Chapter 2
Foundations of the Quantum Chromodynamics

2.1 Origin of QCD

Quantum chromodynamics (QCD) is a theory to describe the strong interaction in hadrons. It was developed in the history of understanding the structure of the hadrons. In the 1950s, a large number of hadrons were discovered in experiments. Some of them are stable, but most are unstable, decaying to more stable particles immediately. This makes it doubtful that all of them could be fundamental particles, and it is proposed that hadrons are composed of more fundamental particles. Later, Gell-Mann et al. discovered that hadrons can be classified according to a method called the eightfold way, which can be explained by an $SU(3)$ flavor symmetry. Consequently, the quark model and three fundamental quarks, called u, d, and s with spin-$\frac{1}{2}$ and fractal charges, were proposed [1–3]. Many experiment results can be understood based on the quark model. However, there are still some phenomena that cannot be explained. For example, the hadron $\Delta^{++}(1232)$ is the ground state composed of three u quarks with spin-$\frac{3}{2}$. As a result, the wave function should be symmetric in both the spin and position spaces. An extra quantum number of the quark, namely *color*, is proposed in order not to violate the spin-statistics theorem. However, the subsequent experiments to discover free quarks all failed, which forced people to presume that the quarks are confined forever in hadrons. In 1972 and 1973, Fritzsch, Gell-Mann and Leutwyler extended the symmetry in the color configuration to $SU(3)$ gauge symmetry, establishing the theory of QCD [4, 5]. This theory can not only explain the properties predicted by the quark model, but can also satisfy the requirement of quark confinement. Later, many other theoretical and experimental developments convinced people that QCD is just the right theory for strong interaction.

One of the most important experiments is the deeply inelastic scattering (DIS) of electrons and protons. This process can be described as $l(p) + N(P) \to l'(p') + X(p_X)$, where l, l' denotes the in-going and outgoing leptons, respectively, N denotes

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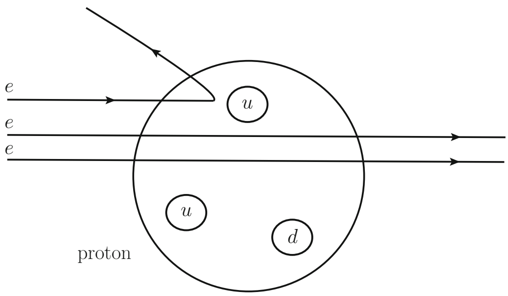


Fig. 2.1 A DIS process

the proton, X represents all the unobserved final states, p, P, p' and p_X are the corresponding momenta. As shown in Fig. 2.1, in the collision of a bunch of electrons with a proton, most electrons go through the proton. Only very few of the electrons change their directions significantly. These processes can be called as the DIS. It happens because the high energy electrons collide with the possible inner small ingredients of the hadron. The ingredients of a hadron are not fixed before any measurement. And the content of the ingredients in a measurement depends on the energy of the measuring particle. The larger the transferred momentum from the lepton $Q \equiv \sqrt{-q^2} = \sqrt{-(p'-p)^2}$ in a DIS, the smaller the measurable ingredients and structure of the hadron. Another important fact in the DIS is that the interaction between the ingredients of a hadron has little impact on the interaction between the lepton and the collided component, because they correspond to interactions with different reaction timescales. For example, if the transferred energy is about 100 GeV, then the interaction of the DIS happens in about 0.67×10^{-26} s, while the interaction between components of hadrons takes place in about 0.67×10^{-22} s after taking into account the time dilation in transferring from the hadron inertial frame to the laboratory frame. This picture was described in the parton (referring to the part of a hadron) model [6], and the scattering cross section of a DIS process is simplified to a sum of contributions from scattering of the lepton with various partons,

$$d\sigma^{\text{DIS}} = \sum_j \int d\xi f_j(\xi) \times d\hat{\sigma}_j, \tag{2.1}$$

where $\sum_j$ is over all partons, $f_j(\xi)$ is the parton distribution function (PDF), and $f_j(\xi)d\xi$ represents the possibility to find a parton j with a momentum fraction of the total momentum of the proton between ξ and $\xi + d\xi$. $\hat{\sigma}_j$ is the scattering cross section between the lepton and partons.

On the other side, according to the general scattering theory, the cross section of the DIS process can be expressed as

$$E' \frac{d\sigma^{\text{DIS}}}{d^3\vec{p}'} \simeq \frac{\pi e^4}{2s} \sum_X \delta^{(4)}(p_X - P - q) |\langle p'|j_\lambda^{\text{lept}}|p\rangle \frac{1}{q^2} \langle p_X|j^\lambda|P\rangle|^2 \tag{2.2}$$

$$= \frac{2\alpha^2}{sQ^4} L_{\mu\nu} W^{\mu\nu}, \tag{2.3}$$

where $\sqrt{s}$ is the hadronic center-of-mass energy, and $\alpha = e^2/(4\pi)$ is the fine structure constant. j^λ is the current

$$j^\lambda = \sum_j e_j \bar{\psi}_j \gamma^\lambda \psi_j, \tag{2.4}$$

where ψ_j denote the different partons and e_j are their charges.

The leptonic part in Eq. (2.3) can be calculated easily as

$$L_{\mu\nu} = \frac{1}{2}\mathrm{Tr}\gamma_\nu \not{p} \gamma_\mu \not{p}' = 2(p_\mu p'_\nu + p'_\mu p_\nu - g_{\mu\nu} p \cdot p'). \tag{2.5}$$

The hadronic part in Eq. (2.3) is complicated, and it can be written as

$$W^{\mu\nu} \equiv 4\pi^3 \sum_X \delta^{(4)}(p_X - P - q)\langle P|j^\mu(0)|p_X\rangle\langle p_X|j^\nu(0)|P\rangle \tag{2.6}$$

$$= \frac{1}{4\pi}\int d^4 z e^{iq\cdot z}\langle P|j^\mu(z)j^\nu(0)|P\rangle. \tag{2.7}$$

The matrix element in the above equation cannot be calculated analytically due to the non-perturbative properties, but must satisfy the following requirements:

- The current is conserved, i.e., $\partial_\mu j^\mu = 0$ and therefore $q_\mu W^{\mu\nu} = q_\nu W^{\mu\nu} = 0$;
- The P-parity is conserved in QED interactions (weak interaction is omitted);
- The scattering amplitude is unitary, i.e., $W^{\mu\nu}$ is Hermitian, and therefore $(W^{\mu\nu})^* = W^{\nu\mu}$.

Then $W^{\mu\nu}$ should be decomposed to

$$W^{\mu\nu} = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right)F_1(x, Q^2) + \frac{(P^\mu - q^\mu P\cdot q/q^2)(P^\nu - q^\nu P\cdot q/q^2)}{P\cdot q}F_2(x, Q^2), \tag{2.8}$$

in which $F_1(x, Q^2)$ and $F_2(x, Q^2)$ are called the structure function, and x is the Bjorken variable, defined as

$$x \equiv \frac{Q^2}{2P\cdot q}. \tag{2.9}$$

Comparing the results in the parton model and the general scattering theory, one obtains

$$F_2(x, Q^2) = \sum_j e_j^2 x f_j(x); \qquad F_1(x, Q^2) = \frac{1}{2x}F_2(x, Q^2). \tag{2.10}$$

From the above equation, the structure function has nothing to do with the transferred momentum Q. This behavior is called Bjorken scaling [7], which was verified in experiments at SLAC in 1969. Bjorken scaling is a result of the parton model, and thus its confirmation also supported the parton model.

After a while, Callan and Gross et al. understood that Bjorken scaling implies that the strong interaction is weak at a short distance [8]. At the same time, it was well known that the strong interaction is strong at a long distance. It follows that the strong interaction becomes weaker and weaker as the interaction distance becomes shorter and shorter, i.e., asymptotic freedom. Gross, Wilczek, and Politzer et al. calculated the anomalous dimension of the QCD coupling, and found that it indeed manifests this behavior [9, 10].

2.2 Lagrangian of QCD and Feynman Rules

QCD is a gauge field theory based on the gauge group $SU(3)$ in color space. The Lagrangian of QCD can be written as

$$\mathcal{L}_{QCD} = \mathcal{L}_B + \mathcal{L}_{GF} + \mathcal{L}_G. \tag{2.11}$$

The basic Lagrangian $\mathcal{L}_B$ is

$$\mathcal{L}_B = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu}_a + \sum_j \bar{q}_{ja}(i\not{D} - m_j)_{ab} q_{jb}, \tag{2.12}$$

where $F^a_{\mu\nu}$ is the strength tensor with the gluon field $\mathcal{A}^a_\mu$

$$F^a_{\mu\nu} = \partial_\mu \mathcal{A}^a_\nu - \partial_\nu \mathcal{A}^a_\mu + g_s f^{abc} \mathcal{A}^b_\mu \mathcal{A}^c_\nu. \tag{2.13}$$

Here a, b, c are color indices, g_s is the QCD coupling, and f^{abc} is the structure constant of $SU(3)$. $\not{D} = \gamma_\mu D^\mu$ with the covariant derivative is defined as

$$D^\mu = \partial^\mu - i g_s t^a \mathcal{A}^\mu_a, \tag{2.14}$$

where t^a, $a = 1, ..., 8$ are the generators of $SU(3)$, satisfying

$$\mathrm{Tr}[t^a t^b] = T_F \delta^{ab} = \frac{1}{2}\delta^{ab}. \tag{2.15}$$

m_j denotes the mass of the quark q_j. $\mathcal{L}_B$ is invariant under the gauge transformation

$$q_a(x) \mapsto q'_a(x) = \exp(i t \cdot \theta(x))_{ab} q_b(x) \equiv \Omega(x)_{ab} q_b(x), \tag{2.16}$$

$$t \cdot \mathcal{A}_\mu \mapsto t \cdot \mathcal{A}'_\mu = \Omega(x) \left(t \cdot A_\mu + \frac{i}{g_s}\partial_\mu \right) \Omega^{-1}(x). \tag{2.17}$$

The number of the gluon's physical degree of freedom is less than that the gauge field $\mathcal{A}^a_\mu$ has. A well-defined propagator for such a field can only be obtained after choosing a specific gauge condition. The covariant gauge is generally used, represented by the gauge-fixing Lagrangian

$$\mathcal{L}_{GF} = -\frac{1}{2\lambda}(\partial^{\mu}\mathcal{A}^{a}_{\mu})^{2}, \tag{2.18}$$

in which λ is a free parameter. Under this gauge, an additional field, called ghost field, is brought out, whose interaction is contained in the ghost Lagrangian

$$\mathcal{L}_{G} = \partial_{\mu}\eta^{a\dagger}(D^{\mu}_{ab}\eta^{b}), \tag{2.19}$$

where η^{a} is a complex scalar field satisfying the anti-communication relation.

When performing the quantization of QCD, the quadratic term of every field is separated to obtain the corresponding propagator. In momentum space, the two-point correlation function of the quark field is

$$\Gamma^{(2)}_{q,ab}(p) = -i\delta_{ab}(\not{p} - m). \tag{2.20}$$

Its inverse gives the quark propagator

$$\Delta^{(2)}_{q,ab}(p) = \frac{i\delta_{ab}}{\not{p} - m + i\epsilon}, \tag{2.21}$$

where $i\epsilon$ with $\epsilon \to 0$ is a prescription for picking poles coincident with causality. The propagator of the ghost field is

$$\Delta^{(2)}_{\eta,ab}(p) = \frac{i\delta_{ab}}{p^{2} + i\epsilon}. \tag{2.22}$$

The two-point correlation function of the gluon field under the covariant gauge is

$$\Gamma^{(2)}_{\mathcal{A},ab,\mu\nu}(p) = i\delta_{ab}\left[p^{2}g_{\mu\nu} - (1 - \frac{1}{\lambda})p_{\mu}p_{\nu}\right]. \tag{2.23}$$

Therefore, the propagator of the gluon field is given as

$$\Delta^{(2)}_{\mathcal{A},ab,\mu\nu}(p) = \frac{i\delta_{ab}}{p^{2}}\left[-g_{\mu\nu} + (1 - \lambda)\frac{p_{\mu}p_{\nu}}{p^{2}}\right]. \tag{2.24}$$

The choice of $\lambda = 1$, corresponding to Feynman gauge, can simplify the calculation significantly in practice. Of course, choosing a general λ and keeping it everywhere in calculation can help to check the correctness of the computation, since the dependence on λ is supposed to be canceled at the end.

There is another kind of gauge for the gluon field, given by the constant gauge-fixing vector n^μ and the associating Lagrangian

$$\mathcal{L}_{GF} = -\frac{1}{2\lambda}(n^\mu \mathcal{A}^a_\mu)^2, \tag{2.25}$$

In this case, no ghost field is needed and the two-point correlation function of the gluon field is

$$\Gamma^{(2)}_{\mathcal{A},ab,\mu\nu}(p) = i\delta_{ab}\left[p^2 g_{\mu\nu} - p_\mu p_\nu + \frac{1}{\lambda}n_\mu n_\nu\right]. \tag{2.26}$$

Therefore, the propagator of the gluon in such a gauge is given as

$$\Delta^{(2)}_{\mathcal{A},ab,\mu\nu}(p) = \frac{i\delta_{ab}}{p^2}\left[-g_{\mu\nu} + \frac{n_\mu p_\nu + n_\nu p_\mu}{n\cdot p} - \frac{(n^2+\lambda p^2)p_\mu p_\nu}{(n\cdot p)^2}\right]. \tag{2.27}$$

If $\lambda = 0$, $n^2 = 0$, i.e., the light-cone gauge, the propagator is

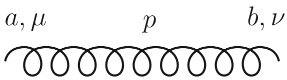
$$\Delta^{(2)}_{\mathcal{A},ab,\mu\nu}(p) = \frac{i\delta_{ab}}{p^2}\left[-g_{\mu\nu} + \frac{n_\mu p_\nu + n_\nu p_\mu}{n\cdot p}\right], \tag{2.28}$$

which agrees with

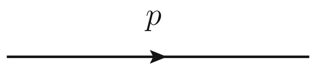
$$n^\mu \Delta^{(2)}_{\mathcal{A},ab,\mu\nu}(p) = 0, \qquad p^\mu \Delta^{(2)}_{\mathcal{A},ab,\mu\nu}(p) = \frac{i\delta_{ab}n_\nu}{n\cdot p}. \tag{2.29}$$

One can see that only two physical polarization degrees of freedom, perpendicular to n^μ and p^μ, are active in the propagator.

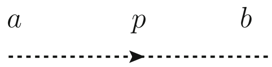
The other Feynman rules for the three- and four-point interactions can be read from the $\mathcal{L}_{QCD}$ by path integration, summarized in Eqs. (2.30–2.36).



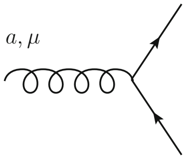
$$\frac{-i\delta_{ab}}{p^2+i\epsilon}\left[g_{\mu\nu} + (\lambda - 1)\frac{p_\mu p_\nu}{p^2}\right] \tag{2.30}$$



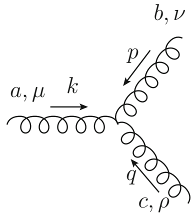
$$\frac{i(\not{p}+m)}{p^2 - m^2 + i\epsilon} \tag{2.31}$$



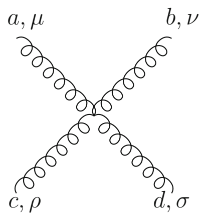
$$\frac{i\delta_{ab}}{p^2 + i\epsilon} \tag{2.32}$$



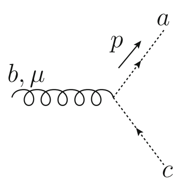
$$ig_s\gamma^\mu t^a \tag{2.33}$$



$$\begin{aligned} g_s f^{abc}[&g^{\mu\nu}(k-p)^\rho \\ &+ g^{\nu\rho}(p-q)^\mu \\ &+ g^{\rho\mu}(q-k)^\nu] \end{aligned} \tag{2.34}$$



$$\begin{aligned} -ig_s^2[&f^{abe}f^{cde}(g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho}) \\ &+ f^{ace}f^{bde}(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\sigma}g^{\nu\rho}) \\ &+ f^{ade}f^{bce}(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma})] \end{aligned} \tag{2.35}$$



$$g_s f^{abc} p^\mu \tag{2.36}$$

2.3 Renormalization

Making use of the Feynman rules of QCD, one can calculate the amplitudes or cross sections of the hadronic scattering processes. Generally, the amplitudes are hard to be calculated analytically, and should be expanded in a series of the coupling of the strong interaction $\alpha_s = g_s^2/(4\pi) \sim 0.12$.

$$\mathcal{M} = \mathcal{M}_0 + \frac{\alpha_s}{4\pi}\mathcal{M}_1 + \left(\frac{\alpha_s}{4\pi}\right)^2 \mathcal{M}_2 + \cdots , \tag{2.37}$$

where $\mathcal{M}_0, \mathcal{M}_1, \mathcal{M}_2$ are referred to as leading order (LO), next-to-leading order (NLO), next-to-next-to-leading order (NNLO) amplitudes, and so forth.

The LO amplitudes are usually easy to calculate. However, the NLO amplitudes are complicated because of the loop integrals in which the momentum of the virtual particles could be infinite, making the integration meaningless. This kind of result is called *ultraviolet-divergent*, a feature of the local field theory. To recover the prediction ability of QCD, renormalization of the theory is needed, which means redefinitions of the parameters in the Lagrangian, such as m_j, g_s. Any observable O is a function of m_j, g_s, i.e., $O(m_j, g_s)$, and meanwhile is finite. Then it is required that the parameters m_j, g_s are also divergent that just cancel the divergences in the loop integrals. It seems unreasonable to use infinite parameters in the calculations. However, if the divergence is universal, which means the structure of infinities in the loop integrals is fixed, then the QCD theory is still predictive after redefinitions of a finite number of parameters. It is remarkable that QCD has been proven to be renormalizable to all orders of α_s [11].

Incorporating infinite parameters in the Lagrangian seems weird at first sight. This is related to the fact that we have considered all fundamental particles as point particles and the interactions are all local.[1] Let us take QED as an example. The electron is charged. If the electron has a finite radius, which has been constrained to be very small experimentally, then the electric potential energy inside the electron would be so large that the individual parts of the electron would be repulsed against each other. As a result, the electron could not be a stable existence. In order to be consistent with reality, the electron is assumed to be point-like. From the uncertainty principle, it is possible that the inside energy of the electron, reflected by the mass, is infinite.

There are two ways to perform renormalization. The first is the bare parameter renormalization. One uses the bare Lagrangian in Eq. (2.11) and its corresponding Feynman rules to calculate the observable $O_1(m_j, g_s)$. Of course, it is divergent. One can use some regularization techniques to represent such divergences, such as $\ln^n\left(\Lambda/m_j\right)$ or $1/\epsilon^n$ with $n = 1, 2, \ldots$. The former is called cutoff regularization and Λ denotes the upper limit of the loop momentum. The latter is called dimensional regularization and the dimension of the loop momentum is extended from 4 to $4 - 2\epsilon$

[1]Here, "local" means that the Lagrangian is a function of fields with the same space-time point. Nonlocal fields and interactions have been discussed in Refs. [12–14].

[15–18]. Taking the renormalization of the mass and coupling as an example, two other observables $O_2(m_j, g_s)$ and $O_3(m_j, g_s)$ are supposed to be calculated before $O_1(m_j, g_s)$ in order to extract the physical or renormalized m_j^{R} and g_s^{R}, which are finite. The relations between the bare and renormalized parameters are obtained as

$$m_j^{\mathrm{R}} = m_j + C + \frac{\alpha_s}{4\pi}\frac{1}{\epsilon} + \left(\frac{\alpha_s}{4\pi}\right)^2 \left(\frac{1}{\epsilon^2} + \frac{1}{\epsilon}\right) + \cdots, \tag{2.38}$$

$$g_s^{\mathrm{R}} = g_s + C + \frac{\alpha_s}{4\pi}\frac{1}{\epsilon} + \left(\frac{\alpha_s}{4\pi}\right)^2 \left(\frac{1}{\epsilon^2} + \frac{1}{\epsilon}\right) + \cdots, \tag{2.39}$$

where C is a finite term and the coefficients of each term have been set to be 1 for simplicity. Then replacing the m_j, g_s in $O_1(m_j, g_s)$ with $m_j^{\mathrm{R}}, g_s^{\mathrm{R}}$, one finds all divergences cancel out and gets a finite $O_1(m_j^{\mathrm{R}}, g_s^{\mathrm{R}})$. Notice that the cancelation of divergences takes place order by order in α_s.

The second way is the Bogoliubov-Parasiuk-Hepp-Zimmermann renormalization scheme [19–21]. Since the observables are finite, it is more natural that the parameters they depend on are finite. Therefore, one can use renormalized parameters, such as $m_j^{\mathrm{R}}, g_s^{\mathrm{R}}$, in the Lagrangian directly and any observable is just a function of renormalized parameters. However, this can be achieved at the cost of adding more interaction terms in the Lagrangian. Explicitly, the fields in the bare Lagrangian should be redefined as

$$q_j = Z_{2,j}^{1/2} q_{j,r}, \tag{2.40}$$

$$A^\mu = Z_3^{1/2} A_r^\mu, \tag{2.41}$$

$$\eta^a = Z_2^{\eta\, 1/2} \eta_r^a, \tag{2.42}$$

and the Lagrangian in Eq. (2.11) can be rewritten as

$$\mathcal{L}_{QCD} = \mathcal{L}_{QCD}^{\mathrm{R}} + \mathcal{L}_{QCD}^{\mathrm{C.T.}}, \tag{2.43}$$

where $\mathcal{L}_{QCD}^{\mathrm{R}} = \mathcal{L}_{QCD}(m_j \to m_j^{\mathrm{R}}, g_s \to g_s^{\mathrm{R}})$, and $\mathcal{L}_{QCD}^{\mathrm{C.T.}}$ contain counterterms,[2]

$$\begin{aligned}
\mathcal{L}_{QCD}^{\mathrm{C.T.}} = & -\frac{1}{4}\delta_3(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a)^2 + \sum_j \bar{q}_j (i\delta_2^j \not{\partial} - \delta_m^j) q_j - \delta_2^\eta \eta^{a\dagger}\partial^2\eta^a \\
& + \sum_j g_s^{\mathrm{R}} \delta_1^j A_\mu^a \bar{q}_j \gamma^\mu q_j - g_s^{\mathrm{R}} \delta_1^{3g} f^{abc}(\partial_\mu A_\nu^a) A_\mu^b A_\nu^c \\
& - \frac{1}{4} g_s^{\mathrm{R}\,2} \delta_1^{4g} (f^{eab} A_\mu^a A_\nu^b)(f^{ecd} A_\mu^c A_\nu^d) - g_s^{\mathrm{R}} \delta_1^\eta f^{abc} \eta^{a\dagger} \partial^\mu A_\mu^b \eta^c.
\end{aligned} \tag{2.44}$$

[2] All the fields here are renormalized but the subscript 'r' is omitted for simplicity.

Here the various new parameters are defined as

$$\begin{aligned}
&\delta_2^j = Z_{2,j} - 1, \quad \delta_3 = Z_3 - 1, \quad \delta_2^\eta = Z_2^\eta - 1, \quad \delta_m^j = Z_{2,j} m_j - m_j^{\mathrm{R}},\\
&\delta_1^j = \frac{g_s}{g_s^{\mathrm{R}}} Z_{2,j} Z_3^{1/2} - 1, \quad \delta_1^{3g} = \frac{g_s}{g_s^{\mathrm{R}}} Z_3^{3/2} - 1,\\
&\delta_1^{4g} = \frac{g_s^2}{g_s^{\mathrm{R}2}} Z_3^2 - 1, \quad \delta_1^\eta = \frac{g_s}{g_s^{\mathrm{R}}} Z_2^\eta Z_3^{1/2} - 1.
\end{aligned} \tag{2.45}$$

Now, all the divergences in $\mathcal{L}_{QCD}$ are implicitly incorporated in $\mathcal{L}_{QCD}^{\mathrm{C.T.}}$. Specifically, δ_2 (including δ_2^j and δ_2^η), δ_m(including δ_m^j), and δ_3 cancel the divergences in the propagators of quarks, ghosts, and gluons, respectively. δ_1 (including δ_1^j, δ_1^{3g}, δ_1^{4g}, and δ_1^η) cancel the divergences associating vertices. Although they are divergent, they cancel out against the divergences in the loop integrals order by order, resulting in finite predictions on the observables. In general, δ_1, δ_2, δ_m, δ_3 can contain arbitrary finite terms. Different finite terms correspond to different renormalized parameters, e.g., m_j^{R}, g_s^{R}. It is only required that the same finite terms be used when comparing the predictions on two observables. This means the absolute value of an observable is meaningless as it depends on the definitions of counterterms in the theory. It is the relationship between observables that is predictable and physical. Any specific choice of the finite terms in the counterterms sets a renormalization scheme. The most used are the modified minimal subtraction ($\overline{\mathrm{MS}}$) [22, 23] and on-shell renormalization schemes. The relation between different renormalization schemes is universal. If one has the results in one renormalization scheme, it is easy to translate them to other schemes.

After calculating the counterterms in QCD, i.e., δ_1, δ_2, δ_m, δ_3, the running behaviors of the renormalized parameters m_j^{R}, g_s^{R} as a function of the scale are also known. Here the scale refers to the magnitude of the energy. The content of a hadron is different when measured by particles with different energy. Thus, the parameters m_j^{R}, g_s^{R} are also different under different energy scale. In QCD, the running equation of g_s^{R}, i.e., the renormalization group equation (RGE), reads as

$$\beta(g_s^{\mathrm{R}}) \equiv \frac{d g_s^{\mathrm{R}}}{d \ln \mu} = g_s^{\mathrm{R}} \frac{d}{d \ln \mu}[- \ln(1 + \delta_1^j) + \ln(1 + \delta_2^j) + \frac{1}{2} \ln(1 + \delta_3)]. \tag{2.46}$$

The scale μ is a result of choosing the dimensional regularization scheme, where one should make the replacement $g_s^{\mathrm{R}} \to g_s^{\mathrm{R}} \mu^\epsilon$ so that the mass dimension of g_s^{R} is still zero. At one-loop level,

$$\beta(g_s^{\mathrm{R}}) = -\frac{g_s^{\mathrm{R}3}}{(4\pi)^2} \left(\frac{11}{3} C_A - \frac{4}{3} n_f T_F \right) = -\frac{g_s^{\mathrm{R}3}}{3(4\pi)^2} \left(33 - 2n_f\right) = -\frac{g_s^{\mathrm{R}} \alpha_s}{4\pi} \beta_0, \tag{2.47}$$

where $C_A = 3$ is the Casimir operator of the adjoint representation in $SU(3)_C$, n_f is the number of active quarks, and $\beta_0 = \left(11 - \frac{2}{3}n_f\right)$. The present experiments show $n_f = 6$. Therefore $\beta(g_s^{\mathrm{R}})$ is negative, which means the strong interaction coupling becomes smaller with the increase of scale. It is just the behavior of asymptotic freedom.

Solving the RGE of $\beta(g_s^{\mathrm{R}})$ above, one obtains the running coupling at one-loop level,

$$\alpha_s(\mu) = \frac{\alpha_s(\mu_0)}{1 + \frac{\alpha_s(\mu_0)}{4\pi}\beta_0 \ln \frac{\mu^2}{\mu_0^2}}. \tag{2.48}$$

Since $\beta_0 > 0$, when $\mu \to \infty$, $\alpha_s(\mu) \to 0$. On the other hand, when $\mu \to \Lambda_{\mathrm{QCD}} \equiv \mu_0 \exp\left(-\frac{2\pi}{\alpha_s(\mu_0)\beta_0}\right)$, $\alpha_s(\mu) \to \infty$. It suggests the perturbative QCD is not applicable any more. Λ_{QCD} denotes the lower energy limit in applying perturbative QCD, and is found to be a few hundreds of MeV.

References

1. M. Gell-Mann, Symmetries of baryons and mesons. Phys. Rev. **125**, 1067–1084 (1962)
2. G. Zweig, An SU(3) model for strong interaction symmetry and its breaking. CERN-TH **401**
3. G. Zweig, An SU(3) model for strong interaction symmetry and its breaking 2. CERN-TH **412**
4. H. Fritzsch, M. Gell-Mann, Current algebra: quarks and what else? in *eConf* C720906V2, pp. 135–165 (1972). arXiv:hep-ph/0208010
5. H. Fritzsch, M. Gell-Mann, H. Leutwyler, Advantages of the color octet gluon picture. Phys. Lett. B **47**, 365–368 (1973)
6. R. Feynman, Photon Hadron Interactions. W.A. Benjamin, New York
7. J. Bjorken, Asymptotic sum rules at infinite momentum. Phys. Rev. **179**, 1547–1553 (1969)
8. J. Callan, Curtis G., D.J. Gross, Bjorken scaling in quantum field theory. Phys. Rev. D **8**, 4383–4394 (1973)
9. D. Gross, F. Wilczek, Ultraviolet behavior of nonabelian gauge theories. Phys. Rev. Lett. **30**, 1343–1346 (1973)
10. D. Gross, F. Wilczek, Asymptotically free gauge theories. 1. Phys. Rev. D **8**, 3633–3652 (1973)
11. G. ’t Hooft, Renormalization of massless yang-mills fields. Nucl. Phys. B **33**, 173–199 (1971)
12. H. Yukawa, Quantum theory of nonlocal fields. 1. Free fields. Phys. Rev. **77**, 219–226 (1950)
13. H. Yukawa, Quantum theory of nonlocal fields. 2: irreducible fields and their interaction. Phys. Rev. **80**, 1047–1052 (1950)
14. A. Pais, G. Uhlenbeck, On field theories with nonlocalized action. Phys. Rev. **79**, 145–165 (1950)
15. G. ’t Hooft, M.J.G. Veltman, Regularization and renormalization of gauge fields. Nucl. Phys. B **44**, 189–213 (1972)
16. C.G. Bollini, J.J. Giambiagi, Lowest order divergent graphs in nu-dimensional space. Phys. Lett. B **40**, 566–568 (1972)
17. J.F. Ashmore, A method of gauge invariant regularization. Lett. Nuovo Cim. **4**, 289–290 (1972)
18. G.M. Cicuta, E. Montaldi, Analytic renormalization via continuous space dimension. Lett. Nuovo Cim. **4**, 329–332 (1972)
19. N. Bogoliubov, O. Parasiuk, Acta Math. **97**, 227 (1957)
20. K. Hepp, Proof of the Bogolyubov-Parasiuk theorem on renormalization. Commun. Math. Phys. **2**, 301–326 (1966)

21. W. Zimmermann, Convergence of Bogolyubov's method of renormalization in momentum space. Commun. Math. Phys. **15**, 208–234 (1969)
22. G. 't Hooft, Dimensional regularization and the renormalization group. Nucl. Phys. B **61**, 455–468 (1973)
23. S. Weinberg, New approach to the renormalization group. Phys. Rev. D **8**, 3497–3509 (1973)

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