

## **Chapter 2**

# **Mechanical Properties of Concrete and Steel Reinforcement**

## **2.1 Strength and Deformation of Steel Reinforcement**

### ***2.1.1 Types and Properties of Steel Reinforcement***

#### **2.1.1.1 Stiff Reinforcement and Steel Bars**

Both stiff reinforcement and steel bars can be used in reinforced concrete members.

Stiff reinforcement includes shape steels (e.g., angle steel, channel steel, I-shape steel, and pipe) and the skeletons fabricated by welding several pieces of shape steel together. Due to its large stiffness, stiff reinforcement can be used in construction as forms or supports to bear the self-weight of structures and construction loads. This can facilitate shuttering (also known as formwork) and speed construction. Also, structural members reinforced by stiff reinforcement possess higher loading capacity than those reinforced by steel bars.

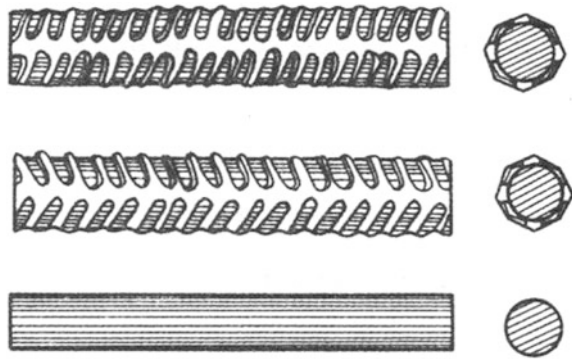
However, steel bars are more frequently used in ordinary reinforced concrete members. Because steel bars are flexible, they are treated as axially loaded elements, while their own stiffness is meaningless in design.

Most design codes and textbooks (including this book) are referring to steel bars when discussing reinforced concrete structures. Answers to questions concerning concrete members reinforced by stiff reinforcement can be found in special textbooks or standards covering steel-reinforced concrete, concrete-filled steel tubes, etc.

#### **2.1.1.2 Surface Profile of Steel Bars**

Steel bars can be classified as plain bars and deformed bars or rebars according to their surface profiles. Deformed bars are the bars with longitudinal and transverse ribs rolled into the surfaces (sometimes without longitudinal ribs). The ribs, which

**Fig. 2.1** Surface profiles of steel bars



may be in the shape of a spiral, chevron or crescent, etc. (Fig. 2.1), can effectively increase the bonding between steel bars and concrete. The cross-sectional area of a deformed bar varies with its length, so the diameter of the deformed bar is a nominal dimension, i.e., an equivalent diameter is the same as that of a plain bar of identical weight. Generally, the diameters of plain bars are 6, 8, 10, 12, 14, 16, 18, 20 and 22 mm, while the diameters of deformed bars are 6, 8, 10, 12, 14, 16, 18, 20, 22, 25, 28, 32, 36, 40 and 50 mm.

Reinforcement steel bars of small diameter (e.g., <6 mm) are also called steel wires, whose surface is generally smooth. If indentations are rolled into the surface of a steel wire to improve the bond, the steel wire is called indented wire.

### 2.1.1.3 Reinforcement Cage

Different kinds of reinforcement in structural members can be strapped or welded into reinforcement cages or wire fabrics before being placed in forms. This not only secures the relative position of the reinforcement, but also helps to bring the reinforcement into full play. Figure 2.2 shows part of a reinforcement cage used to support concrete beams.

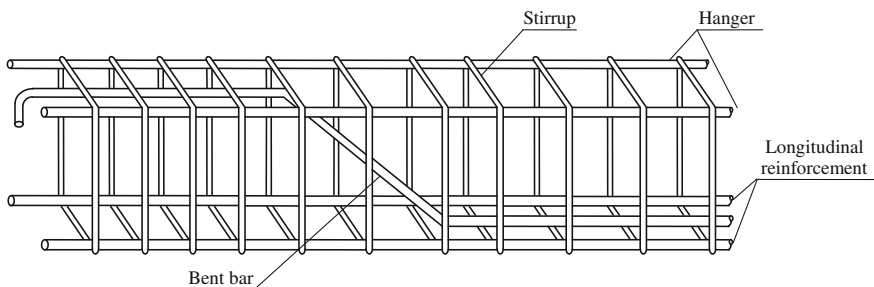


**Fig. 2.2** Reinforcement cage

To prevent a plain bar under tension from slipping in concrete, both ends of the bar should be hooked. Sometimes the intermediate segment of reinforcement should be bent due to the design requirement. The detailings of the bent segments and hooks are listed in relevant design codes or acceptance specifications for construction quality. Figure 2.3 illustrates some hooks and bent segments in reinforced concrete structures.

Hook ends are not necessary for plain bars under compression. Because the cross section of the reinforcement under compression tends to expand, the normal pressure caused by surrounding concrete against such deformation can effectively increase the bond between the reinforcement and the concrete.

The ribs of deformed bars allow the bars to form a better bond with concrete, so hooks are unnecessary at the ends. If a hook end is indispensable for a deformed bar to fulfill the requirement of development length, which will be represented in Chap. 3, a right angle hook rather than a half circle hook will be formed for the convenience of machining.

To ensure that the reinforcement will not crack, fracture, or rupture, a cold bending test is commonly used to check the ductility and internal quality of the reinforcement. The cold bending test is performed by bending the reinforcement around a roll shaft. The reinforcement qualifies as acceptable if there is no crack, delamination, or rupture after being bent to stipulated angles. Readers should refer to relevant national standards for detailed specifications of cold bending tests, such as Metallic materials—Bend test (GB/T 232-2010).

Welded steel cages and wire fabrics are well bonded with concrete. Therefore, installing hooks at their ends is not necessary. Moreover, welded steel cages and wire fabrics are suitable for industrial quantity production and are widely used in precast reinforced concrete construction due to the reduction of in situ reinforcement processing. Reinforcement that needs welding should possess good weldability, i.e., no cracks and excessive deformation are allowed after welding under certain technological conditions.

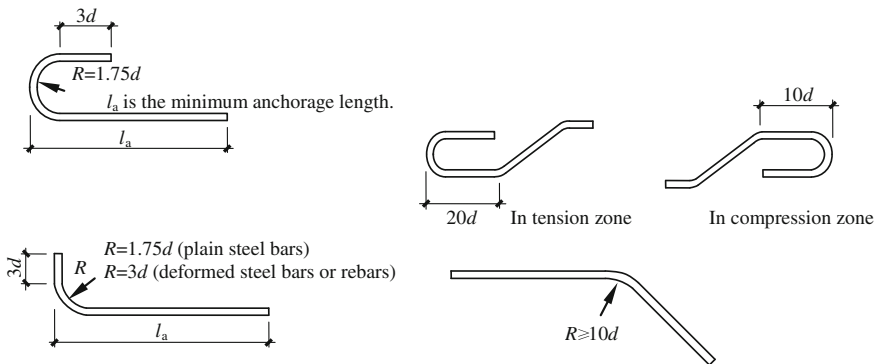


**Fig. 2.3** Hooks and bent steel bars

## 2.1.2 Strength and Deformation of Reinforcement Under Monotonic Loading

### 2.1.2.1 Stress–Strain Curve

Typical stress–strain curves of steel bars used in reinforced concrete structures are obtained from monotonic tension tests, in which the loads are monotonically applied (without any unloading) until the failure of specimens in a short time.

From monotonic tension tests, researchers can evaluate the strength and deformation of steel bars. Figures 2.4 and 2.5 show two stress–strain curves of steel bars with apparent differences.

For hot-rolled low-carbon steel and hot-rolled low-alloy steel, the stress–strain curve in Fig. 2.4 is recorded. The curve exhibits an initial linear elastic portion (segment  $Oa$ ). The stress corresponding to point  $a$  is called the proportional limit. In segment  $ab$ , the strain increases a little bit faster than the stress, although it is not very obvious in the figure. After point  $b$ , the strain increases a lot with little or no increase in the corresponding stress. The curve extends nearly horizontally to point  $c$ . Segment  $bc$  is called the yield plateau. After point  $c$ , the stress again increases

**Fig. 2.4** Stress–strain curve for steel bars with a yield plateau

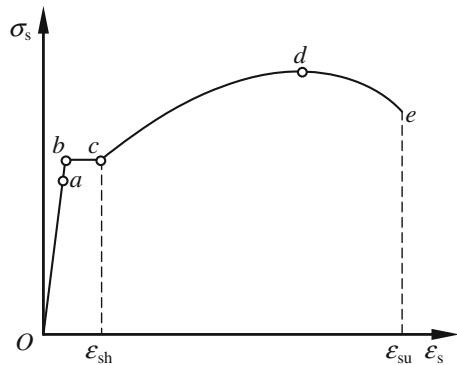
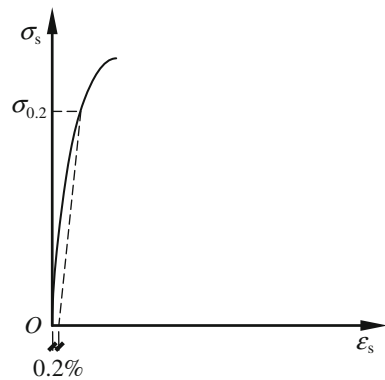


**Fig. 2.5** Stress–strain curve for steel bars without a yield plateau



with the strain until point  $d$ . The stress corresponding to the highest point  $d$  is the ultimate strength of steel bars. Segment  $cd$  is the strain-hardening range. After point  $d$ , the strain increases rapidly accompanied by the area reduction of the weakest cross section, i.e., necking, and finally fracture occurs at point  $e$ .

For high-carbon steel, Fig. 2.5 shows a stress-strain curve, no apparent yield plateau can be observed in the curve. Generally, the stress  $\sigma_{0.2}$  corresponding to the residual strain at 0.2 % is taken as the yield strength. It has been stipulated in Chinese metallurgical standards that the yield strength  $\sigma_{0.2}$  of reinforcement should not be less than 85 % of the ultimate tensile strength  $\sigma_b$  ( $0.85 \sigma_b$ ). Therefore,  $0.85 \sigma_b$  can be taken as the conditional yield point in real engineering applications.

Steel with a well-defined yield plateau is called mild steel, while steel without a well-defined yield plateau is called hard steel.

In reinforced concrete structures, certain ductility of reinforcement is required. The strain corresponding to the end of a yield plateau  $\varepsilon_{sh}$  and the ultimate strain  $\varepsilon_{su}$  are important indices of plasticity, which can ensure apparent warning before either the reinforcement fractures or the members fail.

### 2.1.2.2 Chemical Composition, Grade, and Type of Reinforcement

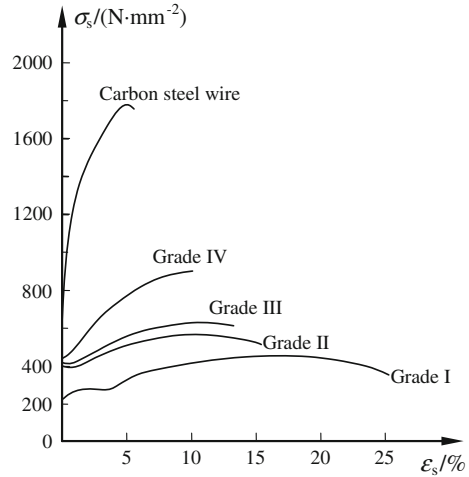
According to chemical composition, steel bars can be categorized as carbon steel bars and ordinary low-alloy steel bars.

Carbon steel contains ferrum and other trace elements, such as carbon, silicon, manganese, sulfur, and phosphorus. Experimental results show that the strength of a steel bar increases with carbon content, but at the cost of plasticity and weldability. Generally, carbon steels with a carbon content of less than 0.25 %, 0.25–6 %, and 0.6–1.4 % are called low-carbon steel, medium-carbon steel, and high-carbon steel, respectively. Low- and medium-carbon steels are mild steels, while high-carbon steel is hard steel.

Ordinary low-alloy steel contains added carbon steel alloying elements such as silicon, manganese, vanadium, titanium, and chromium to efficiently increase strength and improve steel properties. Currently in China, ordinary low-alloy steel can be classified into five classes according to the alloying element addition: manganese class (20MnSi, 25MnSi), silicon–vanadium class (40Si<sub>2</sub>MnV, 45SiMnV), silicon–titanium class (45Si<sub>2</sub>MnTi), silicon–manganese class (40Si<sub>2</sub>Mn, 48Si<sub>2</sub>Mn), and silicon–chromium class (45Si<sub>2</sub>Cr).

According to the processing method, steel bars can be categorized as hot-rolled steel bars, heat treatment steel bars and cold working steel bars. Steel wires include carbon steel wires, indented steel wires, steel strands, and cold stretched low-carbon steel wires. Hot-rolled steel bars can be further classified as hot-rolled plain steel bars HPB300 (also called Grade I reinforcement, denoted by symbol  $\phi$ ), hot-rolled ribbed steel bars HRB335, HRB400, and HRB500 (also called Grade II, III, and IV reinforcement, denoted by  $\phi$ ,  $\Phi$ , and  $\Phi$ , respectively), remained heat treatment ribbed steel bars RRB400 (also called Grade III reinforcement, denoted by  $\Phi^R$ ), and hot-rolled ribbed fine-grained steel bars HRBF335, HRBF400, and HRBF500 (also

**Fig. 2.6** Stress–strain curves of reinforcement for different grades



called Grade II, III, and IV reinforcement, denoted by  $\phi^F$ ,  $\phi^F$ , and  $\phi^F$ , respectively). Cold stretched steel bars are made by mechanically tensioning hot-rolled steel bars in normal temperature. Heat treatment steel bars are ordinary low-alloy steel bars that have undergone the process of heating, quenching, and tempering. High-carbon-killed steel becomes carbon steel wires after several times of cold stretching, stress relieving, and tempering. Rolling indentations into steel wires make indented steel wires, which enables a more secure bond with surrounding concrete. Steel strands are fabricated by twisting several steel wires of the same diameter. Mechanically, tensioning low-carbon steel wires at room temperature makes cold stretched low-carbon steel wires.

As a general rule, steel bars designated as HPB300, HRB335, HRB400, HRB500, and RRB400 can be used as nonprestressed reinforcement, while carbon steel wires, indented steel wires, steel strands, and cold stretched steel bars provide prestressed reinforcement.

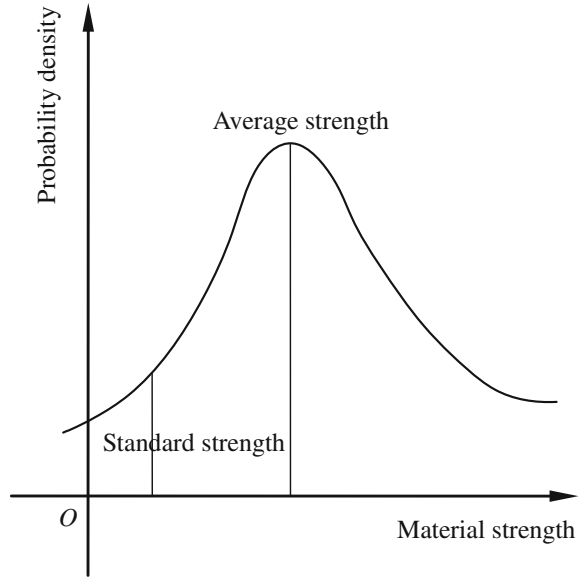
The stress–strain curves of reinforcement for different grades are shown in Fig. 2.6.

### 2.1.2.3 Strength of Reinforcement

Reinforcement undergoes large plastic strain after yielding. The resulting excessive deformation and crack widths of structural members violate the serviceability requirement. So when the capacity of a reinforced concrete member is calculated, yield point (or conditional yield point) is taken as the upper bound.

Reinforcement strength is obtained by tests. But the strengths of different specimens, even when of the same classification or type, are generally different due to the inherent variability of reinforcement materials. Statistical analysis shows that experimental data of reinforcement strength obeys Gaussian distribution (Fig. 2.7).

**Fig. 2.7** Distribution of tested material strength



Therefore, measured strength with a certain degree of confidence can be taken as the standard strength of reinforcement. As shown in Fig. 2.7, if the degree of confidence is 97.73 %, then the standard strength of reinforcement is equal to the average strength minus twice the standard deviation. Tables 2.2 and 2.3 in the addendum list the standard strength of reinforcement (with a 95 % degree of confidence) stipulated in the *Code for Design of Concrete Structures* (GB 50010). Because GB 50010 adopts the probability limit state design method, the standard strength is further reduced by dividing the coefficient  $\gamma_s = 1.1$  to get the design strength of reinforcement. GB 50010 also stipulates that the design strength should be used in the ultimate capacity design of structural members, while the standard strength should be used in the check of deformation and crack width.

Because reinforced concrete design codes in different regions and different engineering professions are based on diverse criteria, such as the *allowable stress design method*, *ultimate strength design method*, and *limit state design method*, they cannot ensure structural members the same safety margin or reliability. Hence, the same reinforcement may be given different strength values under different specifications or even a different name, which will be introduced in detail in the subsequent structural design course.

#### 2.1.2.4 Theoretical Stress–Strain Models of Reinforcement

In theoretical analyses of reinforced concrete structures, the stress–strain relation curves obtained from experiments are seldom directly employed. Theoretical

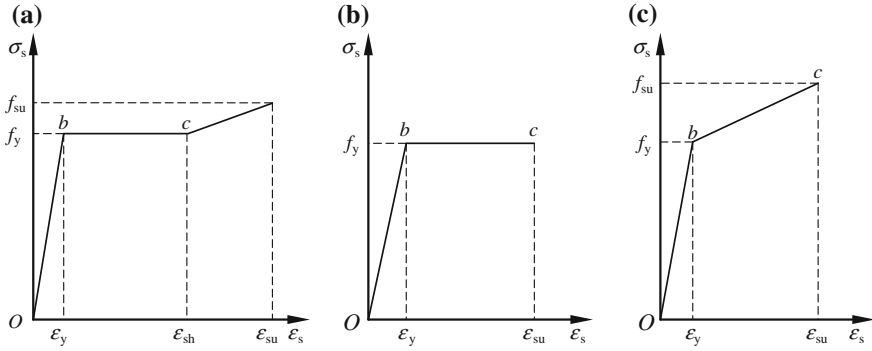


**Fig. 2.8** Theoretical stress–strain models of reinforcement. **a** Trilinear model. **b** Bilinear model (I). **c** Bilinear model (II)

models idealized from the experimental curves are generally preferred. Figure 2.8 shows commonly used theoretical stress–strain models of reinforcement.

The trilinear model (Fig. 2.8a), which is suitable for mild steel with a well-defined yield plateau, can depict the strain-hardening stage and correctly evaluate the stress after reinforcement yielding. If the yield plateau is long, the bilinear model (Fig. 2.8b), i.e., the ideal elastic-plastic model, can give adequate analysis results. Note that the ultimate deformation of concrete at the failure of structural members is limited. Even though the corresponding tensile deformation of reinforcement has entered the strain-hardening stage, the extent of its entrance is still limited. Therefore, in practical engineering, the elastic-perfectly plastic model is commonly employed for ordinary steel bars in theoretical analysis, which can be formulated as:

$$\sigma_s = \begin{cases} E_s \varepsilon_s & (\varepsilon_s \leq \varepsilon_y) \\ f_y & (\varepsilon_s > \varepsilon_y) \end{cases} \quad (2.1)$$

where

$E_s$  the modulus of elasticity of the steel;

$f_y$  the yield strength of the steel; and

$\varepsilon_y$  the yield strain of the steel.

The bilinear model shown in Fig. 2.8c can be used to describe the stress–strain relationship of high-strength steel bars or steel wires, which do not have well-defined yield plateaus.

If the reinforcement does not fail at buckling, the theoretical stress–strain models of steel bars under compression are the same as those under tension.

### 2.1.3 Cold Working and Heat Treatment of Reinforcement

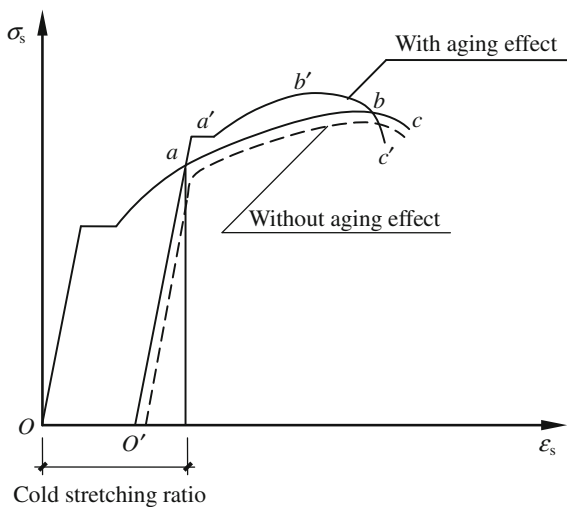
#### 2.1.3.1 Cold Working of Reinforcement

Cold working such as cold stretching and cold drawing can raise the design strength of hot-rolled steel bars (the yielding stress).

Cold stretching is to stretch reinforcement with a well-defined yield plateau into or beyond its stage of yield. As is illustrated by point *a* in Fig. 2.9, a residual strain  $OO'$  will remain and cannot be recovered after the stress is released. And if the reinforcement is stretched again right away, the stress–strain curve will follow the path  $O'abc$  and the yield strength is approximately equal to the stretching stress, which is higher than the yield strength before the cold stretching. But the yield plateau disappears, and the total elongation is reduced from  $Oc$  to  $O'c$ , symbolizing worse plasticity. However, if the reinforcement is stretched again after having been placed in natural conditions for a period of time, the yield point can be further increased from point *a* to point *a'*, i.e., a phenomenon called aging hardening. Moreover, an apparent yield plateau can be observed again, and the stress–strain curve will follow the new path  $a'b'c'$ . The strength increase of steel bars by cold stretching depends on the steel material. The higher the original strength is, the lesser the increase will be. The stretching stress should be rationally selected to keep a certain yield plateau at the same time of increasing the strength. Cold stretching can only increase the tensile strength of reinforcement. When the temperature reaches 700 °C, reinforcement will recover to its original state before cold stretching. So if the reinforcement needs welding, it should be welded first before cold stretching.

Cold drawing is to force reinforcement through a carbide alloy wire drawing die of smaller diameter. The reinforcement will undergo plastic deformation under the

**Fig. 2.9** Stress–strain model of a steel bar after cold stretching



simultaneous action of longitudinal tension and transverse pressure. The cross section of the reinforcement will be reduced, while the length increases. The reinforcement strength is apparently raised due to the internal structural change of the steel. After several times of cold drawing, the plasticity of the reinforcement will significantly decrease, which is proven by the disappearance of the stage of yield. Cold drawing can increase the tensile and compressive strength of reinforcement at the same time.

### **2.1.3.2 Heat Treatment of Reinforcement**

Heat treatment of reinforcement is to make low-alloy steel bars of specific strength experience a series of process, i.e., heating, quenching, and tempering. This can greatly increase the strength of the reinforcement with insignificant decrease of plasticity. Heat treatment steel bars include three types: 40Si<sub>2</sub>Mn, 48Si<sub>2</sub>Mn, and 45Si<sub>2</sub>Cr, of which the stress–strain curves have no obvious yield points.

### **2.1.4 Creep and Relaxation of Reinforcement**

The strain of the reinforcement will increase with time if continuously subjected to high stress. This phenomenon is called *creep*.

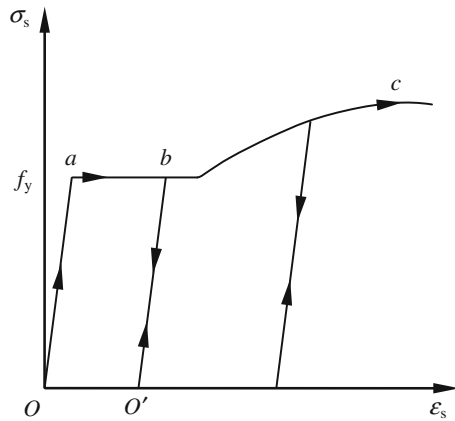
*Relaxation* will occur if the length of a steel bar is kept constant, and the stress of the steel bar will decrease with time. Creep and relaxation have the same physical nature. Creep and relaxation increase with time and depend on initial stress, steel material, and temperature. Generally, high initial stress will cause large creep or great relaxation-induced stress loss. The creep and relaxation of cold stretched hot-rolled steel bars are lower than those of cold stretched low-carbon steel wires, carbon steel wires, and steel strands. If the temperature increases, the creep and relaxation will also increase. The relaxation-induced stress loss in prestressed reinforcement must be considered in prestressed concrete structures.

### **2.1.5 Strength and Deformation of Reinforcement Under Repeated and Reversed Loading**

#### **2.1.5.1 Repeated Loading Behavior**

Repeated loading is to experience a specimen in one direction loading, unloading, reloading, unloading again, and so on. Figure 2.10 shows a stress–strain curve of a steel specimen under repeated loading. If the load is released before failure, the specimen will recover along the linear stress–strain path *bo'* that is parallel to the

**Fig. 2.10** Stress–strain curve of a steel specimen under repeated loading



original elastic portion of the curve  $Oa$ . If loaded again, the stress–strain curve will follow the same path  $O'b$  up to the original curve. The original curve  $bc$  is then closely followed as if unloading had not occurred. Hence, the monotonic stress–strain curve gives a good idealization for the envelope curve of the same specimen being repeatedly loaded.

### 2.1.5.2 Reversed Loading Behavior

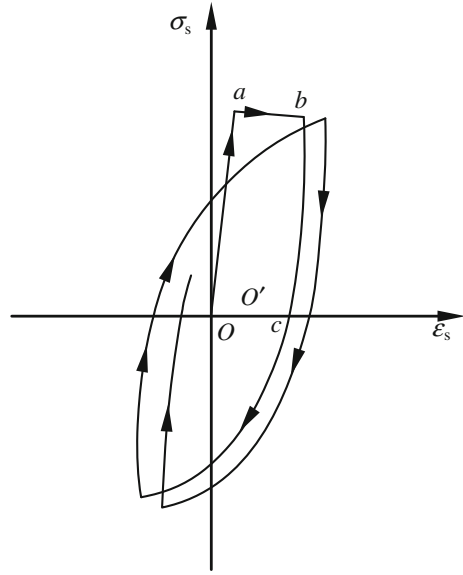
Reversed loading is to load and unload a specimen alternatively in two opposite directions (tension–compression). Figure 2.11 shows a stress–strain curve of a steel specimen under reversed loading. If the stress is released at point  $b$ , which lies in the yield plateau, the specimen will recover along the linear stress–strain path  $bO'$ , that is, parallel to the original elastic portion of the curve  $Oa$ . If loaded again in the opposite direction, the plastic deformation will happen at point  $c$ , which corresponds to a stress much lower than the initial yield strength. This phenomenon is called the Bauschinger effect.

Reversed loading curves are important when considering the effects of high-intensity seismic loading on members.

### 2.1.5.3 Fatigue of Reinforcement

When a steel bar is subjected to periodic loading, even though the maximum stress is lower than the strength value under monotonic loading, the steel bar will fail after a certain number of times of loading and unloading between the minimum stress  $\sigma_{s, \min}^f$  and the maximum stress  $\sigma_{s, \max}^f$ . This is called fatigue failure. In engineering applications, fatigue failure may happen to reinforced concrete members (e.g., crane beams, bridge decks, and sleepers) under repeated loading.

**Fig. 2.11** Stress–strain curve of a steel specimen under reversed loading



The fatigue strength of reinforcement is the maximum stress value of a steel specimen before failing by fatigue after a certain number of cyclic loads with the stress restricted in a stipulated range. The fatigue strength depends on the stress amplitude, i.e., the difference between the maximum stress and the minimum stress. The stress amplitudes for ordinary reinforcement and prestressed reinforcement can be calculated, respectively, as follows:

$$\Delta f_y^f = \sigma_{s, \max}^f - \sigma_{s, \min}^f \quad (2.2a)$$

$$\Delta f_{py}^f = \sigma_{p, \max}^f - \sigma_{p, \min}^f \quad (2.2b)$$

where

$\Delta f_y^f, \Delta f_{py}^f$  the stress amplitudes for ordinary and prestressed reinforcement, respectively;

$\sigma_{s, \max}^f, \sigma_{s, \min}^f$  the maximum stress and the minimum stress of the same layer of ordinary reinforcement at fatigue, respectively; and

$\sigma_{p, \max}^f, \sigma_{p, \min}^f$  the maximum stress and the minimum stress of the same layer of prestressed reinforcement at fatigue, respectively.

In China, the fatigue test is carried out by axially tensioning a steel bar. The times of cyclic loading should be determined in deciding the stress amplitude of reinforced concrete members in service. Two million times of cyclic loading is required in China. In other words, the fatigue strength of reinforcement is usually quantified by the maximum one among all different stress amplitudes that can sustain 2 million times of cyclic loading.

Tables 2.4 and 2.5 in the addendum list the GB 50010 stipulated fatigue stress amplitudes for ordinary reinforcement and prestressed reinforcement, respectively. The fatigue stress ratios in the tables mean the ratios of the minimum stress to the maximum stress of the reinforcement at the same layer.

$$\rho_s^f = \frac{\sigma_{s, \min}^f}{\sigma_{s, \max}^f} \quad (2.3a)$$

$$\rho_p^f = \frac{\sigma_{p, \min}^f}{\sigma_{p, \max}^f} \quad (2.3b)$$

where  $\rho_s^f$  and  $\rho_p^f$  are fatigue stress ratios of ordinary reinforcement and prestressed reinforcement, respectively.

Excluding stress amplitude, other factors such as surface profile, diameter of reinforcement, the processing method, the operating environment, and the loading rate will also influence the fatigue strength of reinforcement. The stress amplitudes listed in Tables 2.4 and 2.5 have already considered the above-mentioned factors.

## 2.2 Strength and Deformation of Concrete

Ordinary concrete is a complex multiphase composite material, i.e., a kind of man-made stone created by mixing cement, aggregates and water. The sands, stones and crystals in cement paste and unhydrated cement grains compose the elastic skeleton of concrete to sustain external loads. Under external loads, the C–S–H gel, micro-voids, and micro-cracks at the interface transition zone between mortar and aggregates make concrete deform plastically. Instinct defects such as voids and initial cracks are generally the origin of concrete failure. The propagation of micro-cracks greatly influences the mechanical properties of concrete.

Because it takes several years for cement paste to harden, the strength and deformation of concrete will vary with time. Furthermore, the deformation will gradually increase with time if the concrete is subjected to sustained loads.

### 2.2.1 Compression of Concrete Cubes

#### 2.2.1.1 Cube Strength of Concrete

The compression test on cubic concrete specimens is very easy and economical. In addition, the measured strength is stable, and cube strength, as an index of evaluation, is deemed as one of the most fundamental indices of concrete strength in China. *National Standard for test method of mechanical properties on ordinary*

concrete (GB/T 50081-2002) stipulates that the cube strength (unit:  $\text{N/mm}^2$ ) of concrete is the compressive strength measured according to the standard test method on standard specimens (150 mm cubes), which have been cured after casting for 28 days in a chamber set at  $20\text{ }^\circ\text{C} \pm 3\text{ }^\circ\text{C}$  with a relative humidity larger than 90 %.

Figure 2.12 shows the setup of the compression experiment on a concrete cube and the failure mode of the specimen. The test method has a great influence on the compressive strength and the failure mode of cubic concrete specimens. After loose concrete is removed, the failed specimen looks like two pyramids connected top against top (Fig. 2.12b). This is because the specimen will shorten vertically and expand laterally when subjected to vertical compression, but the friction between the specimen and the loading plates places both the top and bottom ends of the specimen under a multiaxial loading state, just as if the specimen were restrained by two hoops at the ends. If the top and bottom surfaces of the specimen are greased, the friction between the specimen and the loading plates is significantly reduced. The specimen is almost under a uniaxial compression state. The restriction against lateral expansion of the specimen is approximately constant along the specimen height. Cracks parallel to the loading direction can be observed, and the measured strength is lower than that of the ungreased specimen (Fig. 2.12c). Specimens should not be greased according to the standard test method in China.

The strength of concrete is also related to the strength grade of cement, the water-to-cement ratio, properties of aggregates, methods of forming, age of concrete, environmental conditions during concrete hardening, dimensions and shapes of specimens, and loading rates. Therefore, all nations have their own standard test methods on strength measurement from concrete specimens under uniaxial loading.

For cubic concrete specimens, the faster the loading rate is, the higher the measured strength is. The loading rate is generally specified as  $0.3\text{--}0.5\text{ N/mm}^2$  per minute for concrete specimens with a cube strength lower than  $30\text{ N/mm}^2$  and  $0.5\text{--}0.8\text{ N/mm}^2$  per minute for concrete specimens with cube strength equal to or greater than  $30\text{ N/mm}^2$ .

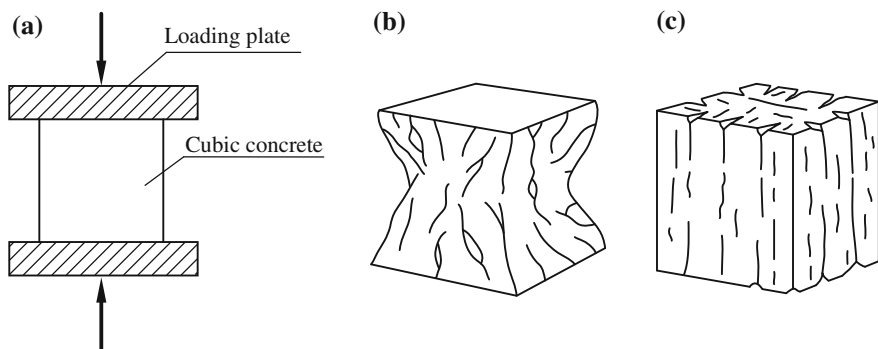


**Fig. 2.12** Test setup and failure mode of cubic concrete. **a** Test setup. **b** Failure mode I. **c** Failure mode II

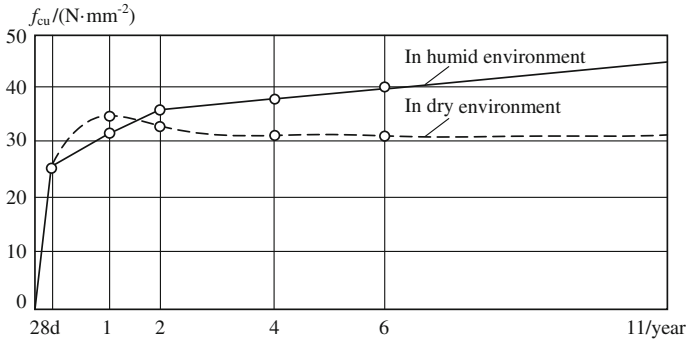


**Fig. 2.13** Changes in cube strength of concrete,  $f_{cu}$  with aging

The cube strength of concrete will increase with its age. The increase in strength is fast at first, but gradually decreases. This process may take several years or even longer for concrete in a humid environment (Fig. 2.13).

The measured strength of concrete should be rectified if nonstandard cubic specimens are adopted. When the cube strength of concrete is less than 60 MPa, conversion factors of 0.95 and 1.05 should be multiplied to 100 mm × 100 mm × 100 mm and 200 mm × 200 mm × 200 mm specimens, respectively. When the cube strength of concrete is larger than 60 MPa, the conversion factor should be determined by experiments.

### 2.2.1.2 Strength Grade of Concrete

The cube strength of concrete is the basis to judge the strength grade of concrete. As stipulated in the *Code for Design of Concrete Structures* (GB 50010), concrete strength grade is determined by the characteristic value of the cube strength (denoted by  $f_{cu,k}$ ). In other words, the strength grade of concrete is the cube strength with a 95 % degree of confidence measured according to the standard test method mentioned above. For example, concrete with the strength grade of C30 means its characteristic strength  $f_{cu,k} = 30 \text{ N/mm}^2$ . The strength grade of concrete in GB 50010 is within the range of C20–C80. And the concrete with the strength grade equal to or larger than C50 is generally called high-strength concrete.

Statistical analysis shows that the measured cube strength of concrete also obeys Gaussian distribution (Fig. 2.7). If the degree of confidence is taken as 95 %, then the standard strength of concrete is equal to the average strength minus 1.645 times the standard deviation.

GB 50010 specifies that the strength grade of concrete in reinforced concrete structures should not be lower than C20 and should not be lower than C25 for structural members using reinforcement of 400 MPa and structural members under repeated loading. For prestressed concrete structures, C30 is the minimum and C40 or larger is preferred.

## 2.2.2 Concrete Under Uniaxial Compression

### 2.2.2.1 Experimental Stress–Strain Curve of Concrete Under Axial Compression

The axial compressive strengths measured on prismatic specimens can obviously reflect the real compression capacity of concrete better than the cube strength. *Standard for test method of mechanical properties on ordinary concrete* (GB/T 50081-2002) stipulates that the standard specimen for axial compressive strength should be a prism in the dimension of 150 mm × 150 mm × 300 mm. The fabrication condition for prismatic specimens is the same as that of cubic specimens, and neither prismatic nor cubic specimens are greased. Figure 2.14 shows the setup of the axial compression test on a prismatic specimen and the corresponding failure mode.

Figure 2.15 illustrates a typical measured full stress–strain curve of concrete under axial compression, which can be divided into two parts, i.e., the ascending branch ( $Oc$ ) and the descending branch ( $cf$ ). In the ascending branch, segment  $Oa$  ( $\sigma_c \leq 0.3f_c$ ) is approximately a straight line. The deformation of concrete is mainly due to the elastic deformation of aggregates and cement crystals, while the influence of the viscous flow of hydrated cement paste and the evolution of initial micro-cracks are small. With the increase of stress ( $0.3f_c < \sigma_c \leq 0.8f_c$ ), the ascending slope of the curve gradually decreases due to the viscous flow of unhardened gel in concrete and the propagation and growth of micro-cracks. When the stress is increased nearly to the axial compressive strength ( $0.8f_c < \sigma_c \leq f_c$ ), large strain energy is stored in the specimen, internal cracks speed their propagation, and the cracks parallel to the axial load link together, which means the specimen is about to fail. Generally, the maximum stress  $\sigma_0$  corresponding to the peak point  $c$  in the stress–strain curve is regarded as the axial compressive strength  $f_c$  of concrete, and the strain at point  $c$  is called the peak strain  $\varepsilon_0$ , whose value approximates 0.002.

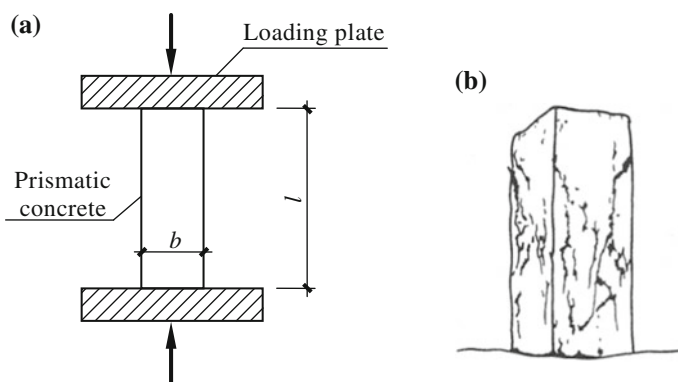


**Fig. 2.14** Test setup and failure mode of prismatic concrete. **a** Test setup. **b** Failure mode

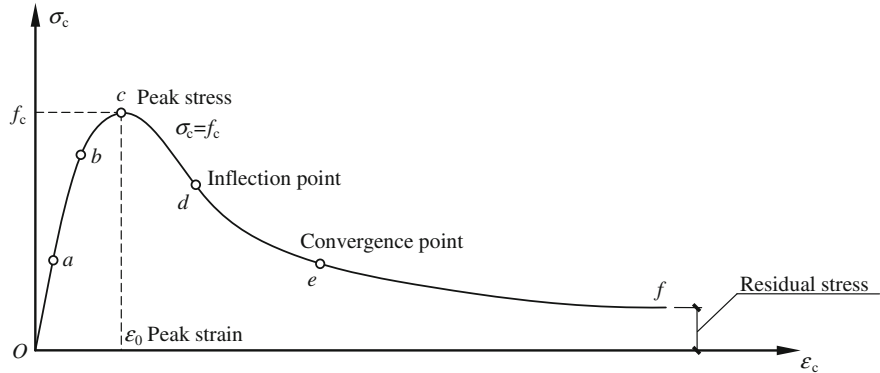


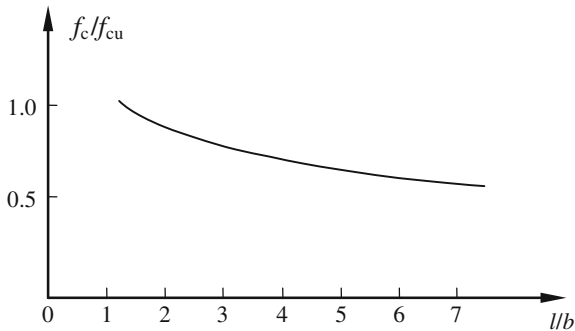
**Fig. 2.15** Stress–strain curve of a prismatic concrete under axial compression

After point  $c$ , further development and connection of continuous cracks damages the prismatic specimen more and more severely, causing the specimen to lose its capacity gradually as shown by the descending branch  $cf$  in Fig. 2.15. The descending branch is hard to record by ordinary test machines because with the decrease of stress, the strain energy stored in the machine is released and the sudden recovery deformation of the machine will surely crush the already severely damaged specimen. Therefore, machines of large stiffness or with certain auxiliary devices must be adopted, and the strain rate should be strictly controlled so as to obtain the descending branch of the stress–strain curve.

### 2.2.2.2 Axial Compressive Strength

The higher a prismatic specimen is, the lesser restriction is placed on the transverse deformation at mid-height of the specimen by the friction between loading plates and the specimen. So as the specimen height/width ratio increases, the axial compressive strength decreases (Fig. 2.16). When determining the dimension of a specimen, the specimen height/width ratio should be large enough so as to

**Fig. 2.16** Effect of aspect ratio on compressive strength



minimize the influence of friction between loading plates and the specimen and to obtain a uniaxial compressive state in the specimen at mid-height. Meanwhile, the height/width ratio cannot be so large as to prevent a large increase in eccentricity, thereby preventing a reduced axial compressive strength from happening before failure. Based on previous research, a height/width ratio of 2–3 is accepted.

GB 50010 stipulates that the measured concrete strength, which is obtained according to the above-mentioned standard test method and has a 95 % degree of confidence, is the characteristic value of the axial compressive strength of concrete, denoted by  $f_{ck}$ . The design value of the axial compressive strength is obtained through dividing the characteristic value by a partial safety factor for material  $\gamma_c = 1.4$ . Table 2.6 in the addendum lists the characteristic values and design values of the axial compressive strength of concrete stipulated in GB 50010.

Similarities in the procedure for measuring the cube strength with nonstandard concrete specimens are evident when measuring the axial compressive strength with nonstandard prismatic specimens, where the measurements must be rectified according to strength grade and dimensions of the specimens. When the strength grade of concrete is less than C60, conversion factors of 0.95 and 1.05 should be multiplied to 100 mm × 100 mm × 300 mm and 200 mm × 200 mm × 400 mm specimens, respectively. When the strength grade of concrete is equal to or larger than C60, the conversion factor should be determined by experiments.

Figure 2.17 shows the relationship between part of the experimental data on axial compressive strength and cube strength obtained in Chinese research institutes. It can be assumed that within a certain range, the axial compressive strength  $f_c$  is approximately proportional to the cube strength  $f_{cu}$ . Based on experimental

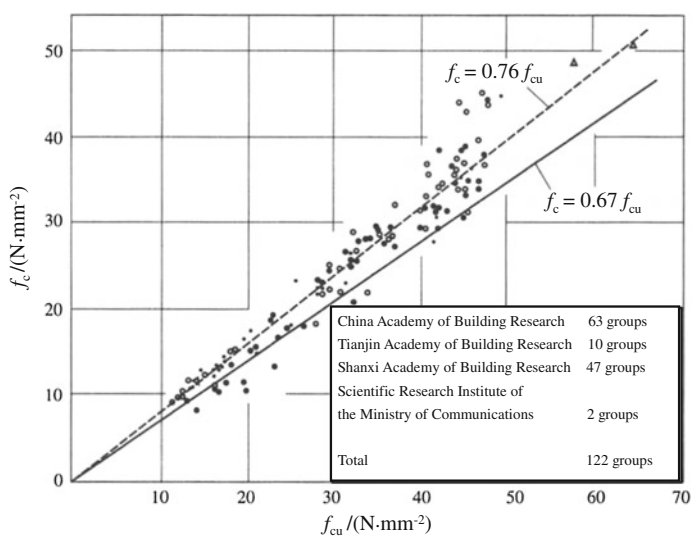


Fig. 2.17 Relationship between axial compressive strength and cubic compressive strength

research, GB 50010 conservatively expresses the relationship between the two strengths as follows:

$$f_{ck} = 0.88\alpha_1\alpha_2f_{cu,k} \quad (2.4)$$

where

- $\alpha_1$  the ratio of the prism strength to the cube strength.  $\alpha_1 = 0.76$  for concrete with the strength grade  $\leq C50$  and  $\alpha_1 = 0.82$  for C80 concrete. The value of  $\alpha_1$  is linearly interpolated between 0.76 and 0.82 for C55–C75 concrete;
- $\alpha_2$  the reduction coefficient considering the brittleness of high-strength concrete.  $\alpha_2 = 1.0$  for C40 concrete and  $\alpha_2 = 0.87$  for C80 concrete. The value of  $\alpha_2$  is linearly interpolated between 1.0 and 0.87 for intermediate grades; and
- 0.88 a parameter to consider the strength differences between laboratory specimens and real structural members because they have different fabrication methods, curing conditions, and loading states.

In some countries or regions, cylinder specimens are chosen to determine the axial compressive strength of concrete. For example, concrete cylinders with the diameter of 6 in. (152 mm) and the height of 12 in. are adopted as the standard specimens for the axial compressive strength in America. The cylinder strength is denoted by  $f'_c$ . Because of the differences in shape and dimension, cylinder strength is different from prism strength.

Based on foreign research results, the relationship between the cylinder strength  $f'_c$  and the cube strength  $f_{cu,k}$  is shown in Table 2.1, in which the ratio of  $f'_c/f_{cu,k}$  increases with the strength grade of concrete if the strength grade is equal to or larger than C60.

The stress–strain curves for concrete of different strength grades are similar in shape but have substantial distinction. From the experimental curves shown in Fig. 2.18, it can be seen that the peak strains for concrete of different strength grades are nearly the same, but the shape of the descending branches varies greatly. The higher the strength grade is, the steeper the descending slope is. So it is generally accepted that concrete of higher strength has a poorer ductility.

Moreover, axial compressive strength is also influenced by the loading rate. As shown in Fig. 2.19, with the decrease of the loading rate, the peak stress (i.e., the axial compressive strength) also decreases a little bit. However, the strain corresponding to the peak stress increases, and the slope of the descending branch becomes gentle.

**Table 2.1** Ratio of  $f'_c$  to  $f_{cu,k}$

| Concrete grade  | Under C60 | C60   | C70   | C80   |
|-----------------|-----------|-------|-------|-------|
| $f'_c/f_{cu,k}$ | 0.79      | 0.833 | 0.857 | 0.875 |

**Fig. 2.18** Stress–strain curves for concrete of different strength grades

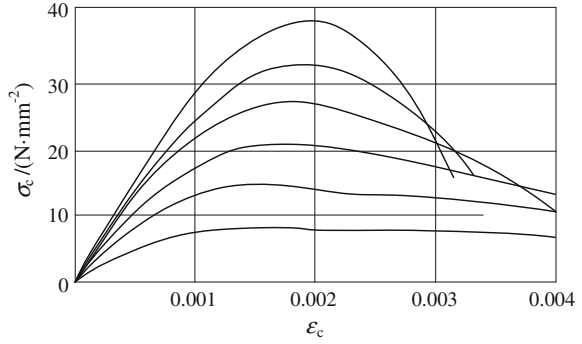
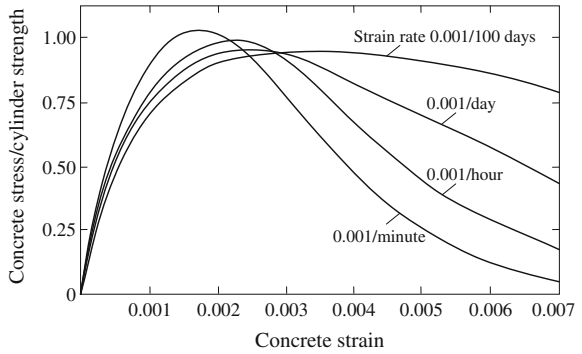


**Fig. 2.19** Stress–strain curves for concrete under different strain rates



### 2.2.2.3 Mathematical Models of the Stress–Strain Relationship of Concrete Under Axial Loading

Researchers at home and abroad have established many mathematical models to describe the stress–strain relationship of concrete under axial loading. Several examples are as follows:

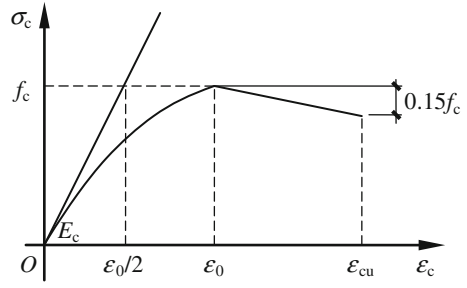
#### 1. Model suggested by Hognestad

The model suggested by Hognestad assumes the ascending branch and the descending branch as a second-order parabola and an oblique straight line (Fig. 2.20), respectively, expressed by Eq. (2.5a and 2.5b).

$$\sigma_c = f_c \left[ 2 \frac{\varepsilon_c}{\varepsilon_0} - \left( \frac{\varepsilon_c}{\varepsilon_0} \right)^2 \right] \quad (\varepsilon_c \leq \varepsilon_0) \quad (2.5a)$$

$$\sigma_c = f_c \left[ 1 - 0.15 \frac{\varepsilon_c - \varepsilon_0}{\varepsilon_{cu} - \varepsilon_0} \right] \quad (\varepsilon_0 < \varepsilon_c \leq \varepsilon_{cu}) \quad (2.5b)$$

**Fig. 2.20** Stress–strain curve suggested by Hognestad



where

$f_c$  the peak stress (the axial compressive strength of concrete);

$\varepsilon_0$  the strain corresponding to the peak stress, taking the value as

$$\varepsilon_0 = 1.8 \frac{f_c}{E_c} \quad (2.5c)$$

$\varepsilon_{cu}$  the ultimate compressive strain, taking the value as 0.0038; and

$E_c$  the modulus of elasticity of concrete, given by empirical equations (omitted herein).

## 2. Model suggested by Rüsçh

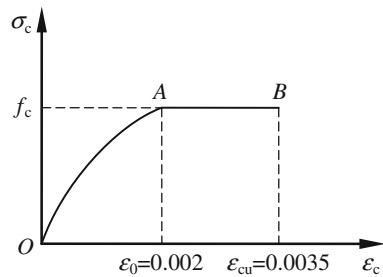
The model suggested by Rüsçh also adopts a parabolic ascending branch, but the descending branch is a horizontal straight line (Fig. 2.21), expressed as

$$\sigma_c = f_c \left[ 2 \frac{\varepsilon_c}{\varepsilon_0} - \left( \frac{\varepsilon_c}{\varepsilon_0} \right)^2 \right] \quad (\varepsilon_c \leq \varepsilon_0) \quad (2.6a)$$

$$\sigma_c = f_c \quad (\varepsilon_0 < \varepsilon_c \leq \varepsilon_{cu}) \quad (2.6b)$$

where the strain corresponding to the peak stress is 0.002 and the ultimate strain is 0.0035.

**Fig. 2.21** Stress–strain curve suggested by Rüsçh



### 3. Model adopted in GB 50010

Referring to Rüsç's model and research results on high-strength concrete in recent years, GB 50010 proposes the following mathematical expression for the stress–strain relationship of concrete under axial loading:

$$\left. \begin{aligned} \sigma_c &= f_c \left[ 1 - \left( 1 - \frac{\varepsilon_c}{\varepsilon_0} \right)^n \right] & \varepsilon_c \leq \varepsilon_0 \\ \sigma_c &= f_c & \varepsilon_0 < \varepsilon_c \leq \varepsilon_{cu} \end{aligned} \right\} \quad (2.7)$$

$$n = 2 - \frac{1}{60}(f_{cu} - 50) \quad (2.8)$$

$$\varepsilon_0 = 0.002 + 0.5(f_{cu} - 50) \times 10^{-5} \quad (2.9)$$

$$\varepsilon_{cu} = 0.0033 - (f_{cu} - 50) \times 10^{-5} \quad (2.10)$$

where

- $\sigma_c$  the stress in concrete corresponding to the compressive strain  $\varepsilon_c$ ;
- $f_c$  the axial compressive strength of concrete;
- $\varepsilon_0$  the compressive strain corresponding to  $f_c$ . If the calculated  $\varepsilon_0 < 0.002$ , take  $\varepsilon_0 = 0.002$ ;
- $\varepsilon_{cu}$  the ultimate compressive strain. If the calculated  $\varepsilon_{cu} > 0.0033$ , take  $\varepsilon_{cu} = 0.0033$ ;
- $f_{cu}$  the cube strength of concrete; and
- $n$  a parameter. If the calculated  $n > 2.0$ , take  $n = 2.0$ ;

When the compressive stress is small (say  $\sigma_c \leq 0.3f_c$ ), approximately take

$$\sigma_c = E_c \varepsilon_c \quad (2.11)$$

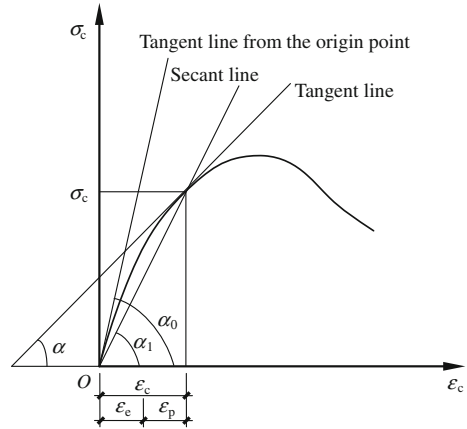
where  $E_c$  is the modulus of elasticity of concrete.

Obviously, when  $f_{cu} \leq 50$  MPa, Eq. (2.7) becomes Eq. (2.6a, 2.6b).

#### 2.2.2.4 Modulus of Elasticity of Concrete

Modulus of elasticity is the ratio of stress to strain. Because the stress–strain relationship of concrete under axial compression is a curve, the modulus of elasticity is a variable. It can be depicted in three ways, i.e., initial modulus of elasticity, secant modulus, and tangent modulus, whose values are  $\tan\alpha_0$ ,  $\tan\alpha_1$ , and  $\tan\alpha$  as shown in Fig. 2.22, respectively.

**Fig. 2.22** Depiction of concrete elastic modulus



The modulus of elasticity of concrete generally means the initial modulus of elasticity, denoted by  $E_c$ . Based on the regression analysis of experimental data, the modulus of elasticity has a relation with cube strength as follows:

$$E_c = \frac{10^5}{2.2 + \frac{34.7}{f_{cu}}} \text{ (N/mm}^2\text{)} \quad (2.12)$$

The moduli of elasticity given by GB 50010 are listed in Table 2.6 in the addendum. It is of practical meaning to use secant modulus or tangent modulus in nonlinear analysis of concrete structures, because the two moduli can better reflect the characteristics of stress–strain relationship curves. And the relation between the secant modulus  $E'_c$  and the initial modulus of elasticity  $E_c$  can be expressed as

$$E'_c = \nu E_c \quad (2.13)$$

where  $\nu$  is a proportional constant, taking the value as 0.4–1.0 for compression and 1.0 for tension failure.

### 2.2.2.5 Transverse Deformation of Concrete

Concrete specimens under uniaxial compression will deform not only longitudinally with the compressive strain of  $\varepsilon_v$ , but also transversely with the transverse strain of  $\varepsilon_h$ . The transverse deformation coefficient (i.e., the Poisson's ratio) is defined as  $\nu_c = \varepsilon_h/\varepsilon_v$ . From the experimental results shown in Fig. 2.23, when the compressive stress is small ( $\sigma_c \leq 0.5f_c$ ),  $\nu_c$  is approximately a constant of the value 1/6, which is the Poisson's ratio corresponding to concrete in the elastic stage. When the compressive stress is large ( $\sigma_c > 0.5f_c$ ),  $\nu_c$  increases apparently due to

**Fig. 2.23** Relationship between compressive stress and Poisson's ratio

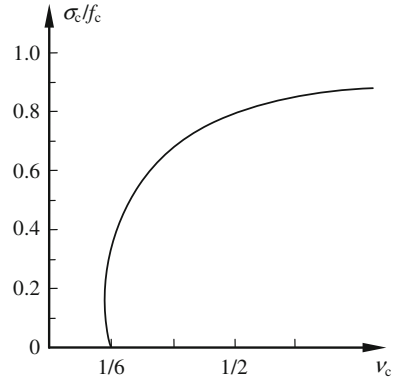
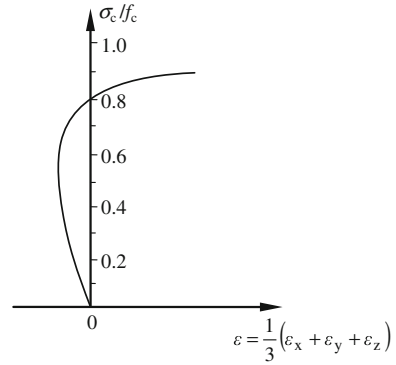


**Fig. 2.24** Relationship between stress and averaged strain



internal crack development, and when concrete is nearly failed,  $\nu_c$  can reach the value as 0.5 or higher.

Figure 2.24 shows the relationship between stress and average strain, which is the average of measured strains in three mutually perpendicular directions. It can be seen that when the stress is small ( $\sigma_c \leq 0.5f_c$ ), the specimen volume decreases with the increase of compressive stress. When the stress is large ( $\sigma_c > 0.5f_c$ ), the compressed volume gradually recovers. When the specimen is nearly failed, the volume may even be larger than the original one.

## 2.2.3 Concrete Under Uniaxial Tension

### 2.2.3.1 Tensile Tests on Concrete

The standard specimens for axial tension test on concrete are prisms with embedded steel bars at both ends (Fig. 2.25).

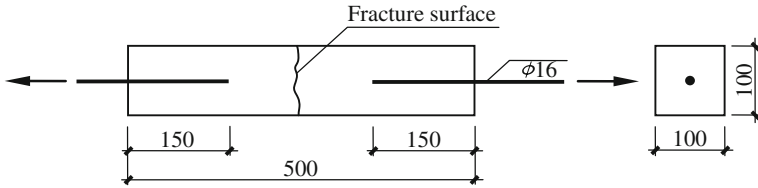


**Fig. 2.25** Schematic diagram of the axial tension test

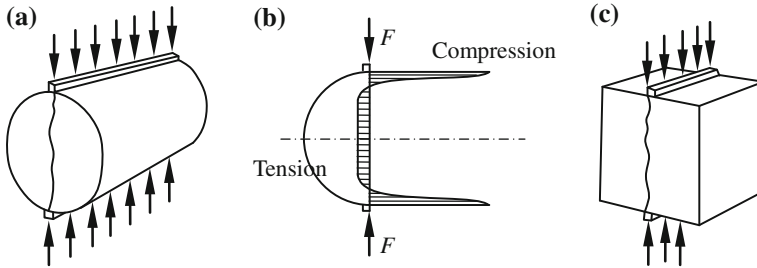


**Fig. 2.26** Schematic diagram of the splitting test

However, using specimens shown in Fig. 2.25 cannot ensure the specimens under perfect axial tension, and an eccentric tension will surely influence the accuracy of the tensile strength measurement. So the much simpler splitting test on cylinders or cubes shown in Fig. 2.26 is widely employed at home and abroad to indirectly measure the tensile strength of concrete. From the theory of elasticity, the splitting tensile strength  $f_{ts}$  can be calculated according to the following equation

$$f_{ts} = \frac{2F}{\pi d_c l} \quad (2.14)$$

where

$F$  the failure load;

$d_c$  the diameter of cylinders or the dimension of cubes; and

$l$  the length of cylinders or the dimension of cubes.

Experimental results show that splitting tensile strength is only slightly higher than the tensile strength obtained from the direct pulling test.

### 2.2.3.2 Axial Tensile Strength

Experimental results of ordinary concrete and high-strength concrete under uniaxial tension indicate the following relation between the axial tensile strength and the cube strength

$$f_t = 0.395f_{cu}^{0.55} \quad (2.15)$$

GB 50010 gives the conversion relation between the characteristic value of axial tensile strength and that of cube strength as follows:

$$f_{tk} = 0.88 \times 0.395f_{cu,k}^{0.55} (1 - 1.645\delta)^{0.45} \times \alpha_2 \quad (2.16)$$

The meanings of 0.88 and the value of  $\alpha_2$  are the same as those in Eq. (2.4). The term of  $(1 - 0.645\delta)^{0.45}$  reflects the influence of dispersion degree of experimental data on confidence degree of characteristic strength.  $\delta$  is the coefficient of variation. Table 2.6 lists the characteristic values and design values of the axial tensile strength of concrete.

The ratio of the axial tensile strength to the cube strength is within 1/17–1/8. The higher the strength grade of concrete is, the smaller the ratio is.

### 2.2.3.3 Stress–Strain Relationship of Concrete Under Axial Tension

The experimental stress–strain curves of concrete under axial tension are much fewer than those under axial compression. The available experimental results show that the stress–strain curves of concrete under axial tension are similar to those under axial compression in shape and also include the ascending and descending branch. But the slope of the descending branch is steep and may become steeper with the increase of the strength grade of concrete.

Because the axial tensile strength is much lower than the axial compressive strength, the stress–strain relationship of concrete under axial tension can be simulated by a bilinear model and the moduli of elasticity of concrete under tension and compression are assumed to be the same.

## 2.2.4 Concrete Under Multiaxial Stresses

In real structural members, concrete is usually subjected to multiaxial stresses rather than the ideal uniaxial loading, so investigating the multiaxial behavior of concrete is of great importance for better understanding the properties of concrete members and improving the design and research of concrete structures.

### 2.2.4.1 Biaxial Behavior of Concrete

The failure curve of concrete under a biaxial stress state (Fig. 2.27) can be obtained by applying normal stresses  $\sigma_1$  and  $\sigma_2$  in two mutually perpendicular directions

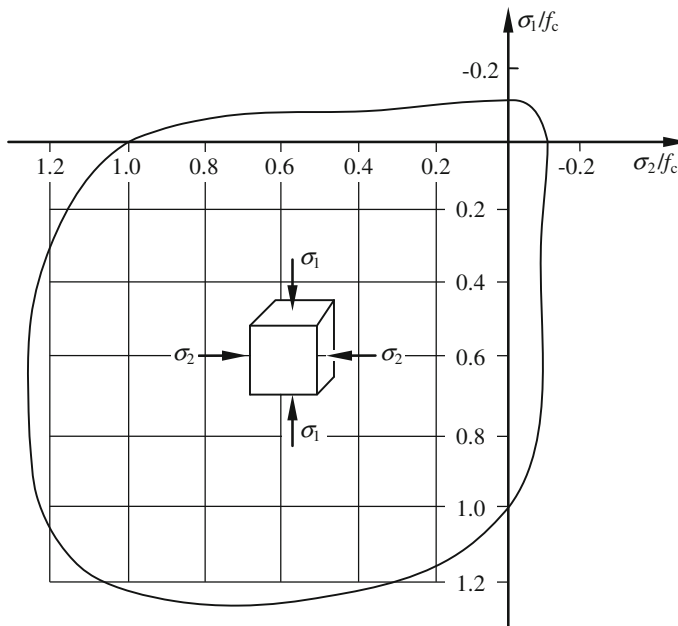


**Fig. 2.27** Concrete strength under biaxial stress

while keeping the normal stress of zero in the third direction perpendicular to the aforementioned two directions and recording the strengths of concrete under different stress ratios ( $\sigma_1/\sigma_2$ ).

It can be seen that under biaxial tension (1st quadrant in Fig. 2.27),  $\sigma_1$  and  $\sigma_2$  do not greatly influence each other, and the biaxial tension strength of concrete is approximately equal to the uniaxial tensile strength. However, for concrete under biaxial compression, the strength in one direction increases with the buildup of compressive stress in another direction (3rd quadrant in Fig. 2.27). The biaxial compressive strength can be as much as 27 % higher than the uniaxial strength. Combined tension and compression loadings reduce both the tensile and compressive stresses at failure (2nd and 4th quadrant in Fig. 2.27).

#### 2.2.4.2 Strength Under Combined Normal and Shear Stresses

Figure 2.28 shows the failure curve of concrete under combined normal and shear stresses. It is found that the shear strength of concrete will increase with the buildup of compressive stress when the latter is small. But after the compressive stress exceeds  $(0.5\text{--}0.7) f_c$ , the shear strength will decrease with the increase of compressive stress. On the other hand, the existence of shear stress reduces the compressive strength of concrete. Similarly, the shear strength decreases with the

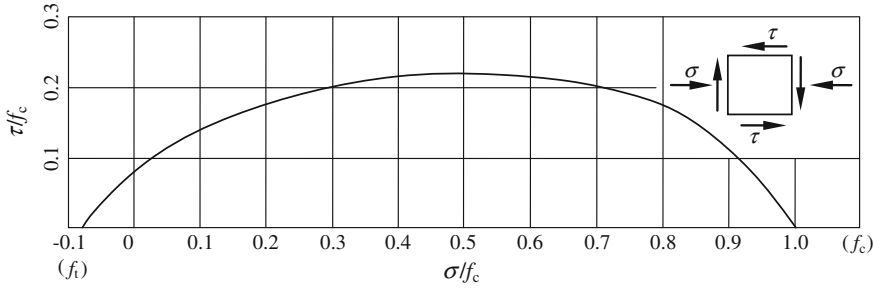


**Fig. 2.28** Concrete strength under combined normal and shear stresses

increase of tensile stress. And the tensile strength of concrete reduces in the presence of shear stress.

### 2.2.4.3 Compressive Strength of Confined Concrete

Overseas experimental research indicates that for axially loaded concrete cylinders, the axial strength will greatly improve if the cylinders are subjected to uniform confining fluid pressure. The increased amplitude is approximately proportional to the confining pressure (Fig. 2.29). When  $\sigma_2$  is not very large, the ultimate compressive strength  $f'_{cc}$  in the direction of  $\sigma_1$  can be expressed as

$$f'_{cc} = f'_c + 4.1\sigma_2 \quad (2.17)$$

where

$f'_c$  the compressive strength of the unconfined concrete cylinder; and  
 $\sigma_2$  the confining pressure.

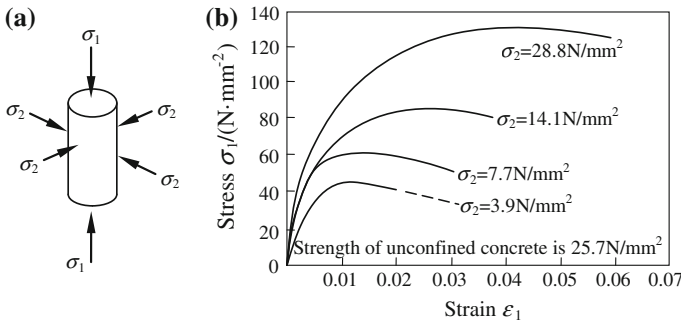


**Fig. 2.29** Stress–strain curves of concrete cylinders under three-dimensional compression

The presence of confining pressure restrains the lateral deformation of concrete cylinders and suppresses the initiation and evolution of internal cracks parallel to the direction of  $\sigma_1$ . Therefore, the ductility of concrete is enhanced simultaneously with the increase of axial compressive strength.

Based on the aforementioned mechanism, the load-carrying capacity and deformation property of concrete columns subjected to compression can be improved by the dense arrangement of circular hoops or spirals at the column periphery to restrain the lateral deformation of internal concrete. The calculation based on the arrangement of circular hoops or spirals will be detailed in Chap. 4.

### 2.2.5 Strength and Deformation of Concrete Under Repeated Loading

The strength and deformation of concrete under repeated loading (several loading and unloading cycles) are greatly different from those under monotonic loading. Fatigue failure can happen to concrete under repeated loading.

Specimens sized  $100 \text{ mm} \times 100 \text{ mm} \times 300 \text{ mm}$  or  $150 \text{ mm} \times 150 \text{ mm} \times 450 \text{ mm}$  are usually used in fatigue tests of concrete. And the compressive stress at which the concrete specimen finally fails after 2 million (or even more) times of repeated loading is called the fatigue strength of concrete.

Figure 2.30a shows the stress–strain curves of a concrete prism subjected to one cycle of loading and unloading, in which  $Oa$  and  $ab$  are loading and unloading curves, respectively. When the stress is decreased to zero after having reached point  $a$ , most of the overall strain  $\varepsilon_c$  corresponding to point  $a$ , i.e.,  $\varepsilon'_e$ , can be recovered instantaneously during unloading, and a small portion of strain  $\varepsilon''_e$  can also be recovered after some time, which is referred to as elastic hysteresis. The unrecovered strain  $\varepsilon'_{cr}$  is called residual strain.

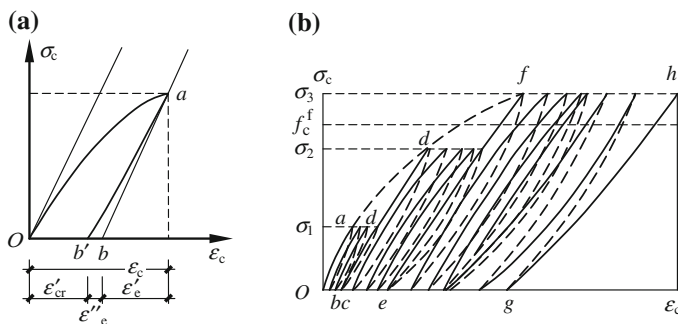


**Fig. 2.30** Stress–strain curves of concrete under repeated loading. **a** Stress–strain curves of a concrete prism subjected to one cycle of loading and unloading. **b** Stress–strain curves of a concrete prism subjected to many cycles of loading and unloading

Figure 2.30b shows the stress–strain curves of a concrete prism subjected to many cycles of loading and unloading. When the loading stress is lower than the fatigue strength of concrete  $f_c^f$ , e.g.,  $\sigma_1$  or  $\sigma_2$  in Fig. 2.30b, the stress–strain curve is similar to that in Fig. 2.30a. But the loop formed by loading and unloading curves in one cycle tends to be closed after several repeated times. However, even if the repeated times were as high as several million, the concrete prism would not be failed by fatigue. If the loading stress exceeds the fatigue strength  $f_c^f$ , say  $\sigma_3$  in Fig. 2.30b, the loading curves are initially convex to the stress axis and gradually become convex to the strain axis after many times of repeated loadings. In the final stage, the loading and unloading curves in one cycle cannot form a closed loop and the slope of the stress–strain curve continuously decreases, indicating the imminent fatigue failure of concrete.

Concrete fatigue originates from internal defects such as micro-cracks and micro-voids. The stress concentration in concrete under repeated loading causes defects to develop and form macro-cracks, which finally lead to concrete failure. Fatigue failure is brittle, i.e., giving no apparent warning before failure. Cracks are not wide, but deformation is large.

GB 50010 stipulates that the design value of concrete fatigue strength is determined by multiplying the design strength  $f_c$  or  $f_t$  by corresponding corrector factors  $\gamma_p$ , which is chosen from Table 2.7 based on the fatigue stress ratio  $\rho_c^f$ .

$$\rho_c^f = \frac{\sigma_{c,\min}^f}{\sigma_{c,\max}^f} \quad (2.18)$$

where  $\sigma_{c,\min}^f$  and  $\sigma_{c,\max}^f$  = the minimum and maximum stresses of concrete in the same fiber, respectively.

## 2.2.6 Deformation of Concrete Under Long-Term Loading

### 2.2.6.1 Creep

Creep is the increase in strain with time due to a sustained load.

Figure 2.31 illustrates the creep curve of a prismatic concrete specimen. An instantaneous strain  $\varepsilon_c$  will be recorded when the specimen is loaded to a certain value of stress, (say  $0.5f_c$  in Fig. 2.31). If the stress is kept constant, the specimen deformation will continuously increase with time, expressed as the creep strain  $\varepsilon_{cr}$ . In the first several months of loading, the creep strain increases rapidly and 70–80 % of the total creep strain can be finished in half a year. Then, the increased rate of the creep strain gradually decreases and will stabilize after a long time. The creep strain measured two years later is 1–4 times the instantaneous strain. If unloaded at this time, the specimen will recover part of the deformation (elastic recovery  $\varepsilon'_c$ ), which is smaller than the instantaneous strain at loading. Another part

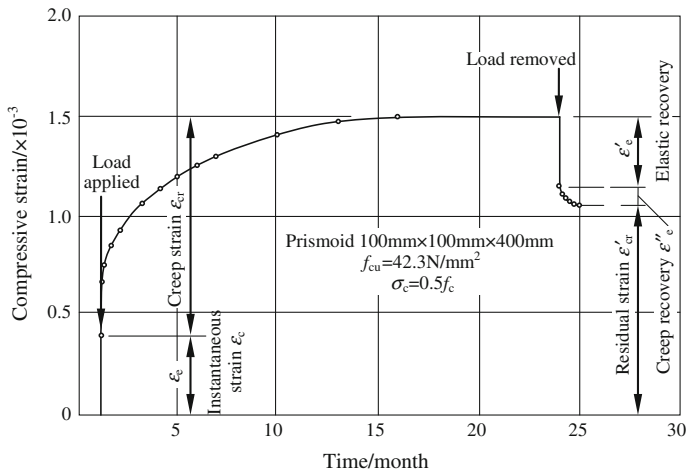


**Fig. 2.31** Concrete creep with time

of strain (creep recovery  $\varepsilon''_e$ ) can be recovered in about 15 days after unloading. However, most of the strain is unrecoverable, which is called the residual strain  $\varepsilon'_e$ .

Creep will greatly influence the properties of concrete structural members. For example, it can enlarge the deformation of structural members, induce stress redistribution in cross sections, and cause prestress loss in prestressed concrete structures.

### 2.2.6.2 Factors Influencing Creep

Creep of concrete will be influenced by many factors, such as stress magnitude, inherent material characteristics, and environmental conditions.

Experiments show that stress magnitude is one of the most important factors. As shown in Fig. 2.32a, when  $\sigma_c \leq 0.5f_c$ , the spacings between any two adjacent creep curves are nearly equal, indicating that the creep of concrete is approximately proportional to the stress. This is called linear creep. Linear creep accelerates quickly at the beginning of loading and then gradually slows down and can be assumed to be fully stopped after about three years.

Figure 2.32b illustrates the influence of large stresses on the creep of concrete. When the stress is large, say  $\sigma_c = (0.5-0.8)f_c$ , the creep is not proportional to the stress any longer, and the increase rate of creep is greater than that of stress. This is called nonlinear creep. When the stress becomes even larger, say  $\sigma_c > 0.8f_c$ , the internal cracks in specimens develop in an unstable way, which causes an intensive increase of nonlinear creep and thus leads to the final failure of concrete. Therefore, it is generally agreed that the compressive strength of concrete under sustained loads is just 75–80 % of its short-term counterpart.

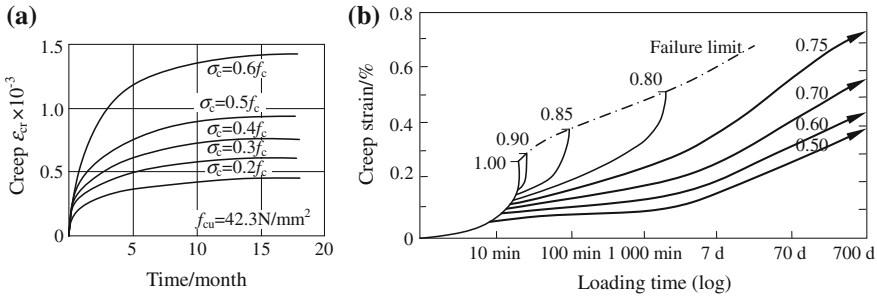


**Fig. 2.32** Relationship between creep and compressive stress

The composition of concrete also affects the creep greatly. The larger the water/cement ratio is, the bigger the creep strain is. The increase of cement usage will enlarge the creep strain too. Moreover, the mechanical properties of aggregates have an apparent influence on the creep of concrete. For example, using maximum size solid aggregates allows an increased modulus of elasticity, and an increased volume ratio of aggregates to concrete helps reduce the creep strain.

The creep may be influenced by fabrication methods and curing conditions of concrete as well. Curing concrete in the conditions of high temperature and humidity can promote the hydration of cement so as to reduce the creep strain. On the other hand, if the temperature is high, but the humidity is low during curing, the creep strain will increase. Additionally, the earlier the load is applied, the larger the creep strain is.

Besides, the shape and dimension of members will affect the creep of concrete. Because the moisture in members of large dimension is hard to evaporate, the creep strain is small. The arrangement of reinforcement may also change the creep strain values.

### 2.2.7 Shrinkage, Swelling, and Thermal Deformation of Concrete

Shrinkage refers to the decrease in the volume of a concrete member when it loses moisture by evaporation. The opposite phenomenon, swelling, occurs when the volume increases through water absorption. Shrinkage and swelling represent the volume change of concrete specimens during hardening irrespective of the external load.

Figure 2.33 shows the experimental results of shrinkage and swelling of concrete members. It can be seen that the shrinkage strain increases quickly at the beginning and becomes almost asymptotic after about one year. The test data of shrinkage strain is very scattering, say  $(2-5) \times 10^{-4}$ , and the value as  $3 \times 10^{-4}$  is generally

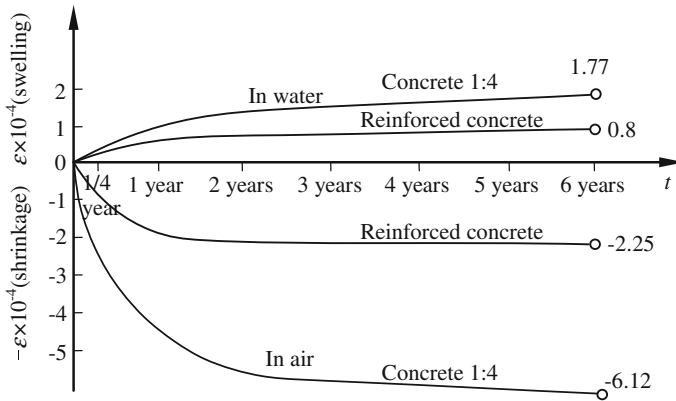


**Fig. 2.33** Shrinkage and swelling of concrete with time

taken. In reinforced concrete members, the shrinkage of concrete is restrained. So its value is half that of plain concrete, i.e.,  $1.5 \times 10^{-4}$ .

There are several factors that affect the magnitude of drying shrinkage.

### 2.2.7.1 Type of Cement

The higher the cement grade is, the larger the shrinkage strain is.

### 2.2.7.2 Amount of Cement

The larger the cement usage or the water/cement ratio is, the bigger the shrinkage strain is.

### 2.2.7.3 Aggregate Property

The larger the modulus of elasticity based on aggregates is, the smaller the shrinkage strain is.

### 2.2.7.4 Ambient Conditions

In the hardening and following service stage of concrete, the larger the ambient moisture is, the smaller the shrinkage strain is. The shrinkage strain will decrease with the increase of curing temperature if the relative humidity is large. And an opposite trend will be observed if the ambient condition is dry.

### 2.2.7.5 Quality of Construction

The denser the concrete is when vibrated, the smaller the shrinkage strain is.

### 2.2.7.6 Ratio of Volume to Surface Area

The larger the ratio of volume to surface area is, the smaller the shrinkage strain is.

If concrete members are not well cured or the contraction is restrained, shrinkage cracks will appear on the surface or in the interior of the members. Cracks will not only affect the appearance, but also adversely influence the serviceability and durability of concrete.

The swelling of concrete is much smaller than the shrinkage and generally beneficial to the structural members, so it is usually not considered.

The linear expansion coefficient of concrete is related to its mix composition and aggregate property. The value  $(1.0-1.5) \times 10^{-5}$  is close to  $1.2 \times 10^{-5}$  of steel. Therefore, the deformation difference between concrete and reinforcement caused by temperature change is small, and no harmful internal stress happens to structural members. However, for mass concrete structures such as pools and chimneys, the influence of thermal stress on structural properties should be taken into account.

#### Questions:

- 2.1 How can reinforcement be classified?
- 2.2 What is the difference between the stress-strain curves of hard steel and mild steel? How is their yield strength determined?
- 2.3 What mathematical models are used to simulate the stress-strain relationship of reinforcement? How can their applicable conditions be described?
- 2.4 What is the influence of cold drawing and cold stretching on mechanical properties of steel?
- 2.5 What are the requirements on properties of steel in reinforced concrete structures?
- 2.6 How are the cube strength, axial compressive strength (i.e., the prism strength), and the axial tensile strength of concrete determined?
- 2.7 How is the strength grade of concrete determined? What is the grade range stipulated in GB 50010?
- 2.8 What is the characteristic of the stress-strain curve of concrete under axial compression? Please provide an example of a frequently used mathematical model used to define the stress-strain relationship.
- 2.9 How is the modulus of elasticity of concrete determined?
- 2.10 What is the fatigue strength of concrete? What is the characteristic of the stress-strain curve of concrete under repeated loading?
- 2.11 What are the shrinkage and creep of concrete? What factors will influence the creep and shrinkage of concrete?
- 2.12 What are the influences of creep and shrinkage on the mechanical properties of reinforced concrete structural members?

Appendix

See Tables 2.2, 2.3, 2.4, 2.5, 2.6, and 2.7.

Table 2.2 Standard strength, design strength, and elastic modulus of normal steel bars

| Type     | Symbol         | Nominal diameter<br><i>d</i> /mm | Standard yield strength<br><i>f<sub>yk</sub></i> /(N·mm <sup>-2</sup> ) | Standard ultimate strength<br><i>f<sub>suk</sub></i> /(N·mm <sup>-2</sup> ) | Design strength/(N·mm <sup>-2</sup> ) |                       | Elastic modulus<br><i>E<sub>s</sub></i> /(N·mm <sup>-2</sup> ) | Ultimate strain<br><i>ε<sub>cu</sub></i> /% |
|----------|----------------|----------------------------------|-------------------------------------------------------------------------|-----------------------------------------------------------------------------|---------------------------------------|-----------------------|----------------------------------------------------------------|---------------------------------------------|
|          |                |                                  |                                                                         |                                                                             | <i>f<sub>y</sub></i>                  | <i>f'<sub>y</sub></i> |                                                                |                                             |
| HPB 300  | ϕ              | 6–22                             | 300                                                                     | 420                                                                         | 270                                   | 270                   | 2.1 × 10 <sup>5</sup>                                          | Not less than 10.0                          |
| HRB 335  | ϕ              | 6–50                             | 335                                                                     | 455                                                                         | 300                                   | 300                   | 2.0 × 10 <sup>5</sup>                                          | Not less than 7.5                           |
| HRBF 335 | ϕ <sup>F</sup> |                                  |                                                                         |                                                                             |                                       |                       |                                                                |                                             |
| HRB 400  | ϕ              | 6–50                             | 400                                                                     | 540                                                                         | 360                                   | 360                   |                                                                |                                             |
| HRBF 400 | ϕ <sup>F</sup> |                                  |                                                                         |                                                                             |                                       |                       |                                                                |                                             |
| RRB 400  | ϕ <sup>R</sup> |                                  |                                                                         |                                                                             |                                       |                       |                                                                |                                             |
| HRB 500  | ϕ              | 6–50                             | 500                                                                     | 630                                                                         | 435                                   | 435                   |                                                                |                                             |
| HRBF 500 | ϕ <sup>F</sup> |                                  |                                                                         |                                                                             |                                       |                       |                                                                |                                             |

Notes When steel bars with the diameter larger than 40 mm are adopted, experimental studies and reliable engineering experience are essential

Table 2.3 Standard strength, design strength, and elastic modulus of prestressed tendons

| Type                                              | Symbol                    | Diameter<br><i>d</i> /mm   | Standard yield<br>strength<br><i>f<sub>pyk</sub></i> /(N·mm <sup>-2</sup> ) | Standard ultimate<br>strength <i>f<sub>puk</sub></i> /<br>(N·mm <sup>-2</sup> ) | Design<br>strength/<br>(N·mm <sup>-2</sup> ) |                        | Elastic<br>modulus<br><i>E<sub>s</sub></i> /(N·mm <sup>-2</sup> ) | Ultimate<br>strain<br><i>ε<sub>pu</sub></i> /‰ |
|---------------------------------------------------|---------------------------|----------------------------|-----------------------------------------------------------------------------|---------------------------------------------------------------------------------|----------------------------------------------|------------------------|-------------------------------------------------------------------|------------------------------------------------|
|                                                   |                           |                            |                                                                             |                                                                                 | <i>f<sub>py</sub></i>                        | <i>f'<sub>py</sub></i> |                                                                   |                                                |
| Prestressed steel<br>wire with medium<br>strength | Smooth<br>spiral rib      | 5, 7, 9                    | 680                                                                         | 800                                                                             | 560                                          | 410                    | 2.05 × 10 <sup>5</sup>                                            | Not less<br>than 3.5                           |
|                                                   |                           |                            | 820                                                                         | 970                                                                             | 680                                          | 410                    |                                                                   |                                                |
|                                                   |                           |                            | 1080                                                                        | 1270                                                                            | 900                                          | 410                    |                                                                   |                                                |
| stress-relieved<br>steel wire                     | Smooth<br>spiral rib      | 5                          | 1330                                                                        | 1570                                                                            | 1110                                         | 410                    | 1.95 × 10 <sup>5</sup>                                            |                                                |
|                                                   |                           |                            | 1580                                                                        | 1860                                                                            | 1320                                         | 410                    |                                                                   |                                                |
|                                                   | Smooth<br>spiral rib      | 7                          | 1330                                                                        | 1570                                                                            | 1110                                         | 410                    |                                                                   |                                                |
|                                                   |                           |                            | 1250                                                                        | 1470                                                                            | 1040                                         | 410                    |                                                                   |                                                |
| Steel strand                                      | 1 × 3 (3-wire<br>twisted) | 6.5, 8.6,<br>10.8,<br>12.9 | 1330                                                                        | 1570                                                                            | 1110                                         | 410                    | 1.95 × 10 <sup>5</sup>                                            |                                                |
|                                                   |                           |                            | 1330                                                                        | 1570                                                                            | 1110                                         | 390                    |                                                                   |                                                |
|                                                   |                           |                            | 1580                                                                        | 1860                                                                            | 1320                                         | 390                    |                                                                   |                                                |
|                                                   | 1 × 7 (7-wire<br>twisted) | 9.5, 12.7,<br>15.2         | 1660                                                                        | 1960                                                                            | 1390                                         | 390                    |                                                                   |                                                |
|                                                   |                           |                            | 1460                                                                        | 1720                                                                            | 1220                                         | 390                    |                                                                   |                                                |
|                                                   |                           |                            | 1580                                                                        | 1860                                                                            | 1320                                         | 390                    |                                                                   |                                                |
| Prestressed rebar                                 | Spiral rib                | 21.6                       | 1660                                                                        | 1960                                                                            | 1390                                         | 390                    | 2.00 × 10 <sup>5</sup>                                            |                                                |
|                                                   |                           |                            | 1460                                                                        | 1720                                                                            | 1220                                         | 390                    |                                                                   |                                                |
|                                                   |                           |                            | 785                                                                         | 980                                                                             | 650                                          | 435                    |                                                                   |                                                |
|                                                   |                           |                            | 930                                                                         | 1080                                                                            | 770                                          | 435                    |                                                                   |                                                |
|                                                   |                           | 18, 25,<br>32, 40,<br>50   | 1080                                                                        | 1230                                                                            | 900                                          | 435                    |                                                                   |                                                |
|                                                   |                           |                            |                                                                             |                                                                                 |                                              |                        |                                                                   |                                                |

Notes (1) When the standard strength values of prestressed tendons do not meet the table specifications, they should be converted to Table 2.3 values as shown  
(2) Compressive strength *f'<sub>py</sub>* is not considered for unbonded prestressed tendons

**Table 2.4** Limitation of fatigue stress range of steel bars in reinforced concrete structures (N·mm<sup>-2</sup>)

| Fatigue stress ratio      | $\Delta f_y^f$ |         |
|---------------------------|----------------|---------|
|                           | HRB 335        | HRB 400 |
| $0 \leq \rho_s^f < 0.1$   | 165            | 165     |
| $0.1 \leq \rho_s^f < 0.2$ | 155            | 155     |
| $0.2 \leq \rho_s^f < 0.3$ | 150            | 150     |
| $0.3 \leq \rho_s^f < 0.4$ | 135            | 145     |
| $0.4 \leq \rho_s^f < 0.5$ | 125            | 130     |
| $0.5 \leq \rho_s^f < 0.6$ | 105            | 115     |
| $0.6 \leq \rho_s^f < 0.7$ | 85             | 95      |
| $0.7 \leq \rho_s^f < 0.8$ | 65             | 70      |
| $0.8 \leq \rho_s^f < 0.9$ | 40             | 45      |

Notes (1) When flash exposure butt joint is adopted in longitudinal tensile steel bars, the design fatigue strength of welded joints should be multiplied by a factor 0.8 based on the table  
(2) Grade RRB 400 steel bars should not be adopted for elements when fatigue check is needed  
(3) Steel bars of Grade HRBF 335, HRBF 400, and HRBF 500 are inappropriate for elements requiring a fatigue check. If necessary, an experimental study should be adopted

**Table 2.5** Limitation of fatigue stress range of prestressed tendons (N·mm<sup>-2</sup>)

| Type                       |                        | $\Delta f_y^f$            |                           |
|----------------------------|------------------------|---------------------------|---------------------------|
|                            |                        | $0.7 \leq \rho_p^f < 0.8$ | $0.8 \leq \rho_p^f < 0.9$ |
| Stress-relieved steel wire | $f_{puk} = 1770, 1670$ | 165                       | 165                       |
|                            | $f_{puk} = 1570$       | 155                       | 155                       |
| Steel strand               |                        | 150                       | 150                       |

Notes (1) When  $\rho_s^f \geq 0.9$ , it is not necessary to check the fatigue strength of prestressed tendons  
(2) When well founded, appropriate adjustments can be made to the fatigue stress range in the table

**Table 2.6** Standard strength, design strength, elastic modulus, and fatigue modulus of concrete ( $\text{N}\cdot\text{mm}^{-2}$ )

| Strength grade | Standard strength |          | Design strength |       | Elastic modulus $E_c$ | Fatigue modulus $E_c^f$ |
|----------------|-------------------|----------|-----------------|-------|-----------------------|-------------------------|
|                | $f_{ck}$          | $f_{tk}$ | $f_c$           | $f_t$ |                       |                         |
| C15            | 10.0              | 1.27     | 7.2             | 0.91  | $2.20 \times 10^4$    |                         |
| C20            | 13.4              | 1.54     | 9.6             | 1.10  | $2.55 \times 10^4$    | $1.10 \times 10^4$      |
| C25            | 16.7              | 1.78     | 11.9            | 1.27  | $2.80 \times 10^4$    | $1.20 \times 10^4$      |
| C30            | 20.1              | 2.01     | 14.3            | 1.43  | $3.00 \times 10^4$    | $1.30 \times 10^4$      |
| C35            | 23.4              | 2.20     | 16.7            | 1.57  | $3.15 \times 10^4$    | $1.40 \times 10^4$      |
| C40            | 26.8              | 2.39     | 19.1            | 1.71  | $3.25 \times 10^4$    | $1.50 \times 10^4$      |
| C45            | 29.6              | 2.51     | 21.1            | 1.80  | $3.35 \times 10^4$    | $1.55 \times 10^4$      |
| C50            | 32.4              | 2.64     | 23.1            | 1.89  | $3.45 \times 10^4$    | $1.60 \times 10^4$      |
| C55            | 35.5              | 2.74     | 25.3            | 1.96  | $3.55 \times 10^4$    | $1.65 \times 10^4$      |
| C60            | 38.5              | 2.85     | 27.5            | 2.04  | $3.60 \times 10^4$    | $1.70 \times 10^4$      |
| C65            | 41.5              | 2.93     | 29.7            | 2.09  | $3.65 \times 10^4$    | $1.75 \times 10^4$      |
| C70            | 44.5              | 2.99     | 31.8            | 2.14  | $3.70 \times 10^4$    | $1.80 \times 10^4$      |
| C75            | 47.4              | 3.05     | 33.8            | 2.18  | $3.75 \times 10^4$    | $1.85 \times 10^4$      |
| C80            | 50.2              | 3.11     | 35.9            | 2.22  | $3.80 \times 10^4$    | $1.90 \times 10^4$      |

**Table 2.7** Correction factor  $\gamma_p$  for concrete fatigue compressive strength under different fatigue stress ratios  $\rho_c^f$

| $\rho_c^f$ | $0 \leq \rho_c^f < 0.1$ | $0.1 \leq \rho_c^f < 0.2$ | $0.2 \leq \rho_c^f < 0.3$ | $0.3 \leq \rho_c^f < 0.4$ | $0.4 \leq \rho_c^f < 0.5$ | $\rho_c^f \geq 0.5$ |
|------------|-------------------------|---------------------------|---------------------------|---------------------------|---------------------------|---------------------|
| $\gamma_p$ | 0.68                    | 0.74                      | 0.80                      | 0.86                      | 0.93                      | 1.0                 |

*Notes* If steam curing is adopted, the temperature should not be above 60 °C; If exceeds, the calculated design strength of concrete should be multiplied by a factor 1.2

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