

Chapter 2

Basic Theory of FEL

Abstract This chapter shows the basic theory of FEL. We start with the energy exchange equation that describes the interaction between the electrons and radiation. From these deductions, we get the resonant condition and pendulum equation of the FEL. As this thesis is mainly focus on FEL with external seed lasers, equations for harmonic generation schemes include HGHG and EEHG are derived in detail to get the final bunching factor of these schemes.

Keywords Resonant condition • Pendulum equation • Bunching factor

2.1 Energy Exchange Equation

Here we take the planar undulator as an example to introduce the basic theory of FEL. As shown in Fig. 2.1, considering a planar undulator with period of λ_u and a sinusoidal magnetic field in the y direction:

$$B_y(z) = B_0 \sin(k_u z) \quad (2.1)$$

where $k_u = 2\pi/\lambda_u$, and the electron motion in the x-z plane can be written as

$$m\gamma \frac{d^2 x}{dt^2} = e(v_y B_z - v_z B_y) = -eB_0 c \sin(k_u z) \quad (2.2)$$

where m , γ and e is the mass, relativistic energy and charge of an electron, respectively. B_0 is the peak magnetic field of the undulator, c is the speed of light in vacuum, and v_z is the velocity of the electron. When the electron energy is high enough, $v_z \approx c$ and we can change the independent variable from t to z with

$$dt = \frac{dz}{v_z} \approx \frac{dz}{c}. \quad (2.3)$$

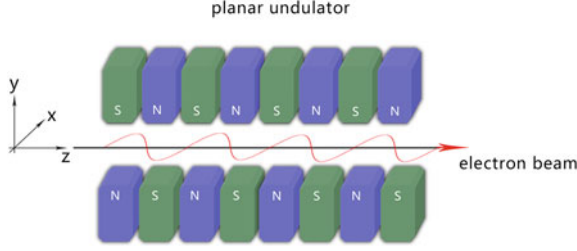


Fig. 2.1 Electron beam trajectory in a planar undulator

And Eq. (2.2) is changed to

$$x'' = -\frac{eB_0}{\gamma mc} \sin(k_u z). \quad (2.4)$$

Integrate Eq. (2.4) along z , we can get

$$x' = \frac{eB_0}{\gamma mck_u} \cos(k_u z) \equiv \frac{K}{\gamma} \cos(k_u z) \quad (2.5)$$

where $K = eB_0 / mck_u \approx 0.934B_0[\text{Tesla}]\lambda_u[\text{cm}]$ is the undulator parameter. Integrate Eq. (2.5) again, we arrive at

$$x = \frac{K}{\gamma k_u} \sin(k_u z) \quad (2.6)$$

The radiation field generated by the electrons propagates collinearly with the electron beam:

$$\mathbf{E} = \hat{x}E_0 \cos(k_s z - \omega_s t + \theta_0) \quad (2.7)$$

where $\omega_s = ck_s = 2\pi c/\lambda_s$ is the radiation frequency. The electron's transverse velocity induced by the undulator magnet is

$$v_x = \frac{Kc}{\gamma} \cos(k_u z). \quad (2.8)$$

Since the average velocity of the electron is approximately constant

$$\mathbf{v} = c\sqrt{1 - 1/\gamma^2} \quad (2.9)$$

v_z can be calculated by:

$$v_z = c \left(1 - \frac{1 + K^2/2}{2\gamma^2} \right) - \frac{K^2 c}{4\gamma^2} \cos(2k_u z), \quad (2.10)$$

So the average electron longitudinal velocity is

$$\bar{v}_z = c \left(1 - \frac{1 + K^2/2}{2\gamma^2} \right). \quad (2.11)$$

The energy exchange between the electrons and the field should be

$$\begin{aligned} mc^2 \frac{d\gamma}{dt} &= e\mathbf{E} \cdot \mathbf{v}_x = \frac{eE_0 K c}{\gamma} \cos(k_u z) \cos(k_s z - \omega_s t + \phi) \\ &= \frac{eE_0 K c}{2\gamma} \{ \cos[(k_s - k_u)z - \omega_s t + \phi] + \cos[(k_s + k_u)z - \omega_s t + \phi] \} \end{aligned} \quad (2.12)$$

The energy will transfer from electrons to radiation field when $e\mathbf{E} \cdot \mathbf{v}_x < 0$. When $e\mathbf{E} \cdot \mathbf{v}_x > 0$, the energy of the field will be transferred to the electron, which is used for the inverse FEL (iFEL).

2.2 Phase Evolution Equation

From Eq. (2.12), we can get the rate change of the ponderomotive phase θ can be written as

$$\frac{d\theta}{dt} \approx 2k_u c, \quad (2.13)$$

where

$$\theta = (k_s + k_u)z - \omega_s t + \phi \quad (2.14)$$

Derivation of Eq. (2.14),

$$\frac{d\theta}{dt} = (k_s + k_u)v_z - ck_s \quad (2.15)$$

Using Eq. (2.11) and $k_u/k_s \ll 1$, we get

$$\frac{d\theta}{dt} = (k_s + k_u)\bar{v}_z - ck_s = ck_s \left(\frac{k_u}{k_s} - \frac{1 + K^2/2}{2\gamma^2} \right) \quad (2.16)$$

2.3 Resonance Condition and Pendulum Equation

In order to obtain a stationary phase $d\theta/dt = 0$, we need

$$\frac{k_u}{k_s} = \frac{\lambda_s}{\lambda_u} = \frac{1 + K^2/2}{2\gamma^2} \quad (2.17)$$

This is the famous resonant condition. For a given resonant energy γ_r , the output wavelength can be written as

$$\lambda_s = \frac{\lambda_u}{2\gamma_r^2} (1 + K^2/2). \quad (2.18)$$

Here we introduce the normalized electron energy variable parameter

$$\eta = \frac{\gamma - \gamma_r}{\gamma_r} \ll 1, \quad (2.19)$$

and the phase equation is changed to

$$\frac{d\theta}{dt} = 2k_u c \eta \quad (2.20)$$

Equation (2.12) can be written as

$$\frac{d\eta}{dt} = \frac{1}{\gamma_r} \frac{d\gamma}{dt} = -\frac{eE_0 K [JJ] c}{2\gamma \gamma_r m c^2} \sin \theta, \quad (2.21)$$

where

$$[JJ] = J_0 \left(\frac{K^2}{4 + 2K^2} \right) - J_1 \left(\frac{K^2}{4 + 2K^2} \right). \quad (2.22)$$

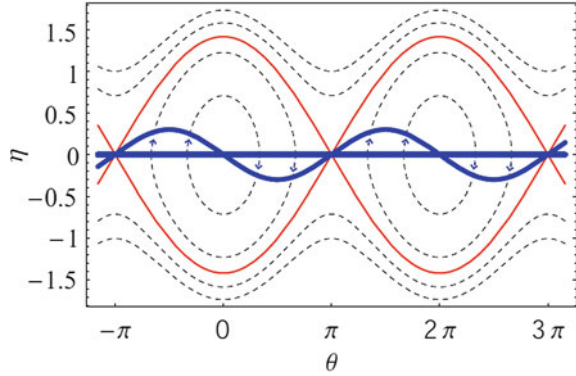
Use $cdt \approx dz$, Eqs. (2.20) and (2.21) can be rewritten as

$$\begin{cases} \frac{d\theta}{dz} = 2k_u \eta \\ \frac{d\eta}{dz} = -\frac{eE_0 K [JJ]}{2\gamma_r^2 m c^2} \sin \theta \end{cases} \quad (2.23)$$

From Eq. (2.23) we can get the familiar pendulum equation

$$\frac{d^2\theta}{dz^2} + \Omega_s^2 \sin \theta = 0 \quad (2.24)$$

Fig. 2.2 Electron motion in the longitudinal phase space due to the presence of the radiation field and the undulator



where $\Omega_s^2 = eE_0K[JJ]/2k_0mc^2\gamma_r^2$ is the synchrotron rotation frequency. In the low-gain regime, the synchrotron rotation frequency can be considered as a constant. Intergration of Eq. (2.24) and we get

$$\left(\frac{d\theta}{dz}\right)^2 / 2 - \Omega_s^2 \cos \theta = U = \text{Const} \quad (2.25)$$

Figure 2.2 shows the electron's phase space trajectory in the combined system of the radiation field and the undulator. The dashed lines are the phase-space trajectories, and the solid red line is the separatrix, which is the boundary separating trapped and un-trapped trajectories. The region inside the separatrix is called the "bucket". The FEL interaction causes the electrons to gain or lose energy, depending on their ponderomotive phase. Electrons with ponderomotive phase between $[2n\pi, (2n+1)\pi]$ lose energy and move to the bottom of the bucket. Electrons with ponderomotive phase between $[(2n-1)\pi, 2n\pi]$ gain energy and move to the top of the bucket. The resulting energy modulation causes the electrons to develop density modulation with the period of the radiation wavelength.

2.4 Equations for Harmonic Generation Seeded FEL Schemes

As we have mentioned in Chap. 1, harmonic generation schemes with external seed lasers hold the ability to generate stable and fully coherent radiation at short wavelength. In these schemes, the electron beam is manipulated in a nonlinear way to create density modulations at higher harmonics of the seed frequency. Benefit from the coherent density modulation, seeded FEL schemes can be used to increase the stability and temporal coherence over a SASE FEL. In this section, we give the basic theory for both HGHG and EEHG.

2.4.1 HGFG-FEL [1, 2]

The HGFG scheme consists of two undulators separated by a small chicane. In the modulator, the electron beam interacts with a seed laser pulse to generate the energy modulation, which have a maximal value of

$$\Delta\gamma = \frac{k_1 a_1 a_m l_m [JJ]}{\gamma}, \quad (2.26)$$

where k_1 is the wave number of the seed laser, l_m is the length of the modulator, a_1 and a_m is the vector potential of the laser and the undulator, respectively. Here we assume an initial Gaussian beam energy distribution with an average energy $\gamma_0 mc^2$ and use the variable $p = (\gamma - \gamma_0)/\sigma_\gamma$ for the dimensionless energy deviation of a particle, where σ_γ is the initial beam energy spread. Thus the initial longitudinal phase space distribution should be written as

$$f_0(p) = \frac{N_0}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}p^2\right), \quad (2.27)$$

where N_0 is the number of electrons per unit length of the beam. After passing through the modulator, the electron beam is modulated with amplitude $A = \Delta\gamma/\sigma_\gamma$, where $\Delta\gamma$ is the energy modulation depth induced by the seed laser, and the dimensionless energy deviation of the electron beam becomes

$$p' = p + A \sin(k_1 s), \quad (2.28)$$

where s is the longitudinal position. And the longitudinal beam distribution evolves to

$$f_1(\zeta, p) = \frac{N_0}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}(p - A \sin \zeta)^2\right], \quad (2.29)$$

After the dispersion section, the energy modulation will be converted into density modulation. The longitudinal position will be changed to

$$s' = s + R_{56} p \sigma_\gamma / \gamma \quad (2.30)$$

where R_{56} is the strength of the chicane and p is the beam energy after the energy modulation. The distribution function can be written as

$$f_{HGFG}(\zeta, p) = \frac{N_0}{\sqrt{2\pi}} \exp\left\{-\frac{1}{2}[p - A \sin(\zeta - Bp)]^2\right\}, \quad (2.31)$$

where $B = R_{56}k_1\sigma_\gamma/\gamma$ is the dimensionless strength of the dispersion section. Integration of Eq. (2.31) over p gives the beam density distribution

$$N(\zeta) = \int_{-\infty}^{\infty} dp f_{HGHG}(\zeta, p) \quad (2.32)$$

The density modulation of the electron beam can be quantified by the bunching factor, which has a maximum value of unity:

$$b = \frac{1}{N_0} |\langle e^{-ia\zeta} N(\zeta) \rangle| \quad (2.33)$$

And the bunching factor at k th harmonic can be written as:

$$b_k = J_k(kAB) \exp\left(-\frac{1}{2}k^2B^2\right) \quad (2.34)$$

The bunched electron beam will be sent into the radiator for high harmonic radiation. In the first two gain lengths of the radiator, the FEL works in the coherent harmonic generation regime, where the harmonic field grows linearly with the distance traversed in the radiator z , and the peak power grows as z^2 . So the coherent radiation power after two gain lengths can be written as

$$P_{coh} = \frac{Z_0 I_p^2 K^2 b_k^2 [JJ]^2}{32\pi \Sigma_A \gamma^2} (2L_G)^2 \quad (2.35)$$

where $Z_0 = 377\Omega$, I_p is the current of the electron beam. After about two gain lengths, the longitudinal dynamic of the electron beam induced by the radiated fields become important, and the radiation power will be exponentially amplified along the radiator until saturation. The saturation power can be estimated by

$$P_{sat} = 1.6\rho \times \left(\frac{L_{G1D}}{L_{G3D}}\right)^2 \frac{\gamma m_e c^2 I_p}{e}, \quad (2.36)$$

where ρ is the Pierce parameter [3], L_{G1D} and L_{G3D} is the one-dimensional and three-dimensional gain length of the FEL, respectively. From Eqs. (2.35) and (2.36), we get the saturation length of the HGHG-FEL:

$$L_{sat} = L_{G3D} \left[\ln\left(\frac{P_{sat}}{P_{coh}}\right) + 2 \right] \quad (2.37)$$

2.4.2 EEHG-FEL [4, 5]

The EEHG technique employs two pairs of modulator and dispersion section to introduce the echo effect into the electron beam phase space for enhancing the frequency multiplication of the current modulation with a relatively small energy modulation.

After sending the electron beam through the first modulator and dispersion section, the longitudinal beam distribution is similar to Eq. (2.31). Here we assume the energy modulation amplitude induced by the first seed laser is A_1 , and the strength of the first dispersion section is B_1 , then Eq. (2.31) can be rewritten as

$$f_2(\zeta, p) = \frac{N_0}{\sqrt{2\pi}} \exp \left\{ -\frac{1}{2} [p - A_1 \sin(\zeta - B_1 p)]^2 \right\} \quad (2.38)$$

After the second energy modulation, the beam energy is changed to

$$p' = p + A_2 \sin(k_2 s + \varphi) \quad (2.39)$$

where A_2 is the energy modulation amplitude induced by the second seed laser, k_2 is the wave number of the second seed laser and φ is the relative phase of the second seed laser to the first seed laser. After passing through the second dispersive chicane, the longitudinal position of the electrons change to

$$s' = s + R_{56}^{(2)} p \sigma_\gamma / \gamma \quad (2.40)$$

where p now refer to the electron energy the entrance to the second dispersion section. The strength of the second dispersion section is $B_2 = R_{56}^{(2)} k_1 \sigma_\gamma / \gamma$, and we get the longitudinal phase space distribution at the entrance to the radiator

$$f_{EEHG}(\zeta, p) = \frac{N_0}{\sqrt{2\pi}} \exp \left[-\frac{1}{2} \{ p - A_2 \sin(K\zeta - KB_2 p + \varphi) - A_1 \sin[\zeta - (B_1 + B_2)p + A_2 B_1 \sin(K\zeta - KB_2 p + \varphi)] \}^2 \right] \quad (2.41)$$

where $K = k_2/k_1$. Integration of Eq. (2.41) over p gives the beam density distribution, and the bunching factor can be written as

$$b_{n,m}(\zeta, p) = \left| J_m [-(Km + n)A_2 B_2] \times |J_n \{ -A_1 [nB_1 + (Km + n)B_2] \} \exp \left\{ -\frac{1}{2} [nB_1 + (Km + n)B_2]^2 \right\} \right| \quad (2.42)$$

The up-conversion harmonic number is $k = n + Km$.

Analysis shows that the n th bunching factor attains its maximum when $n_1 = \pm 1$ and $n_2 = n_3 \cdot n_1$ is chosen to be -1 to make B_1 and B_2 have the same sign. For given A_1 and A_2 , the maximal bunching factor is achieved when $B_2 = (n_2 + 0.81n_2^{1/3})/nA_2$ and B_1 is the solution of

$$A_2[J_0(A_1\xi) - J_2(A_1\xi)] = 2\xi J_1(A_1\xi) \quad (2.43)$$

where $\xi = B_1 - (Km - 1)B_2$. There are infinite number of roots of Eq. (2.43), however, only two roots that have minimal absolute value maximize the expression. The maximal bunching factor of EEHG is

$$b_k \approx \frac{0.39}{(k+1)^{1/3}} \quad (2.44)$$

The FEL gain process in the radiator is similar with that in the HGHG. The saturation power and saturation length of EEHG can also be calculated by Eqs. (2.36) and (2.37).

References

1. Yu LH (1991) Generation of intense UV radiation by sub harmonically seeded single-pass free-electron lasers. *Phys Rev A* 44(8):5178–5193
2. Yu LH, Wu J (2002) Theory of high gain harmonic generation: an analytical estimate. *Nucl Instrum Methods Phys Res Sect A* 483(1):493–498
3. Bonifacio R, Pellegrini C, Narducci LM (1984) Collective instabilities and high-gain regime in a free electron laser. *Opt Commun* 50(6):373–378
4. Stupakov G (2009) Using the beam-echo effect for generation of short-wavelength radiation. *Phys Rev Lett* 102(7):074801
5. Xiang D, Stupakov G (2009) Echo-enabled harmonic generation free electron laser. *Phys Rev Spec Top Accel Beams* 12(3):030702

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Feng, C.

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