

Preface to the Second Edition

The production of this second edition of *Scattering Theory* has provided the opportunity to accommodate several corrections and updates. One particular advance that is worth drawing attention to, is the modified effective-range expansion for potentials with an attractive inverse-cube behaviour at large distances. This result, derived by Tim-Oliver Müller, was at the verge of publication when the first edition of *Scattering Theory* came out in 2013. The result published by Müller in *Phys. Rev. Lett.* **110**, 260401 (2013) appears as Eq. (4.129) in Chap. 4.

I repeat my thanks to all mentioned in the preface to the first edition and would like to add thanks to Dr. Thorsten Schneider, Ute Heuser and Birgit Münch from Springer-Verlag for their efficient cooperation and assistance.

Garching, Germany
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Preface to the First Edition

This book presents a modern and concise coverage of “scattering theory”, a topic with a long history dating back to the first half of the twentieth century. It is motivated by the observation that most important books on the subject were written several decades ago, from the first edition of *Theory of Atomic Collisions* by Mott and Massey (Clarendon Press, Oxford, 1933), over *Scattering Theory* by John R. Taylor (John Wiley & Sons, Hoboken (NJ), 1972), to the second edition of *Scattering Theory of Waves and Particles* by Roger G. Newton (Springer-Verlag, New York, 1982). Several such classics have been reprinted in recent years, but new original publications are rare.¹

The need for a fresh look at scattering theory follows from the fact that experimental advances have shifted and broadened the scope of applications where concepts from scattering theory are used. Beyond traditional scattering experiments, which continue to be performed with increasing refinement and precision, a deeper understanding of scattering theory has become important in other contexts as well. One example is provided by the field of ultracold atoms and molecules, which has been experiencing enormous growth in recent years, largely triggered by the successful realization of Bose–Einstein condensates of dilute atomic gases in 1995. A sound comprehension of the physics of weakly bound states just below the continuum threshold and low-energy scattering states just above threshold is important for understanding the phenomena observed in this ultracold regime.

The book starts with a chapter on the classical theory describing the scattering of a projectile particle from a target particle. This is helpful, because important concepts such as cross sections can already be explained in this context. Furthermore, the relation between classical mechanics and quantum mechanics and the behaviour of quantum systems in the “semiclassical” or “anticlassical” limits are subjects of lasting interest.

The second chapter contains a detailed formulation of the quantum mechanical description of elastic scattering by a conservative potential. Particular attention is

¹One new book has just been published: *Scattering Theory of Molecules, Atoms and Nuclei* (World Scientific, Singapore, 2013) by L. Felipe Canto and Mahir S. Hussein.

given to the influence of the large-distance behaviour of the interaction potential, which is often well known in realistic situations. The concept of channels, which are related to internal degrees of freedom of the projectile and/or the target, is introduced for the example of internal orbital angular momenta and spins.

The third chapter generalizes this ansatz to arbitrary internal excitations, which enables the description of inelastic scattering via the appropriate coupled-channel equations. This chapter contains the theory of Feshbach resonances, as opposed to single-channel shape resonances, and an account of multichannel quantum-defect theory, which is a powerful tool for describing Coulombic systems with attractive interactions falling off as $1/r$ for large values of the projectile-target separation r .

Chapter 4 addresses some special topics which are particularly relevant for current research involving cold atoms and molecules. Section 4.1 on deep potentials falling off faster than $1/r^2$ at large distances contains a general theory for the description of near-threshold bound and continuum states, which is well suited for application to realistic binary systems such as diatomic molecules or molecular ions. Section 4.2 connects the well established theory of Feshbach resonances with the empirical description that has become widely used in the cold-atoms community, and it formulates a threshold-insensitive parametrization of the Feshbach resonances which is relevant for the analysis of current experiments. The last section contains a short treatise on two-dimensional scattering, which reveals significant differences to the 3D case, in particular in the low-energy, near-threshold regime.

The first appendix describes scaling properties of classical and quantum systems governed by a conservative potential depending homogeneously on the coordinates. This is particularly useful for understanding if and where a given quantum mechanical system has a semiclassical or an anticlassical, extreme quantum limit. The second appendix contains a brief summary of the definitions and some important properties of special mathematical functions occurring in the solutions of various versions of the Schrödinger equation throughout the book.

The book is intended for advanced students and researchers. It is hoped that it will be useful for theorists and experimentalists alike. The level of abstraction is kept as low as at all possible, and deeper questions related to mathematical foundations of scattering theory, as discussed e.g. in *Methods of Modern Mathematical Physics, III. Scattering Theory* by Reed and Simon (Academic Press, New York, 1979), are passed by. The present book should be understandable for anyone with a basic knowledge of nonrelativistic quantum mechanics.

The conception of scattering theory presented in this book has grown over four decades of active involvement in the subject. For the comprehension of the recent developments, I have profited considerably from the input of talented young, unbiased students. In particular, with respect to the theory described in Chapter 4 of this book, original contributions of many students (and post-docs), and their critical views of previous work, have played a major role for achieving a mature formulation of rather general validity. In this context I would like to thank Johannes Trost, Petra Meerwald, Thomas Purr, Michael Moritz, Christopher Eltschka, Georg Jacoby, Carlo Meister, Alexander Jurisch, Eskender Mesfin, Florian Arnecke, Patrick Raab, Johannes Eiglsperger, Tim-Oliver Müller, Martin Fink, Alexander Kaiser, Frauke

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