Preface

This book arose from my interest in disordered systems. It was known, for some time, that disorder in a one-particle Hamiltonian usually leads to localized states in one-dimensional chains. Anderson had argued that in higher-dimensional systems, there may be regions of localized and extended states, separated by a mobility edge. In 1979 and 1980, it became clear that this Anderson transition could be described in terms of a nonlinear sigma model. Lothar Schäfer and myself reduced the model to one described by interacting matrices by means of the replica trick. Efetov, Larkin, and Khmel'nitskii performed a similar calculation. They, however, started from a description by means of anticommuting components. In 1982 Efetov showed that a formulation without the replica trick was possible using supervectors and supermatrices with equal number of commuting and anticommuting components.

I had the pleasure of giving many lectures and seminars on disordered systems and critical systems, and also on fermionic systems, where Grassmann variables play an essential role. Among them were seminars in the Sonderforschungsbereich (collaborative research center) on *stochastic mathematical models* with mathematicians and physicists and in the Graduiertenkolleg (research training group) on *physical systems with many degrees of freedom* and seminars with Heinz Horner and Christof Wetterich. In particular, I remember a seminar with Günther Dosch on Grassmann variables in statistical mechanics and field theory.

Some of the applications of Grassmann variables are presented in this volume. The book is intended for physicists, who have a basic knowledge of linear algebra and the analysis of commuting variables and of quantum mechanics. It is an introductory book into the field of Grassmann variables and its applications in statistical physics.

The algebra and analysis of Grassmann variables is presented in Part I. The mathematics of these variables is applied to a random matrix model, to path integrals for fermions (in comparison to the path integrals for bosons) and to dimer models and the Ising model in two dimensions.

Supermathematics, that is, the use of commuting and anticommuting variables on an equal footing, is the subject of Part II. Supervectors and supermatrices, which contain both commuting and Grassmann components, are introduced.

In Chaps. 10–14, the basic formulae for such matrices and the generalization of symmetric, real, unitary, and orthogonal matrices to supermatrices are introduced. Chapters 15–17 contain a number of integral theorems and some additional information on supermatrices. In many cases, the invariance of functions under certain groups allows the reduction of the integrals to those where the same number of commuting and anticommuting components is canceled.

In Part III, supersymmetric physical models are considered. Supersymmetry appeared first in particle physics. If this symmetry exists, then bosons and fermions exist with equal masses. So far, they have not been discovered. Thus, either this symmetry does not exist or it is broken. The formal introduction of anticommuting space-time components, however, can also be used in problems of statistical physics and yields certain relations or allows the reduction of a disordered system in d dimensions to a pure system in $d-2$ dimensions. Since supersymmetry connects states with equal energies, it has also found its way into quantum mechanics, where pairs of Hamiltonians, $Q^\dagger Q$ and $QQ^\dagger$, yield the same excitation spectrum. Such models are considered in Chaps. 18–20.

In Chap. 21, the representation of the random matrix model by the nonlinear sigma model and the determination of the density of states and of the level correlation are given. The diffusive model, that is, the tight-binding model with random on-site and hopping matrix elements, is considered in Chap. 22. These models show collective excitations called diffusions and if time-reversal holds, also cooperons. Chapter 23 discusses the mobility edge behavior and gives a short account of the ten symmetry classes of disorder, of two-dimensional disordered models, and of superbosonization.

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