the measured surface (the real one) has the shape that results from production technology, and differs from the assumed nominal shape. Therefore to describe it more precisely, there should be identified as many points that represent it as possible, an almost infinite number. However, based on technical and economic criteria, their number is optimized to an indispensable minimum. This number is still substantially higher than the number that results from a geometric description of the assumed shape, because the ideal profile is usually calculated by the Gauss least squares method or minimum zone by Chebyshev. The difference in the location of the reference (ideal) surfaces creating an “image” of the measured object in relation to the location of the real surface determines a measurement error. Therefore in the analysis of errors, attention should be paid to the basic problem, which is the difference between the location of the measuring point, indicated in the coordinate system and used later to calculate the locked reference area, and its real location. The difference appears due to a whole complex of factors that can be broadly defined as the sources and causes of errors that occur in measurements with the use of coordinate machines. The CMT opened up new opportunities for automated quality control, finding usage in research laboratories, measurement chambers, and above all, in modern production engineering, where characteristics such as flexibility in making the measuring tasks, speed, and accuracy are of essential importance. Because of that CMT becomes an essential part of

the whole quality assurance system. According to the experience of the last several years it can be said that the organization of today's dimensionally shaped control systems in the machine construction field should definitely be based on this technique. It also comes from the need to control more and more parts designed and produced using the computer techniques (CAx) that enable the development of information linkages within the production system.

2.2 Measurement Uncertainty

Measurement of any quantity is correlated with the need for its evaluation. The measurement is always an inaccurate operation, due to physical conditions, and it means that the value obtained as a result of the measurement (determined in the process of measurement) differs from its true value. The measure of this difference is measurement accuracy. Therefore its determination is one of the basic tasks that should be performed in the measurement process. The result of measurement in the form of a standalone single measured value has little practical significance; only after measurement accuracy is added to it, can the comprehensive meaning of the measurement be obtained. The generally accepted interpretation of the accuracy is the range on the value axis, set around the true value x_t as

$$[x_t - \Delta_{\min}x, x_t + \Delta_{\max}x] \quad (2.1)$$

where:

$$\Delta_{\min}x, \Delta_{\max}x > 0$$

In this range the previously obtained individual measurement results are kept. Those values of differences between these results, established during the process of measurement, and the true value, are known as the *error of measurement*. In coordinate metrology we use the term *deviation*, understood as the difference between the indicated and the reference value. The obtained limit value of deviation from the true value (or the one accepted instead) is known as the *maximum measurement error* [20, 92], or (taking the specified range of variability of obtained values, in accordance with [21]), as the measurement uncertainty range. In the case of coordinate measurements, the axial-range interpretation of measurement accuracy is extended to a spatial vector, maintaining the directionally axial interpretation (coordinates) [106]. According to [92] *measurement accuracy* is defined as closeness of agreement between a measured quantity value and a true quantity value.

Concluding the reflections on the accuracy of determining the measurement result, it can only be said about the true value x_t of the measured quantity that it lies within a given designable area. From the theory of measurement point of view, its mathematical description bears significant importance, that is, measurement accuracy modeling, because it determines whether the resulting data—results of

measurements—can be used for the identification of the measured object. Determination of the measurement error can be understood as a mathematical model of measurement accuracy and in accordance with [21] it can be divided into two types: deterministic and random models.

The *deterministic model* [21] assumes that the unknown true value of the measured quantity x_i lies within an area of uncertainty. The deterministic model is used to describe the measurement accuracy instruments. For the given instrument (or instrument type) the broad enough uncertainty range is determined (constant or as a value function of the measured quantity) which for certain ranges of variation of influential quantities (e.g., temperature) contains all errors of measurements performed with the given instrument in the entire measurement range. Maximum error describing the uncertainty range in a deterministic model is a component of the metrological characteristics of the given measuring instrument. Determination of the maximum error of measurement is associated with the concept of measurement traceability, which requires parameters characterizing the inaccuracy of measurement and instrument having a reference to generally accepted primary standards, implemented as national or international standards.

The *random model* [21] assumes that in addition to the uncertainty range the level of confidence is also determined. It is the probability p that the unknown true value of the measured quantity x_i lies within the uncertainty range, and resolves the error of a single measurement into two components: *systematic error* understood as the difference between the average of an infinite number of measurement results of the same measured quantity carried out under repeatability conditions, and the true value of the measured quantity; and *random error* defined as the difference between the measurement result and the average of an infinite number of measurement results of the same measured quantity carried out under repeatability conditions. Systematic error is the same for all observations or changes according to a certain rule together with the change of conditions. *Random error*, equal to the difference between the total error and systematic error is generally different for individual measurements and takes positive and negative values, therefore it is modeled with a random variable.

In industrial practice, the measurement with CMM is generally performed only once. Therefore the use of the random model requires acceptance of a hypothetical method of measurement repetition, which can be used in the case of simulation of real measurements (while maintaining in this simulation the conditions of repeatability and reproducibility). In the random model of measurement accuracy the following cases may be specified:

- (a) *Direct multiple measurement with random error only.* This is typical for repetitive measurements, which are dedicated to set a coordinate of a single measuring point, assuming the elimination of systematic errors, for example, for the study of a random part of a contact probe error.
- (b) *Direct multiple measurement with random and systematic error.* It is possible only to determine a single measuring point and is used practically in the research environment.

- (c) *Indirect measurement with the use of directly measured quantities, the accuracy of which is described by a deterministic or random model.* This applies to measurements carried out on CMM in industrial conditions, in which a single measurement dominates; in this case determination of parameters of measured shape elements is based on directly determined coordinates of measuring points.

A random model including both random and systematic error is a model already adopted in the standard PN-71/N-02050. However, the classical metrology, in the opinion of [21] does not provide a clear recipe enabling the user to carry out calculations of maximum error of measurements affected by errors with a systematic and random component. In fact, the classical approach requires identification and removal of the systematic error in the form of appropriate corrections and if it is not possible to determine the systematic error, the value of which is “sufficiently small” in comparison with measurement accuracy, systematic errors in the calculation of uncertainty should be considered as random. In coordinate technology the adoption of a random model with the possibility of eliminating systematic error, typical of the work of Lotze and Hartmann [19, 26, 27, 61–65, 125], again becomes important because of the intensive development of CMM error correction techniques by the CAA method [2, 5, 7, 8, 16, 22, 72, 79, 81, 96, 99, 106, 137, 138]. Nowadays, because of the development of correction technology based on CAA, a significant reduction of the impact of a systematic error without its technical elimination is possible [6, 29, 128]. Some arguments call into question the usefulness of systematic error elimination [29, 48, 51, 79, 131] instead of performing the software correction, because in consequence the residual systematic errors are treated as random ones during measurement uncertainty calculation.

An already classical theory of measurement accuracy (measurement error) has been replaced by the so-called *uncertainty theory*, the formally recognized and universal method for determining measurement accuracies, adopted by all international metrological organizations.

In general, *measurement uncertainty* is a range around the measurement result, which with some given probability contains the true value of the measured quantity. Therefore the measurement uncertainty consists of two values, with the real value located somewhere in between.

The basic set of definitions commonly used in Poland is “International vocabulary of metrology—Basic and general concepts and associated terms,” (VIM) the PKN ISO/IEC Guide 99 revised in 2010 [92], in terms of measurement uncertainties determination, supplemented by the ISO *Guide, Guide to the Expression of Uncertainty in Measurement (GUM) ISO 1993/1995*, and published in 1999 by the Central Office of Measures in Polish version—*Wyrażanie niepewności pomiaru przewodnik*, Główny Urząd Miar, Warszawa 1999 z dodatkiem do wydania polskiego, J.M. Jaworskiego [21]. This guide contains a description of the measurement uncertainty model and guidelines of methods for its determination, the usage of which provides complete comparability of measurement results.

Modern understanding of the measurement uncertainty concept appeared in the German metrological literature, and its continuation is the definition of this term by the German standard DIN 1319 [11] “Fundamentals of Metrology” developed in 1997 (already based on GUM), but referring to the earlier definition. Although this standard has taken from GUM concepts and mathematical models for determining measurement uncertainty, the concept of uncertainty itself is slightly different from the definition adopted there. In the DIN 1319 standard uncertainty is defined as “The parameter obtained during the measurement along with the measurement result and is used to define the variation of true value of the measured quantity.” This definition coincides with the conventionally true value of the measured quantity, whereas GUM defines uncertainty as “The parameter related to the result, which characterizes the spread of values that can be substantially attributed to the measured quantity.” The definition describing the term “uncertainty” in the DIN 1319 standard states that uncertainty is not only a statistical dispersion of measurement results, which can be reduced by performing a large number of measurements and calculating the average value, but it contains a systematic part (known and unknown), associated with, for example, a reference standard. It means the confirmation of a connection between the measured value and national/international standards, an important component in industrial practice. According to the ISO 10012 standard [84], uncertainty is defined as the result of measurements made to estimate the range that contains a true value of the measured quantity, usually with a given reliability. In Poland, the uncertainty has already been defined by PN-71/N-02050 [91] as the dispersion of measurement results determined by maximum errors (Fig. 2.3).

Finally, according to the VIM, measurement uncertainty is defined as a non-negative parameter characterizing the dispersion of the quantity values being attributed to a (*measurand*) quantity true value/quantity intended to be measured based on the information used. This parameter could be, for example, the standard deviation, the standard uncertainty (or its multiple), as well as half of the range that has a certain expanded probability. In accordance with the above-mentioned

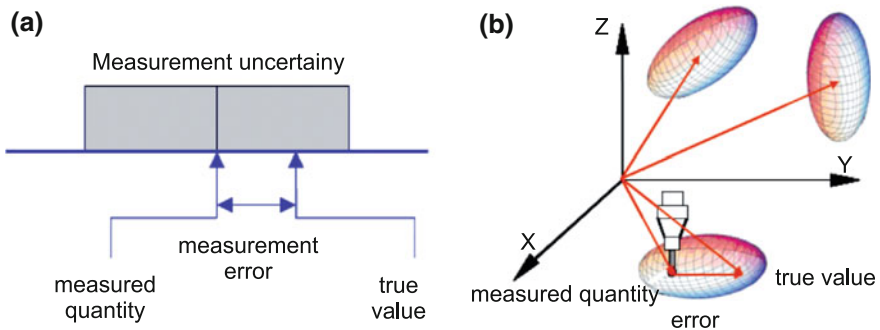


Fig. 2.3 Graphical representation of the meaning of error and measurement uncertainty: the axial system (scalar) (a), spatial (vector) (b)

information of measurement uncertainty cited after DIN 1319, VIM accepts that uncertainty contains components from systematic influences, such as components associated with corrections and values assigned to reference standards and definitional uncertainty. Sometimes estimated systematic effects are not corrected, and instead the measurement uncertainty components corresponding to them are introduced. It is therefore assumed that measurement accuracy is described by its uncertainty. The practical development of this definition in the form of guidelines for evaluation of measurement uncertainty is given in the GUM Guide [21]. It introduces the concept of the *standard uncertainty* u , understood as the uncertainty of the measurement result expressed as a standard deviation and *expanded uncertainty* U , given as a quantity determining the range around the measurement result, which covers a large part of the value distribution that reasonably can be attributed to the measured quantity (resulting from the assumed confidence level).

$$U = k \cdot u \quad (2.2)$$

where:

U expanded uncertainty

u standard uncertainty

k coverage factor that depends on the assumed confidence level p (determined from the t distribution or normal distribution)

It is generally assumed that $k = 3$ for $p \approx 99.73\%$ or $k = 2$ for $p \approx 95.45\%$.

In practical conditions of measurements $t = 2$ is assumed, because the confidence level $p = 95\%$ is usually assigned.

When the measurement result is obtained from values of other quantities, then the standard uncertainty is called *combined standard uncertainty* u_c : it is an estimate of variance and assuming n independent components of standard uncertainty u_i it is determined in accordance with the uncertainty propagation rules stated as Eq. (2.3):

$$u_c^2 = k_1 u_1^2 + k_2 u_2^2 + \cdots k_i u_i^2 + \cdots k_n u_n^2 \quad (2.3)$$

where:

u_i standard uncertainty components

k_i weight factors for i th uncertainty factor

Standard uncertainty or combined standard uncertainty components may be determined, as suggested by the definition of uncertainty [21, 92], by two methods:

- (1) Based on statistical analysis of a series of measurement results
- (2) Based on assumptions about the possible variation of a given uncertainty component (type of distribution, the variation range), allowing the estimation of standard deviation

In accordance with the recommendations of [21], uncertainty components are divided into two categories, depending on their calculation method:

- (A) Uncertainties that have been calculated by statistical methods
- (B) Uncertainties that have been estimated by other methods

This division aims to indicate two different ways of calculating the uncertainty components and facilitating discussion; the classification does not aim to indicate the differences in the nature of the components calculated by different methods. Both methods of calculation are based on probability distribution, and uncertainty components calculated by both the first and the second method are defined in terms of the variance or standard deviations.

Therefore, the standard uncertainty that is, the measurement uncertainty expressed as a standard deviation or as an estimate of this deviation, determined as *standard uncertainty type A*, is calculated from the probability density function obtained from the observed frequency distribution, whereas the *standard uncertainty type B* is calculated on the basis of the assumed probability density function, based on the confidence degree for probability of the given event appearance.

Therefore the determination of the type A uncertainty is of experimental character. Based on repeating n measurements of a given quantity w , the individual results of which are marked as w_i , their arithmetic mean $\bar{w}$, as the best estimation of the expected value μ_w should be determined. As a measure of the variability, the estimate of the variance of the random variable is taken. In the case of coordinate measurements, if particular coordinates of the measuring points were treated as independent random variables, the variance of the random variable w for each independent coordinate of the measuring point (x, y, z) of the probability function $p(w)$ would be determined by the expression:

$$\sigma^2(w) = \int (w - \mu_w)^2 p(w) dw \quad (2.4)$$

where w takes the appropriate values for particular coordinates: x, y, z , and μ_w is the expected value of w and takes appropriate values μ_x, μ_y, μ_z .

As the estimator of the variance $\sigma^2(w)$ the experimental standard deviation $s^2(w_i)$ is accepted. It characterizes, as has already been mentioned, the variability of observed values w_i , or more precisely, their dispersion around the average $\bar{w}$:

$$s^2(w_i) = \frac{1}{n-1} \sum_{i=1}^n (w_i - \bar{w})^2 \quad (2.5)$$

where w_i means the i th from n independent observations of a random variable w . As the estimator of expected value μ_w , the arithmetic mean $\bar{w}$ from n observations is taken:

$$\bar{w} = \frac{1}{n} \sum_{i=1}^n w_i \quad (2.6)$$

If as a result of the measurement the average of the number of repetitions is taken, then the correct measure of the uncertainty of the measurement result is the variance of the arithmetic mean. The variance of the arithmetic mean of a series of n independent observations w_i of a random variable w is determined by the expression:

$$\sigma^2(\bar{w}) = \frac{\sigma^2(\bar{w}_i)}{n}$$

and its estimate is given by the experimental variance of the mean:

$$s^2(\bar{w}) = \frac{s^2(w_i)}{n} = \frac{1}{n(n-1)} \sum_{i=1}^n (w_i - \bar{w})^2 \quad (2.7)$$

Experimental variance of an average $s^2(\bar{w})$ and experimental standard deviation of an average $s(\bar{w})$ determine numerically how correctly the expected value μ_w of variable w is estimated. Each of them can be used as a measure of the standard uncertainty $\bar{w}$. According to [21] (Sect. 4.2.3), the expressions $u(w) = s(w_i)$ are called the *standard uncertainty type A*.

In contrast to the type A method, which is based on frequency distributions, the *type B method* for calculation of uncertainty components is based on the a priori data distributions, which means that for the estimate w_i of the input quantity w (not determined from repeated observations), the estimate of its variance $u^2(w_i)$ or the standard uncertainty $u(w_i)$ is determined by scientific analysis based on all available information about the possible variation of the input quantity w .

In relation to the coordinate measurement uncertainty determination done using the B method, the set of this information may include:

- Previous measurement data
- Measuring experience based on the knowledge of phenomena and characteristics of the standards or instruments used
- Manufacturers' specifications
- Data obtained during calibration

However, it should be noted that the determination of standard uncertainty by the type B method can be as reliable as the calculation carried out by the type A method. That situation may occur especially when calculation by the type A method is based on a relatively small amount of independent information [82].

The type *B* method of determining the uncertainty uses an error propagation process (random variables). Each input quantity is described by its best estimate and standard deviation as the standard uncertainty associated with the estimate, and then for these data the probability distribution propagation is used. In this way, the estimate of output quantity is obtained and associated with its standard uncertainty. This model, however, has one major defect. Namely, the output quantity is usually described by a random variable that has a Gaussian or *t* distribution. As is known, these distributions are symmetric, and their extreme is achieved for the expected value. In practice, however, it often happens that an output quantity does not have symmetrical distribution. In this case much better results may be had by the use of the Monte Carlo (MC) method [70]. The MC method is a numerical method developed by Stanislaw Ulam, implemented for mathematical modeling of complex processes where the value is difficult to determine by an analytical method. It is suitable for uncertainty estimation, which confirms the GUM supplement describing its application [124]. The use of the MC method for probability propagation is a random sampling from the distribution function. The propagation process is also worth mentioning (adding random variables). The method is based on determination of the probability distribution function for the output quantity by analyzing the function of distributions assigned to the input quantities, which have a certain influence on output quantity.

The use of the MC method can be presented as shown in Fig. 2.4 [124]. There are three main stages highlighted: formulation of the problem, the propagation, and the summary stage.

1. Formulation of the problem:

- (a) Determination of the output quantity
- (b) Determination of the input quantities, those on which the output quantity is dependent
- (c) Construction of a model (mathematical), linking the input quantities with the output quantity
- (d) Assigning appropriate probability density functions (PDF) to input quantities, based on the knowledge and known information

2. Propagation:

PDF sampling from input quantities and use of mathematic measurement model for obtaining the PDF for the output quantity

3. Summary:

PDF application for the output quantity in order to determine:

- (a) The expected value of this quantity being its estimate
- (b) The standard deviation of this quantity being its standard uncertainty
- (c) The coverage interval containing the values of input quantity with appropriate probability *p*

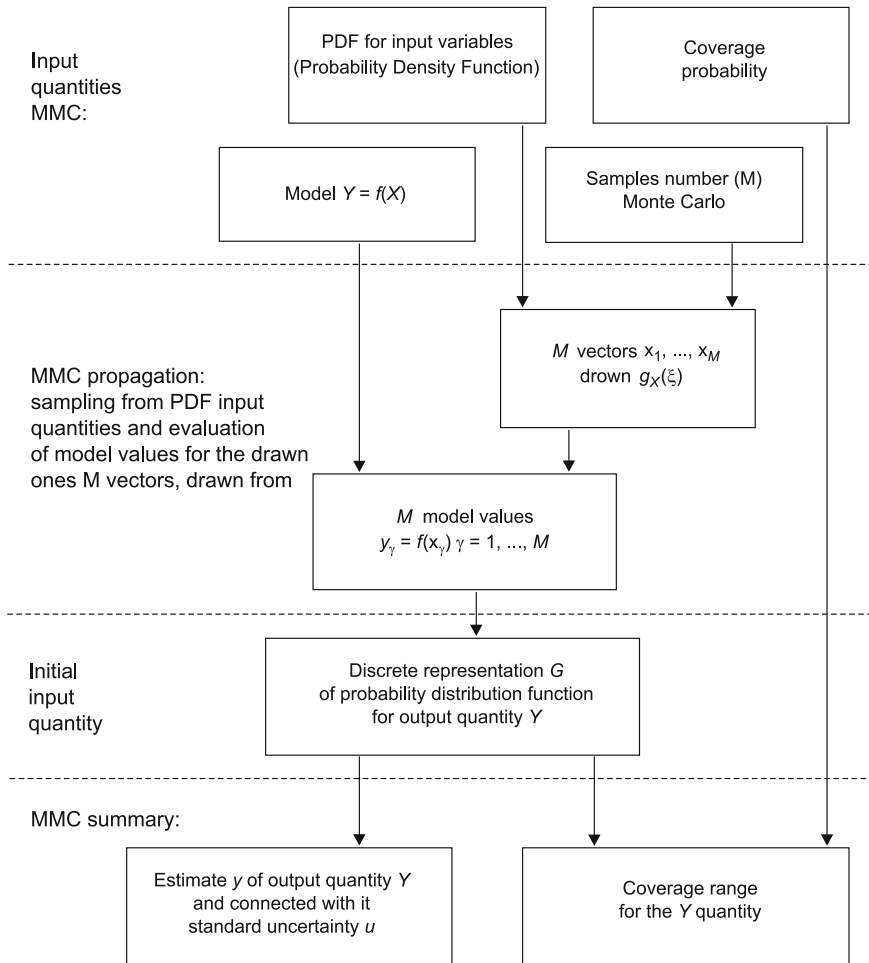


Fig. 2.4 MC method usage stages [116]

The main advantages of the application of the MC method in the uncertainty estimation are:

- Efficiency of calculations for complex or nonlinear uncertainty models
- Increase of accuracy of the Y estimate determination for nonlinear models
- Improvement of accuracy of standard uncertainty estimation for Y estimate in nonlinear models, especially when PDFs without a Gaussian distribution or t distribution are assigned to input quantities
- Accurate determination of the coverage interval in the case when the output quantity is not described by a Gaussian or t distribution

2.3 Vector Concept of Describing Coordinate Measurement Accuracy: Measuring Point Reproducibility Error

Coordinate measurement is essentially the determination of the basic measurement information quantum in the form of measuring point coordinates. This task is carried out as a direct measurement, and the other tasks of this metrological process are indirect measurements, and that is why the construction of the coordinate measuring system accuracy model should be based on it. However, the process of determination of any point coordinate and the analysis of the errors should be described first. In the case of systems of the contact measuring point identification as a basis for model construction, the reproducibility error of the measuring point (REMP) formulated in [99] was adopted. It is characterized by a vector $\bar{P}_a$, which represents a difference between the position vector of the probe tip contact point $\bar{P}_{rz}$ with the measured surface, and a position vector of the point indicated by the machine $\bar{P}_m$, treating $\bar{P}_{rz}$ as a position vector of the real measuring point. From the metrological point of view vector $\bar{P}_a$ is a vector that characterizes CMM accuracy in a given point of its measuring volume, as shown in Fig. 2.5.

$$\bar{P}_a = \bar{P}_{rz} - \bar{P}_m \quad (2.8)$$

However, to make the model complete, the concept of position error given in papers [51, 74, 76, 99] and probe error has to be recalled. The position error $\bar{P}_p$ ¹ in accordance with [51] and [106] is the difference between a position vector of a contact tip (being at the limit of sending the impulse of contact) and measurement surface $\bar{P}_{km}$ common point, and a vector $\bar{P}_m$ of a point indicated by the machine. Therefore the $\bar{P}_p$ vector cumulates inside the errors associated with the kinematics (21 error components) and the errors of displacement measuring system:

$$\bar{P}_p = \bar{P}_{km} - \bar{P}_m \quad (2.9)$$

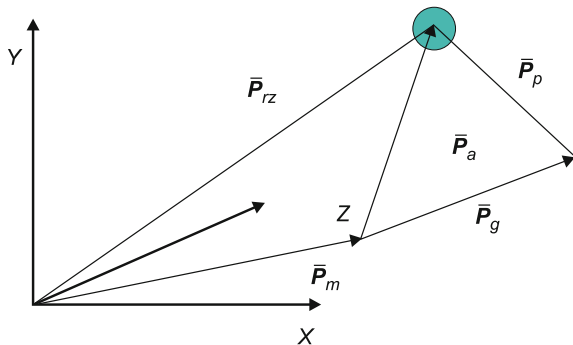
According to this definition, the shape deviation of the contact tip sphere, the impact of the measured object, and most of all the variable errors of the probe, the values of which are described by the FBG (α β , BG) function (Sect. 4.3) are not included here. Therefore the model for a single measuring point, with probe errors included and reported here as vector coordinates $\bar{P}_g(x, y, z)$, appears as

$$\bar{P}_a = \bar{P}_p + \bar{P}_g = \bar{P}_{rz} - \bar{P}_m \quad (2.10)$$

In the case of multistylus probes, indexing probes, and redundant systems (measuring arms and laser tracer) styli, this record should be supplemented with an

¹Note: Do not confuse with the positioning error defined in Sect. 4.1.

Fig. 2.5 Accuracy error and components of the reproducibility error of the measuring point (REMP) [105]



orientation vector P_o , which together with the vector of probe errors defined before creates a vector P_{go} . It is defined as their sum (2.11), as shown in Fig. 2.6:

$$\bar{P}_{go} = \bar{P}_g + \bar{P}_o \quad (2.11)$$

However, the total (defined for the whole measuring space) CMM accuracy model can be adopted using the differentiator given above in the form of a vector $\bar{P}_a$, applying it to the whole volume V of dimensions R^3 . If we assume that to each point $P(x, y, z) \in V \subset R^3$ a vector $\bar{P}_a$ is assigned, a vector of the CMM error field will be characterized, with its components in the form of:

$$\bar{P}_a(p) = \bar{i}Pa_x(p) + \bar{j}Pa_y(p) + \bar{k}Pa_z(p) \quad (2.12)$$

where:

$\bar{P}_a(p)$ vector field of measuring point reproducibility errors at the p point in measuring volume V of given CMM
 Pa_x, Pa_y, Pa_z components of error field in Cartesian CMM coordinate system
 $\bar{i}, \bar{j}, \bar{k}$ versors of coordinate system axes

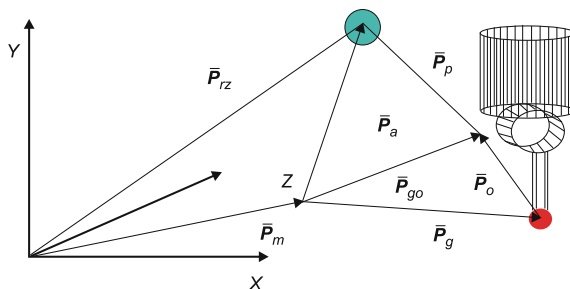


Fig. 2.6 Accuracy error and reproducibility error components including probe error and orientation of the measuring stylus for probes with indexing mechanisms and redundant systems, for example, measuring arms

The creation of the error model is therefore based on the determination of relation (2.12). However, it should be considered that in real terms, apart from systematic factors, measuring point reproducibility errors are influenced by random factors. It is therefore necessary to characterize the random error field, which for the CMM measuring volume can be considered to be continuous with a discrete parameter, according to [51, 106, 133] (See Fig. 2.7). If the coordinate system U is specified in a three-dimensional volume, then a random vector field $\bar{U}(P_a)$ can be written as

$$\bar{U}(P_a) = \bar{i}U_x(P_a) + \bar{j}U_y(P_a) + \bar{k}U_z(P_a) \quad (2.13)$$

where

U_x, U_y, U_z = component uncertainties in x, y, z system

On the basis of the findings described previously [75, 77, 99, 106] we can formulate the following statement: the part of error that comes from the contact probe head dominates in random component error.

Analyzing the issue of random error participation, the work [26, 63, 65] of Lotze and Hartmann should be adduced. They deal with the determination of the uncertainty area for two-dimensional issues, only in [106] the uncertainty area for spatial (three-dimensional) measurements was determined, assuming that the random error for coordinate measurement has vector character [13, 99, 106, 107] and is usually described by a matrix of variances and covariances [3, 106]:

$$\mathbf{S} = \begin{bmatrix} \sigma_x^2 & \rho_{xy}\sigma_x\sigma_y & \rho_{xz}\sigma_x\sigma_z \\ \rho_{xy}\sigma_x\sigma_y & \sigma_y^2 & \rho_{yz}\sigma_y\sigma_z \\ \rho_{xz}\sigma_x\sigma_z & \rho_{yz}\sigma_y\sigma_z & \sigma_z^2 \end{bmatrix} \quad (2.14)$$

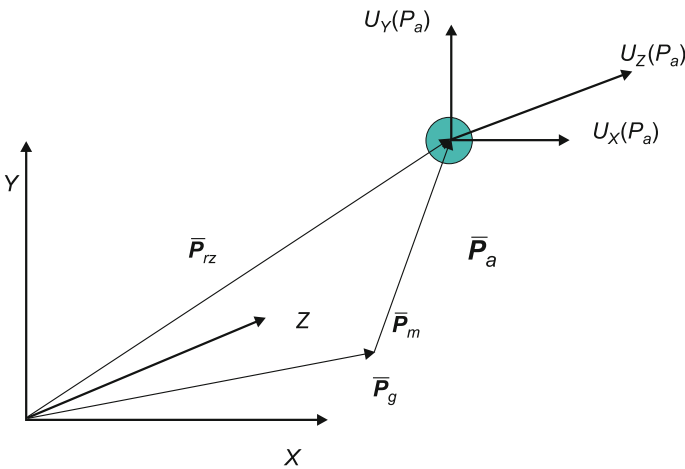


Fig. 2.7 Random components $U_x(P_a), U(P_a), U_z(P_a)$ of the measuring point reproducibility error $\bar{P}_a$

where:

σ_x standard deviation of error

ρ_{xz} covariance between coordinates

In practice the described matrix has the form of estimates of variances and covariances:

$$\mathbf{S} = \begin{bmatrix} s_x^2 & r_{xy}s_xs_y & r_{xy}s_xs_z \\ r_{xy}s_xs_y & s_y^2 & r_{yz}s_ys_z \\ r_{xz}s_xs_z & r_{yz}s_ys_z & s_z^2 \end{bmatrix} \quad (2.15)$$

Random errors during multirepetition of measurements, under set conditions, change in unpredictable ways both in the sign and in absolute value. Considering the performance of a typical contact probe head and other components of CMM error it can be concluded that random errors are the sum of many influences, and have not been included either because of the small effect on the behavior of the machine, or because it has not been possible to measure them. Their impact appears during the contact process and affects the determined measuring point coordinate. In the case of random errors the following assumptions can be adopted:

- Positive and negative errors are equally probable (symmetry).
- Possibility of making the big mistake is less probable than the small one (concentration).

Because of these two reasons, it can be concluded that random errors are of normal distribution. The equation of the distribution (probability density function) for the general n -dimensional case by [3, 106, 107] takes the form:

$$\begin{aligned} (x) &= \frac{1}{\sqrt{\det(S)} \cdot (2\pi)^n} \exp\left(-\frac{Q}{2}\right) \\ Q(x) &= (x - \mu)^T \cdot S^{-1} (x - \mu) \end{aligned} \quad (2.16)$$

for which:

$$x = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \quad \mu = \begin{bmatrix} \mu_1 \\ \vdots \\ \mu_n \end{bmatrix}$$

where:

x random vector

μ expected value of random error

If the case of measurement in the flat system (two-dimensional) is taken as in [3, 106, 107], then relations (4.7) and (4.8) take the form:

$$\mathbf{S} = \begin{bmatrix} s_x^2 & r_{xy}s_x s_y \\ r_{xy}s_x s_y & s_y^2 \end{bmatrix}$$

$$\varphi(x, y) = \frac{1}{2\pi s_x s_y \sqrt{1 - r_{xy}^2}} \exp\left(-\frac{Q}{2}\right) \quad (2.17)$$

$$Q(x, y) = \frac{1}{1 - r_{xy}^2} \left[\frac{(x - \mu_x)^2}{s_x^2} - 2r_{xy} \frac{(x - \mu_x)(y - \mu_y)}{s_x s_y} + \frac{(y - \mu_y)^2}{s_y^2} \right]$$

If the coordinate system is moved to the point (μ_x, μ_y) and rotated so that the covariance r equals zero, Eqs. (2.16) turn into:

$$\mathbf{S} = \begin{bmatrix} s_I^2 & 0 \\ 0 & s_{II}^2 \end{bmatrix}$$

$$\varphi(x, y) = \frac{1}{2\pi s_I s_{II}} \exp\left(-\frac{Q}{2}\right) \quad (2.18)$$

$$Q(x, y) = \frac{x^2}{s_I^2} + \frac{y^2}{s_{II}^2}$$

When determining values of the function $\varphi(x, y)$ for points $(s_I, 0)$, $(-s_I, 0)$, $(0, s_{II})$, $(0, -s_{II})$ we obtain:

$$\varphi(-s_I, 0) = \frac{1}{2\pi s_I s_{II}} \exp\left(-\frac{\left(\frac{-s_I}{s_I}\right)^2}{2}\right) = \frac{1}{2\pi s_I s_{II}} \exp\left(-\frac{1}{2}\right)$$

$$\varphi(s_I, 0) = \frac{1}{2\pi s_I s_{II}} \exp\left(-\frac{\left(\frac{s_I}{s_I}\right)^2}{2}\right) = \frac{1}{2\pi s_I s_{II}} \exp\left(-\frac{1}{2}\right) \quad (2.19)$$

As can be seen, the value of the function at these points is the same. Now by cutting function $\varphi(x, y)$ with plane parallel to the XY plane at the height of this point, we obtain:

$$\begin{aligned} \varphi(x, y) &= \frac{1}{2\pi s_I s_{II}} \exp\left(-\frac{\frac{x^2}{s_I^2} + \frac{y^2}{s_{II}^2}}{2}\right) \\ &= \frac{1}{2\pi s_I s_{II}} \exp\left(-\frac{1}{2}\right) \exp\left(-\frac{\frac{x^2}{s_I^2} + \frac{y^2}{s_{II}^2}}{2}\right) \\ &= \exp\left(-\frac{1}{2}\right) - \frac{\frac{x^2}{s_I^2} + \frac{y^2}{s_{II}^2}}{2} \\ &= -\frac{1}{2} \frac{x^2}{s_I^2} + \frac{y^2}{s_{II}^2} = 1 \end{aligned} \quad (2.20)$$

It is an ellipse with axes that describe main standard deviations. In the case of a three-dimensional Eq. (2.16), after moving the coordinate system origin to the point μ , they take the form:

$$\begin{aligned} \varphi(x, y, z) &= \frac{1}{2\sqrt{2\pi^3} \cdot s_x s_y s_z \sqrt{1 - r_{xy}^2 - r_{yz}^2 - r_{xz}^2 + 2r_{xy}r_{yz}r_{xz}}} \exp\left(-\frac{Q}{2}\right) \\ Q(x, y, z) &= \frac{1}{1 - r_{xy}^2 - r_{yz}^2 - r_{xz}^2 + 2r_{xy}r_{yz}r_{xz}} \quad (2.21) \\ &\quad \left(\frac{(1-r_{yz})x^2}{s_x^2} + \frac{(1-r_{xz})y^2}{s_y^2} + \frac{(1-r_{xy})z^2}{s_z^2} \right. \\ &\quad \left. - 2 \frac{(r_{xy}-r_{yz}r_{xz})xy}{s_x s_y} - 2 \frac{(r_{yz}-r_{xy}r_{xz})yz}{s_y s_z} - 2 \frac{(r_{xz}-r_{xy}r_{yz})xz}{s_x s_z} \right) \end{aligned}$$

After determining the values and eigenvectors $\mathbf{S}$, and then transforming the system in a system in which versors are eigenvectors of a matrix of variances and covariances, Eqs. (2.19) take the form:

$$\begin{aligned} \mathbf{S} &= \begin{bmatrix} s_I^2 & 0 & 0 \\ 0 & s_{II}^2 & 0 \\ 0 & 0 & s_{III}^2 \end{bmatrix} \\ \varphi(x, y, z) &= \frac{1}{2\pi\sqrt{2\pi}s_I s_{II} s_{III}} \exp\left(-\frac{Q}{2}\right) \quad (2.22) \\ Q(x, y, z) &= \frac{x^2}{s_I^2} + \frac{y^2}{s_{II}^2} + \frac{z^2}{s_{III}^2} \end{aligned}$$

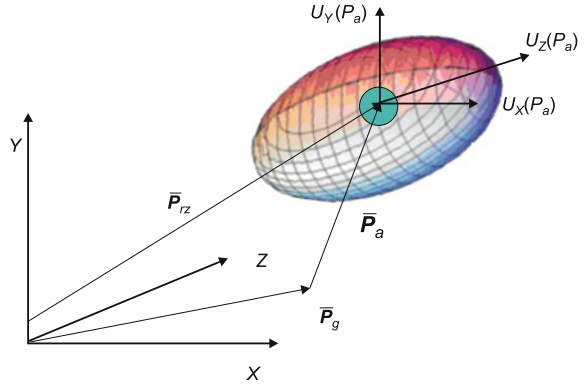
When values of the function $\varphi(x, y, z)$ are determined for points on the axes having values corresponding to standard deviations, the formula describing the ellipsoid with axes s_I, s_{II}, s_{III} is obtained (2.22).

If the coordinate system is not rotated by matrix $\mathbf{S}$ eigenvectors, the equation of an ellipsoid of axes rotated relative to coordinate system axes is obtained. It has been proved that for the case of a three-dimensional measurement, the obtained shape is the ellipsoid with axes s_I, s_{II}, s_{III} . It can be concluded that the random field area, corresponding to the uncertainty area, takes the shape of an ellipsoid (Fig. 2.8). This is compatible with suggestions about the shape of this area contained in the literature [50, 74, 99, 106, 107, 136].

It can be said that the ellipsoid represents the area in which the probability of finding the true value of the measured point is pp . Therefore inasmuch as the standard deviation values constitute a reference point for the uncertainty area, the ellipsoid illustrates this state in three-dimensional volume.

In order to verify the developed equations, the studies based on gaining knowledge of the impact of random errors have to be carried out. They are based on checking the real performance of the measuring machine probe head and redundant

Fig. 2.8 The shape of the uncertainty area around the measuring point defined in the form of an ellipsoid



system stylus (articulated arm coordinate measuring machine, AACMM) for different approach angles.

In the case of AACMM the sphere was measured at 15 points, and the measurements were repeated 32 times. The sphere was placed in the center of the AACMM measuring volume to minimize the effect of systematic components of measurement error.

The variance, standard deviation, and covariance of coordinates of the points determining the center of the sphere were calculated:

$$\begin{aligned}\text{Var}(X) &= 2.961 \times 10^{-3} \text{Stdev}(X) = 0.054 \\ \text{Cvar}(X, Y) &= -4.809 \times 10^{-4} \\ \text{Var}(Y) &= 1.698 \times 10^{-3} \text{Stdev}(Y) = 0.041 \\ \text{Cvar}(X, Z) &= 2.591 \times 10^{-4} \\ \text{Var}(Z) &= 2.548 \times 10^{-3} \text{Stdev}(Z) = 0.050 \\ \text{Cvar}(Y, Z) &= -5.585 \times 10^{-5}\end{aligned}$$

where:

Var variance

Stdev standard deviation

Cvar covariance

A matrix of random errors for the AACMM has been designated:

$$S = \begin{pmatrix} 2.961 \times 10^{-3} & -1.078 \times 10^{-6} & 7.119 \times 10^{-7} \\ -1.078 \times 10^{-6} & 1.698 \times 10^{-3} & -1.162 \times 10^{-7} \\ 7.119 \times 10^{-7} & -1.162 \times 10^{-7} & 2.548 \times 10^{-3} \end{pmatrix}$$

Eigenvalues of the $\mathbf{S}$ matrix, which are ellipsoid axes:

$$w = \begin{pmatrix} 1.698 \times 10^{-3} \\ 2.961 \times 10^{-3} \\ 2.549 \times 10^{-3} \end{pmatrix}$$

Eigenvectors of the $\mathbf{S}$ matrix, which are the versors of the axes system:

$$z = \begin{pmatrix} 8.536 \times 10^{-4} & 1 & -1.723 \times 10^{-3} \\ 1 & -8.538 \times 10^{-4} & -1.354 \times 10^{-4} \\ 1.359 \times 10^{-4} & 1.723 \times 10^{-3} & 1 \end{pmatrix}$$

Therefore the ellipsoid of the uncertainty equation characterizing the AACMM random errors field has the form [69] shown in Fig. 2.9.

Analyzing random errors of AACMM [80, 109], the following regularities are observed.

- The value of the z variable is on the same level as x and y , which may mean that the friction and adhesion forces do not affect the measurement.
- The ellipsoid is rotated in relation to the reference system; it is associated with an orientation error of the stylus.
- The axis parallel to the direction of the approach in AACMM is comparable to y , conversely to the case of the conventional coordinate measuring machine (CMM), where the axis parallel to the direction of approximation is shorter by an order of magnitude.
- The area of random errors confirms that the device may be used interchangeably with conventional CMM.

As for AACMM in the case of the measuring machine, the sphere was measured on PMM12106 at 15 points and 32 times. After calculation in accordance with the equations given above, graphs shown in Figs. 2.10, 2.11 and 2.12 were drawn. They show projections of points and the determined uncertainty area. The uncertainty ellipsoid is shown by projections on the planes XY , YZ , and XZ , respectively.

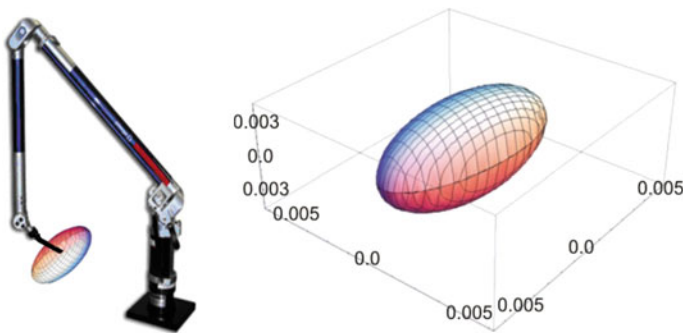


Fig. 2.9 AACMM uncertainty ellipsoid

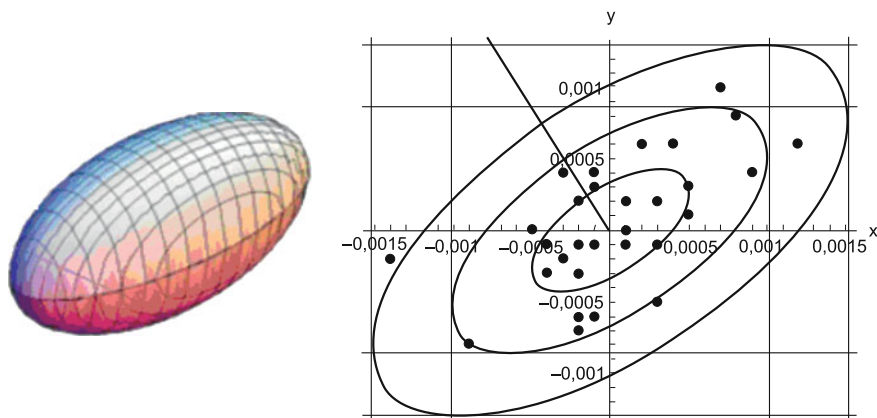


Fig. 2.10 Projection of ellipsoid of uncertainty on the XY plane [106]

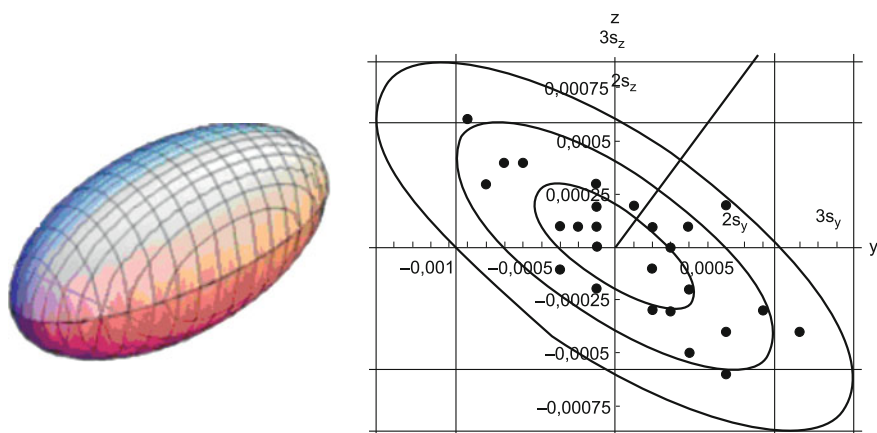


Fig. 2.11 Projection of ellipsoid of uncertainty on the YZ plane [106]

The analysis of several simulations of the implementation of the probe performance during contact with the curved surface (the case of simultaneous impact of sources of random errors in all axes on the random error field formation), taking into account the change in the angle between the measurement direction and the surface normal, allows us to draw these conclusions:

- The uncertainty ellipsoid axes take locations in accordance with the direction of measurement movement.
- The axis parallel to the direction of approximation is the shortest one.
- The standard deviations (uncertainty ellipsoid axes) of measuring point coordinates in the orthogonal directions to the measurement direction are several times, even several dozen times, greater than in the parallel axis, and the largest ones are perpendicular to the measurement plane.

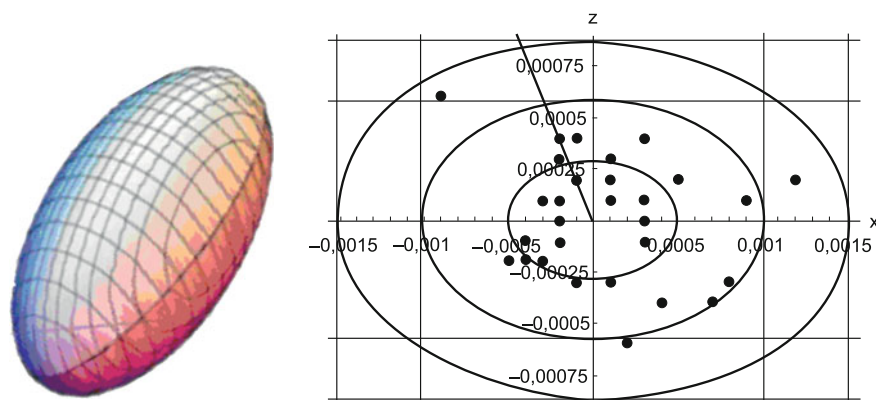


Fig. 2.12 Graph of uncertainty ellipsoid in the XZ plane [106]

For these reasons, the process of probe head model development can be really focused on modeling interactions in the contact direction (tip approach). This confirms the adoption of the PEF identification concept [99, 106] (in accordance with Sect. 3.5) and the assumption of the reasonable simplification in the form of transferring the matter of CMM random error identification to the group of errors independent of the location.

However, from the concept and the model of REMP analysis and in accordance with the works [106, 107] it can be stated that the basic difficulty, from a technical point of view, is a determination of the value and direction of the measuring point reproducibility error for any point of the so-defined error field, and of the dimensions of the uncertainty field for its determination. It is also clear that the REMP determination for all points from the measuring range of the machine is impossible, because each point can be achieved from any number of directions and reaching it is limited by the location and shape of the measuring tips. It is therefore necessary to define some, possibly small, number of points, for which it is necessary to determinate error vectors that will most precisely characterize a given coordinate system in metrological terms. This is possible if the method of CMM accuracy identification were based on the determination of REMP vectors for a selected number of measuring points.

Accepting the multisensor direction of the development of coordinate systems (with contact and noncontact determination of the coordinate of a single point or its cloud), it can be clearly said that the essence from the measurement theory point of view is the determination of the measuring point as a direct measurement. Actually this should be the basis for the measuring system accuracy determination, more than the already known length measurement. This idea was formulated for the first time by the author in [99] and developed in [106], and now with the methods and measuring systems development the technical capabilities of its implementation have been established. The most general conception of description and assessment of the coordinate system accuracy, including all contact and noncontact systems, such as computed tomography (CT) and optical scanners, was described for the first time

in [106] in the form of the MM. This issue is discussed in Sect. 4.4 of this book. The MM is based on the determination of the set (grid) of reference points, and its aim is to define its own coordinate measuring system error field. The reference points are obtained as points on the material standard surface or as the centers of constant curvature elements, for which an explicitly repeatable measurement strategy can be defined. A better explanation of this concept gives the idea based on using a standard cube for the CT system calibration presented in Fig. 2.13. It reveals the relationship of the REMP described by Eq. (2.8) with its random components. The uncertainty area is defined in the form of an ellipsoid created around the measuring point as shown in Fig. 2.8. This idea is so universal that it corresponds well with the concept of metrological use of CT and the single voxel as a measure of its resolution.

It can also be successfully used for optical scanner error identification. Such research was carried out in the framework of [98] and published in [120, 121]. However, the modern concept of assessing the accuracy of CMS in the reference points keeps waiting for comprehensive implementation and it should become a subject of detailed study in scientific work.

From a theoretical point of view, to determine the value and the direction of a so-defined error, the most appropriate measuring system is an independent system, “external” in relation to the checked device, and with higher base accuracy. In technical terms, the implementation of such a system is still difficult. Such a system, analogous to the (GPS) global positioning system, also known as internal GPS, requires reference points and the possibility to measure the distance using the above-mentioned multilateration method [2, 13, 14, 17, 18, 33, 72, 73, 78, 97, 126, 127, 131, 139, 140]. The biggest hopes are put on laser tracking systems LTS [14, 32, 33, 49, 59, 60, 73, 97], the accuracy of which is currently sufficient, for example, for large coordinate measuring machines. In the case of more accurate systems, the

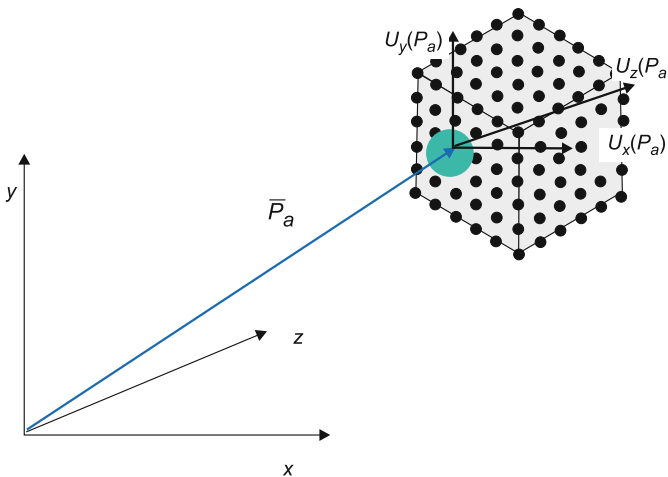


Fig. 2.13 Reproducibility error of measuring point for errors field defined as a reference point grid for coordinate measuring system

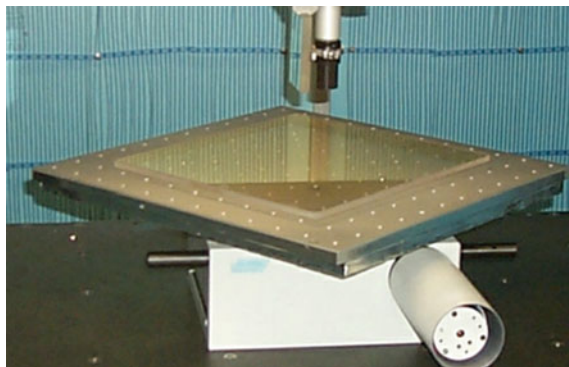


Fig. 2.14 PTB plate standard made of a thermostable material: Zerodur [83]

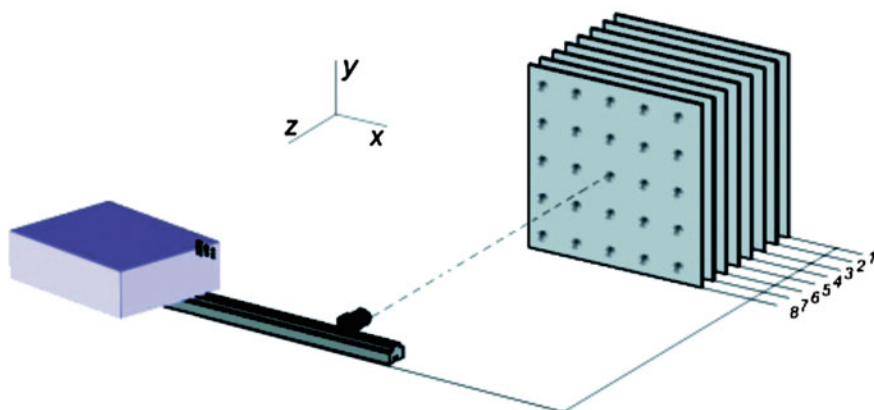
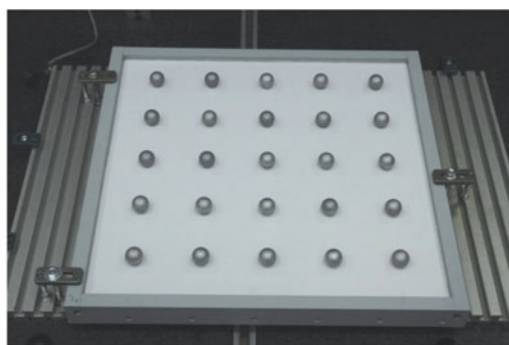
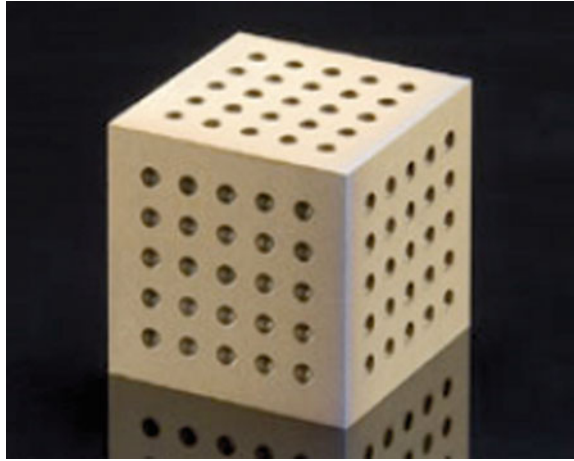


Fig. 2.15 Plate standard for structured-light optical scanners and the construction method of measuring point reference grid for system accuracy assessment [98]

Fig. 2.16 Standard for computed tomography, which allows the construction of the reference grid for accuracy assessment of these systems [15]



LaserTracer with accuracy comparable to the most accurate step gauges can be applied. For optical systems flat and spatial standards are sufficient. In two-dimensional systems a standard used by the PTB could be applied. The example of such a standard is shown in Fig. 2.14. In the case of structured-light optical scanner sphere plates, such as proposed in [58, 98], described in [120, 121], and shown in Fig. 2.15, could be applied. Another solution, consistent with the concept shown in Fig. 2.12, is a standard developed by Feinmess [15] in the form of a hole cube made of material with specific X-ray permeability, presented in Fig. 2.16. Such a standard enables us to carry out the accuracy identification of a coordinate measuring system at the reference points for the CT system. Of course all these systems implement the MM idea [106].

2.4 Coordinate Measurement Uncertainty and Regulatory Requirements

In addition to the GUM guide, among descriptions having great practical importance for measurement assessment, both in the area of quality control and also in the accuracy assessment of measuring instruments, the ISO 14253 standard—Inspection by measurement of workpieces and measuring equipment [85]—should be mentioned. It consists of three parts:

- ISO 14253-1: Decision rules for proving conformity or nonconformity with specifications
- ISO 14253-2: Guidance for the estimation of uncertainty in GPS measurement, in calibration of measuring equipment and in product verification
- ISO 14253-3: Guidelines for achieving agreements on measurement uncertainty statements

This standard is dedicated to geometric quantity measurements and is based on the indications contained in [21].

The ISO 14253-1 standard introduces the concept of specification that here means the tolerance of an object property (dimension). According to GPSs (ISO/TR 14638:1995, *GPSs—Masterplan*) [36, 86] it contains the decision rules that require narrowing the tolerance by the measurement uncertainty to prove conformity with specifications, and extending the tolerance by the measurement uncertainty to prove nonconformity with the specifications. In technical drawings tolerance limits are usually clearly defined. However, when we intend to determine that the real object is set inside or outside the tolerance range, and additionally the measurement is loaded with an uncertainty, then this definition may not be clear (Figs. 2.12 and 2.13).

The purpose of the ISO 14253-1 standard is to regulate these concerns, which may contribute to costly disputes between customers and buyers. Therefore, the standard defines three zones, which are shown in Figs. 2.17 and 2.18.

The compliance zone with the tolerance is limited by the measurement uncertainty at the area borders. Noncompliance zones are set outside the measurement uncertainty ranges. Uncertainty ranges are areas where the compliance or non-compliance cannot be determined.

Measurement uncertainty always acts against the one who performs a measurement, and therefore presents the judgment of compliance or noncompliance. In the ISO 14253-2 [87] there is a suggestion of an algorithm that enables the determination of geometric quantity measurement uncertainty based on the acceptance of an iteration-determined uncertainty budget (called PUMA, procedure of uncertainty management), Fig. 2.19.

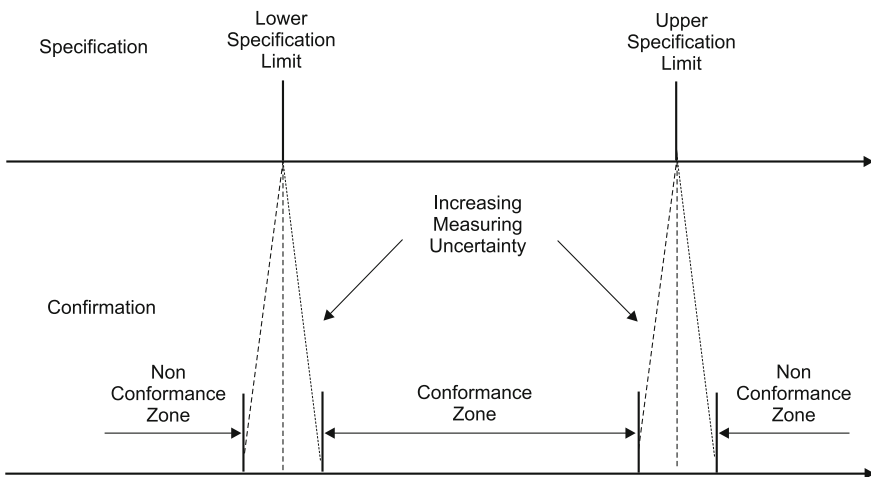


Fig. 2.17 Specification with two clear limits down and top and the method of its conformation [86]

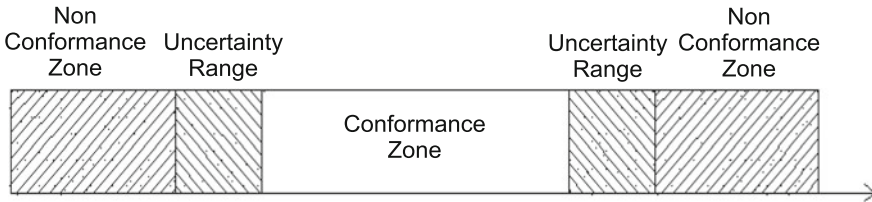


Fig. 2.18 Conformance and nonconformance zones with the specification by ISO 14253-1 [86]

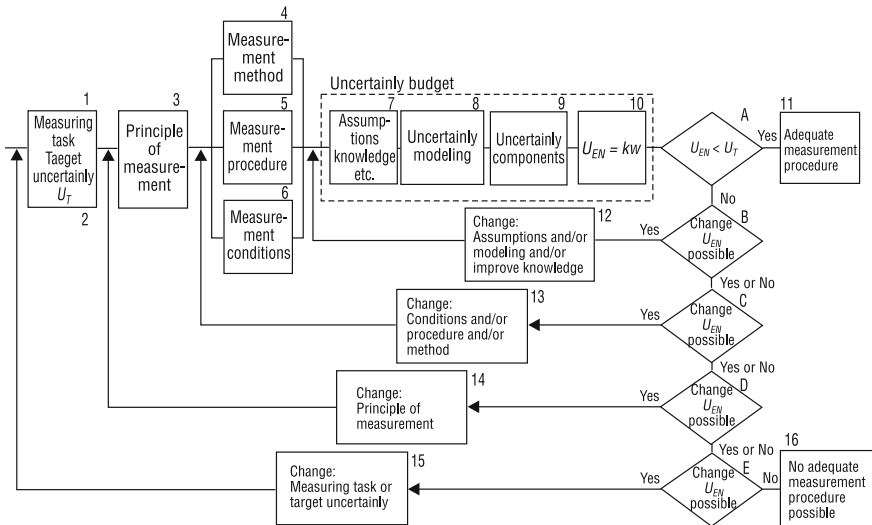


Fig. 2.19 Iterative method of measurement uncertainty evaluation, PUMA [87]

The iterative method for determining uncertainty consists of four steps visualized in Fig. 2.18. After the development of the uncertainty budget, according to requirements set by the quality system, they constitute these variables:

- Assumptions and/or model and/or increase knowledge
- Conditions and/or procedure and/or method
- Measurement principle
- Measuring task or the aim of uncertainty determination

The ISO 9000 series of standards [34] distinguishes in many places the problem of measurements and monitoring product quality and, related to it, inspection of measuring equipment and its calibration. These standards have contributed to the implementation of the concept of uncertainty in industry as a measure of the accuracy of production, the basic concept of determination of product compliance with requirements.

PN/EN ISO 9001:2009 [89] standard in Sect. 7.6—Control of monitoring and measuring equipment states:

... The organization shall determine the monitoring and measurement to be undertaken and the monitoring and measuring equipment needed to provide evidence of conformity of product to determined requirements. The organization shall establish processes to ensure that monitoring and measurement can be carried out and are carried out in a manner that is consistent with the monitoring and measurement requirements. Where necessary to ensure valid results, measuring equipment shall:

- be calibrated or verified, or both, at specified intervals, or prior to use, against measurement standards traceable to international or national measurement standards. Where there are no such standards, the basis used for calibration or verification shall be recorded,
- be adjusted or re-adjusted as necessary;
- have identification in order to determine its calibration status;
- be safeguarded from adjustments that would invalidate the measurement result;
- be protected from damage and deterioration during handling, maintenance and storage.

In addition, the organization shall assess and record the validity of the previous measuring results when the equipment is found not to conform to requirements. The organization shall take appropriate action on the equipment and any product affected.

Records of the results of calibration and verification shall be maintained ...

In addition, the ISO 10012–1 standard contains the requirements to carry out and control calibration on a regular basis, increasing its importance in ensuring product quality.

Furthermore, the standard contains the requirement to carry out documentation of the uncertainty determination method for the measurements. When determining the uncertainty the ISO 10012-1 [84] standard does not permit any derogations and at the same time refers to ISO 9004-1 [90] Chap. 13. There are also recommendations related to measuring device quality assurance and their connections with units by a system of calibrations. Among these recommendations an undisturbed chain of links can be created through a system of independent accredited institutions (calibration laboratories of measuring equipment; manufacturers do not provide this independence, which is a common reason for failures to meet system requirements). In conclusion, it should be noted that the system requirements are fully compliant with the essential requirements contained in the GUM guide [21]. Chapter 6 describes the ISO 10360 standard for controlling coordinate system accuracy. A set of German recommendations VDI/VDE 2617 commenting on the standard determinations and often outpacing its determinations is also of significant importance.

In industrial plants, particularly in the automotive industry, different internal directives are used for measurement uncertainty management in the quality management system, consisting of examination of the measurement capability. For example, companies that apply such directives are Bosch, Daimler Benz, and Ford [119]. The foundation for the process of measuring resource ability examination is the development of proprietary recommendations. In the case of the Bosch and Daimler Benz companies it is the ratio of the measurement uncertainty to the

Table 2.1 Confrontation of measurement capability calculation procedures on example of the three companies

Przedsiębiorstwo	c_g	c_{gk}	Wymaganie
Bosch	$\frac{0.2 \cdot T}{6 \cdot s_M}$	$\frac{0.1 \cdot T - a_s}{3 \cdot s_M}$	>1.33
Ford	$\frac{0.15 \cdot 6\sigma_p}{3 \cdot s_M}$	$\frac{0.075 \cdot 6\sigma_p - a_s}{3 \cdot s_M}$	>1.00
Daimler Benz	$\frac{0.133 \cdot T}{2 \cdot s_M}$	$\frac{0.133 \cdot T}{a_s + 2 \cdot s_M}$	>1.33

Symbols

T Tolerance of checked dimension

σ_p Production process standard deviation

s_M Measurement results standard deviation for repeated measurements

a_s Sum of measurement result systematic errors derived from the standard calibration

tolerance, and in Ford the dispersion of the production process. Although their definitions in the recommendations are identical, the rules of their calculation are different. The principle of the capacity calculation is shown for the example of three producers in Table 2.1.

Although the index c_g covers only the dispersion of the measured value, the component c_{gk} takes into account the deviation of the average value from the standard calibration value, and thus takes into account the systematic deviations of the measurement result.

Essentially in all discussions on measurement capability it is required that the result be corrected of systematic errors. Systematic errors of measuring resources are included in the form of measurement capability component c and thus added to the dispersion of the measurement result. This shows that in industry there is a need to close systematic errors in measurement uncertainty [40, 111]. A request formulated in the GUM [21] to correct systematic errors often turns out to be impractical from the measuring technique point of view. For many of the measuring resources such an eventuality is impossible to achieve, or uneconomical, however, confrontations of various detection methods are carried out and calculations are made in order to assess the measuring equipment.

To confirm the capability of the measurement process, tasks connected with measurement uncertainty must be known and remain within an acceptable relation to corresponding tolerances of an object. Measurement uncertainty depends not only on the measurement process itself, but also on dispersion of measured object material properties and on production conditions [135]. The measurement process capability is marked by g_{pp} . It is the ratio of the expanded uncertainty U ($k = 2$) to the tolerance T of measured value:

$$g_{pp} = 2 \times \frac{U}{T} \quad (2.23)$$

The g_{pp} parameter permits us to state that measurement-expanded uncertainty is consistent with the specification. This uncertainty is specific for a given measuring task, so that the tolerance T associated with the product specification applies only to the given task.

Assessment of the measurement capacity can be defined as the sum of knowledge of the error sources in the measurement process, which is an important requirement for the assessment of the task of uncertainty determination. In coordinate measurements error sources are specified, and the examples are presented in Fig. 3.1. The measurement process capability is confirmed when the ratio of the expanded uncertainty U and tolerance T does not exceed the G_{pp} limit value [135]:

$$g_{pp} = 2 \times \frac{U}{T} \leq G_{pp} \quad (2.24)$$

Generally, the limit value of G_{pp} is between 0.2 and 0.4. However, higher values can be adopted due to the specificity of tools and measuring systems or because of difficulties in measurement realization. Also in accordance with the historical Berndt principle G_{pp} can be taken at 0.5. However, the primary principle is here: the smaller the limit value is, the greater the cost of the measurement. The final choice of the upper limit values should be made with full awareness of the fact that the regulation of the production process may be insufficient. Therefore assessment of the real measurement accuracy gains key importance. In the case of coordinate measurements the determination of their accuracy is a difficult task, but the developed methods are:

- Method using calibrated artifact or standard (ISO/TS 15530–3) [37]
- Multiposition method with the use of noncalibrated artifact, based on specification ISO CD TS 15530–2 [35]
- Error budget: uncertainty estimation based on the identification of the sources of their individual components
- Analytical method [42–47]
- Expert method based on relations from the model of maximum geometric errors for the measuring task [12, 23–25, 31, 38, 93]
- Simulation methods, [1, 4, 9, 12, 28, 30, 39, 41, 48, 52–57, 66–68, 71, 94, 95, 100–106, 108–110, 112–115, 117, 118, 122, 123, 128–132, 134]

On the basis of the implementation method and availability, we can distinguish classical methods available for each user, and simulation methods, based on the use of a virtual measuring system, the design and installation of which are described in Chap. 5.

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