

# Chapter 2

## Mathematical Model and Simulation of an Improved Magnetically Controlled Reactor

Yakun Li, Teng Li, Yonggang Ma and Wei Zhang

**Abstract** The configuration and operation principle of an improved magnetically controlled reactor are introduced in this paper, which has the advantages such as low noise, low loss, and wide range for the overload ability. The state equations are presented according to the corresponding working states. On the basis of simulation, the characteristics of the reactor are analyzed.

**Keywords** Magnetically controlled reactors (MCRs) · Mathematical model · Simulation

### 2.1 Introduction

Magnetically controlled reactors (MCRs), as a kind of shunt reactors, have offered flexible ways regulating the reactive power in the power system. Consequently, MCR causes the widespread concern in recent years and has been applied in many practical fields. The equivalent circuit is a nonlinear one because of the nature of iron core, and it uses DC excitation to control the saturation degree and change AC windings' inductance value [1, 2].

This paper analyzed an improved structure of magnetic-valve controlled reactor (MVCR), it is also belong to MCRs, which can be quickly adjusted [3]. The special design of iron core can reduce the core loss. The improved MCR is with characters of simple structure, easy maintaining, and high stability [4].

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Y. Li (✉) · T. Li

School of Electrical Engineering, Beijing Jiaotong University,  
No.3 Shang Yuan Cun, Hai Dian District, Beijing, China  
e-mail: 12121511@bjtu.edu.cn

Y. Ma

Urumqi Railway Power Supply Department, Urumqi, China

W. Zhang

Beijing Sifang Tengtai Electric Power Technology Co, Beijing, China

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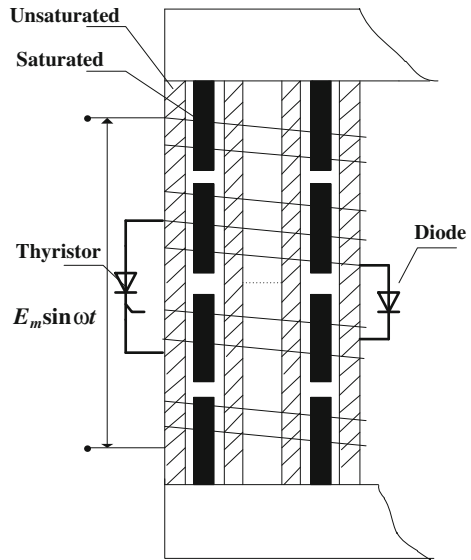
## 2.2 Mathematical Model

The idea of MCR is derived from the magnetic magnifier [5]. By controlling the winding, the DC current is changed; thus, the iron core of the MCR is deeply saturated, and the reactance of the MCR will be changed accordingly. This working principle of the conventional MCR is presented in detailed in literature [6, 7].

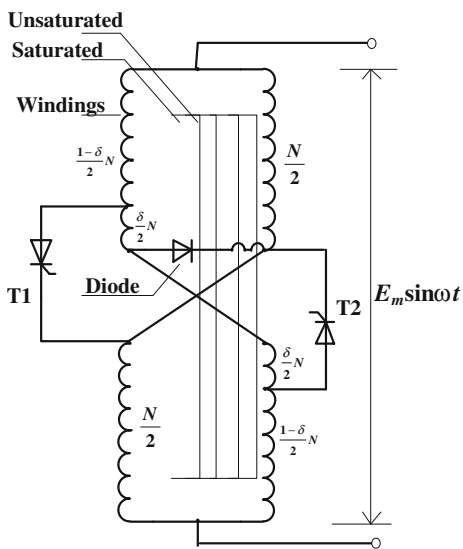
### 2.2.1 Configuration of MCR

The structure of the improved magnetically controlled reactor is shown in Fig. 2.1. The iron core of the MCR is specially designed. The core includes two parts, the unsaturated region and saturated region, and these two regions are paralleled in magnetic circuits. Furthermore, the improved structure does not need to set up separate magnetic shielding device, or to attach magnetic shield structure in metal structure components because of the unique core design. The distribution of silicon steel is different compared to that of the traditional MCR. Sheets belong to the saturated and unsaturated regions are staggered with each other. The purpose of the design is to reduce the core loss since the magnetic flux cannot horizontally pass through the silicon steel sheets. The magnetic flux leakage of the iron core saturated region can be absorbed by the unsaturated region through this design. The overload

**Fig. 2.1** Configuration of core structure and windings



**Fig. 2.2** Principle diagram of the equivalent electric circuit



ability of the reactor will be enhanced up to 150 % compared to traditional reactor. In general, this improved MCR is called “paralleled magnetic circuit and magnetic flux leakage self-shielded” controlled reactor.

In one cycle, thyristors T1 and T2 are generating excitation current when alternatively conduct in the loop. Magnitude of excitation current depends on the thyristor conduction angle, the smaller the angle, the greater the excitation current, and the magnetization intensity of the unsaturated region and the saturated region strengthened at the same time. So by adjusting conduction angle, we can control the added DC excitation current to adjust the reluctance (or the area) of the unsaturated and saturated regions, to change their degree of saturation. In this way, the reactance value can be continuously and rapidly adjusted. By increasing the DC excitation current, the speed of excitation can be improved, and the dynamic performance of controlled reactor becomes better.

The connection of two thyristors to the taps of the windings is different from the conventional reactor, as shown in Fig. 2.2. It makes the structure of circuits' principle changed, so the equation and mathematical model based on traditional literature [1, 2] cannot be directly applied.

### 2.2.2 Electromagnetic Equations of the Reactor

The electromagnetic equations can be summarized as follows from the newly designed reactor circuit topologies. The reactor has five basic operation states in one period (T1 turned on, T2 and D turned off; T1 and D turned on, T2 turned off;

D turned on, T1 and T2 turned off; T2 turned on, T1 and D turned off; T2 and D turned on, T1 turned off). The working state of magnetic circuit is symmetric since the two windings are in parallel, so just three basic states are listed in here and the other two can be derived accordingly. The three basic operation states and the corresponding loop circuit equations are listed in (2.1), (2.3), and (2.5), respectively. Thyristors and diodes are assumed to be ideal switching elements, and the switching transient process can be neglected.

When T1 is conducting, T2 and D are turned off:

$$\left\{ \begin{array}{l} E_m \sin \omega t = \left( \frac{1-\delta}{2} + \frac{1}{2} \right) NA \frac{dB_1}{dt} + \left( \frac{1-\delta}{2} + \frac{1}{2} \right) Ri + \frac{1-\delta}{2} Ri_1 + \frac{1}{2} Ri_2 \\ 0 = \left( \frac{1-\delta}{2} \right) NA \frac{dB_1}{dt} - \frac{1}{2} NA \frac{dB_2}{dt} + \left( \frac{1-\delta}{2} \right) Ri + \left( \frac{1-\delta}{2} + \frac{1}{2} \right) Ri_1 \\ 0 = \left( \frac{1-\delta}{2} \right) NA \frac{dB_1}{dt} - \frac{1}{2} NA \frac{dB_2}{dt} + \frac{1}{2} Ri + \left( \frac{2+\delta}{2} \right) Ri_2 \end{array} \right. \quad (2.1)$$

$$\left\{ \begin{array}{l} F_1 = \left( \frac{1-\delta}{2} + \frac{1}{2} \right) Ni + \left( \frac{1-\delta}{2} \right) Ni_1 + \left( \frac{1-\delta}{2} \right) Ni_2 \\ F_2 = -\frac{1}{2} Ni_1 - \frac{1}{2} Ni_2 \end{array} \right. \quad (2.2)$$

When T1 and D are conducting, T2 is turned off:

$$\left\{ \begin{array}{l} E_m \sin \omega t = \left( \frac{1-\delta}{2} + \frac{1}{2} \right) NA \frac{dB_1}{dt} + \left( \frac{1-\delta}{2} + \frac{1}{2} \right) Ri + \frac{1-\delta}{2} Ri_1 + \frac{1}{2} Ri_3 \\ 0 = \left( \frac{1-\delta}{2} \right) NA \frac{dB_1}{dt} - \frac{1}{2} NA \frac{dB_2}{dt} + \left( \frac{1-\delta}{2} \right) Ri + \left( \frac{1-\delta}{2} + \frac{1}{2} \right) Ri_1 \\ 0 = \left( \frac{1-\delta}{2} \right) NA \frac{dB_1}{dt} - \frac{1}{2} NA \frac{dB_2}{dt} + \frac{1}{2} Ri + \left( \frac{2+\delta}{2} \right) Ri_3 - \frac{\delta}{2} Ri_2 \\ 0 = \frac{\delta}{2} NA \frac{dB_1}{dt} + \frac{\delta}{2} Ri_2 - \frac{\delta}{2} Ri_1 \end{array} \right. \quad (2.3)$$

$$\left\{ \begin{array}{l} F_1 = \left( \frac{1-\delta}{2} + \frac{1}{2} \right) Ni + \left( \frac{1-\delta}{2} \right) Ni_1 - \frac{\delta}{2} Ni_2 + \left( \frac{1-\delta}{2} \right) Ni_3 \\ F_2 = -\frac{1}{2} Ni_1 - \frac{1}{2} Ni_3 \end{array} \right. \quad (2.4)$$

When D is conducting, T1 and T2 are turned off:

$$\begin{cases} E_m \sin \omega t = NA \frac{dB_1}{dt} + Ri + \frac{1}{2} Ri_1 + \frac{1}{2} Ri_2 \\ 0 = \frac{1}{2} NA \frac{dB_1}{dt} - \frac{1}{2} NA \frac{dB_2}{dt} + \frac{1}{2} Ri + Ri_1 \\ 0 = \frac{1}{2} NA \frac{dB_1}{dt} - \frac{1}{2} NA \frac{dB_2}{dt} + \frac{1}{2} Ri + Ri_2 \end{cases} \quad (2.5)$$

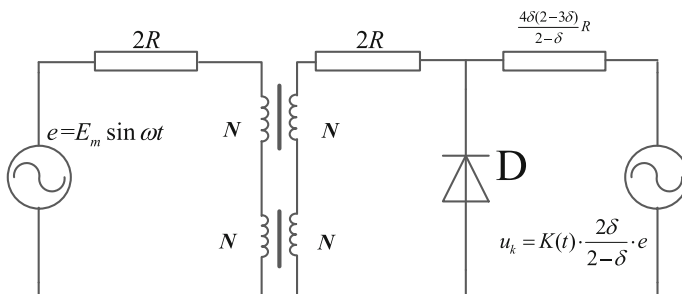
$$\begin{cases} F_1 = Ni + \frac{1}{2} Ni_1 + \frac{1}{2} Ni_2 \\ F_2 = -\frac{1}{2} Ni_1 - \frac{1}{2} Ni_2 \end{cases} \quad (2.6)$$

### 2.2.3 Equivalent Circuit

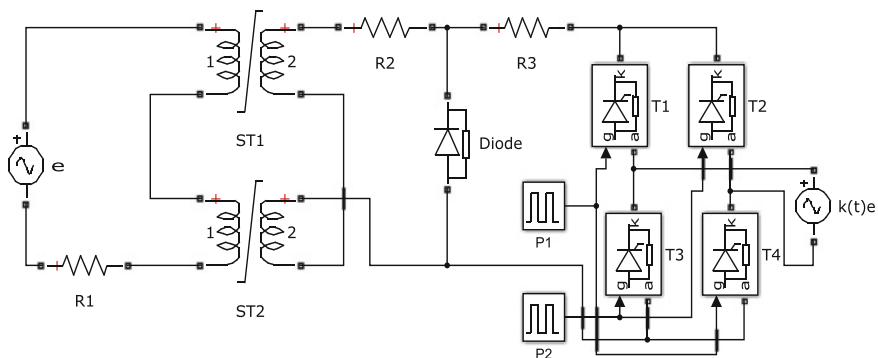
The control loop equation can be derived as in (2.7) from the above equations.

$$\frac{2\delta E_m \sin \omega t}{(2-\delta)} = \left[ 1 + \frac{4\delta - 6\delta^2}{(2-\delta)^2} \right] \frac{2R}{2N} (F_1 - F_2) + NA \left( \frac{dB_1}{dt} - \frac{dB_2}{dt} \right) \quad (2.7)$$

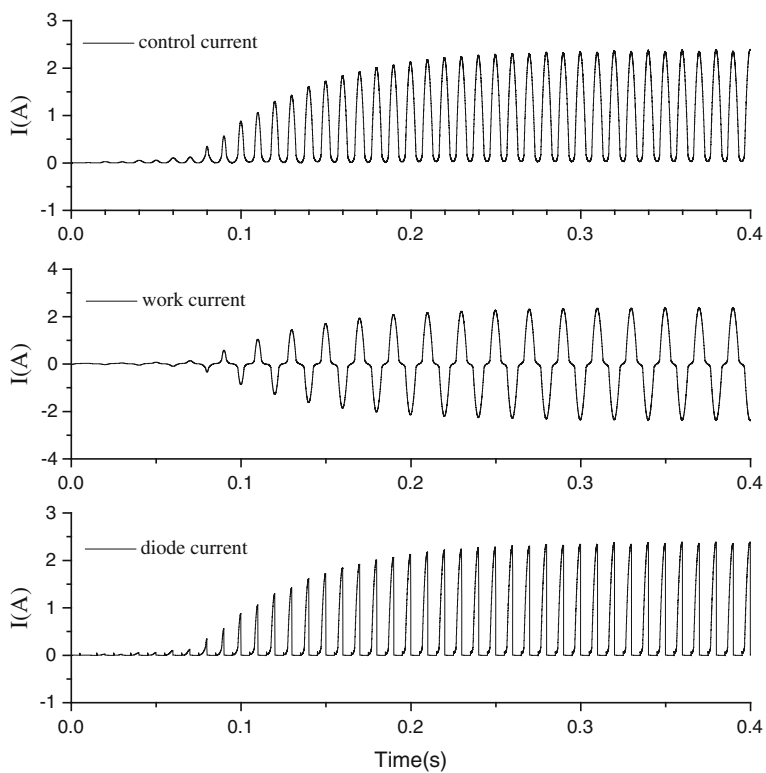
From Eq. (2.7), the equivalent electric circuit is shown in Fig. 2.3. And  $K(t) = 1(0)$  is the state of T1 conducted. The equivalent circuit is almost the same as the conventional MCR. After comparing with the equivalent circuit of the traditional MCR in literature [8], it can be found that just the coefficient of the resistance in the control circuit is different. The coefficient contains  $\delta$ , which is the key parameter to affect the response time.



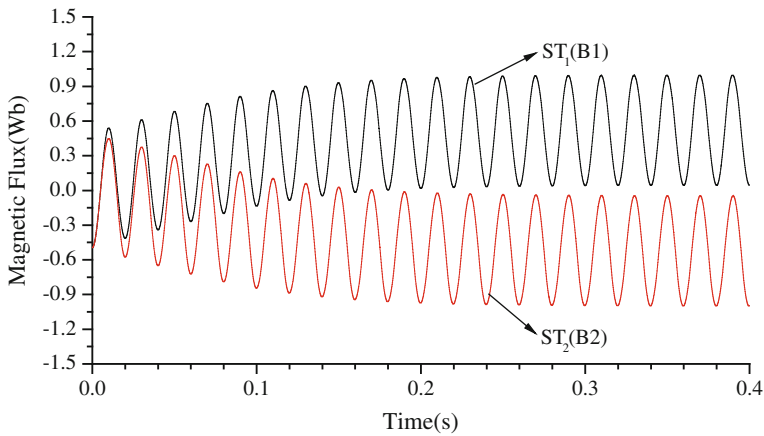
**Fig. 2.3** Equivalent circuit



**Fig. 2.4** Magnetically controlled reactor simulation model in MATLAB/Simulink



**Fig. 2.5** The output current with switching angle of  $90^\circ$



**Fig. 2.6** Magnetic fluxes (B1 and B2) in the iron cores of the MCR

## 2.3 Simulation and Results

The simulation of the circuit in MATLAB/Simulink based on the equivalent electric circuit is shown in Fig. 2.3. In this paper, the single-phase MCR is modeled using two saturation transformer modules in Simulink. The primary windings of the saturation transformers are connected with different polarity, and secondary windings are connected with the same polarity. The DC current controller is composed of a single-phase full-bridge controlled rectifier. As the switching angle of the thyristors is increasing, the DC control current in the control winding decreases and the reactance of the reactor increases. The circuit design is shown in Fig. 2.4.

The output current with the thyristor switching angle of  $90^\circ$  is shown in Fig. 2.5.

The magnetic flux waveform of iron cores is shown in Fig. 2.6. Curve B1 begins to increase and B2 begins to decrease at the same time ( $t = 0.03$  s), and both stop changing when  $t = 0.21$  s. This result explains the working principle of MCR: The AC magnetic flux changes smoothly by regulating the switching angle of the thyristors, and a continuous output current can be obtained.

## 2.4 Conclusion

In this paper, theoretical analysis on the structure of an improved magnetically controlled reactor prototype and mathematical derivation on the equivalent circuit equation is studied. Its mathematic modeling is established. Based on the modeling, the equivalent circuit is derived. It generates less core loss and has auto-supplied excitation DC system and has advantages such as low noise and wide range for the

overload ability. In the future, more work will be focused on the improvement of the reactor response speed.

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