Introduction

Modern mathematical literature is characterized by a far-reaching distinction, which all of you must have noticed: The interests and ways of thinking of the theoreticians differ considerably from the methods that are effectively used in mathematical applications. Not only does the scientific training of the individual suffer from this discrepancy, but also the standing of this science itself. It seems extremely important to try to counteract the grievances arising from this situation. The lecture course that I start here is aimed at providing a contribution to this issue, in that it tries to establish a clear relationship – so to say from an epistemological point of view – between the different kinds of mathematical questions, as they arise in some occasions according to their very nature. You must, on the one hand, try to understand the interests of modern theoreticians, but on the other hand you must form an opinion on which parts of mathematical speculation have a direct meaning in applications. I do not doubt that you will find the resulting contraposition of different points of view interesting and beneficial. My hope is that this lecture course will eventually even result in you, on your part, contributing in the future to re-shape the considerably single-sided development of our science towards an all-round, harmonious one!

However, the program that I am setting up is too broad to be developed in all directions in one semester. Therefore I will bring to the foreground only *one* branch of mathematics, namely *geometry*. The practical part of geometry includes geometrical drawing and measurement, which is confronted – from a theoretical point of view – with the abstract treatment of geometrical problems started by the Greeks. I will treat the questions relevant for both sides in analytic form. This is actually not necessary; one could also proceed without leaving the field of geometry. However, the development of the questions that I have to discuss has, until now, mainly taken place in connection with *analysis*: We will consider in particular the development and meaning of differential and integral calculus.

I cannot give any textbook as reference. I will give single references case-
[2] by-case and for the rest I prefer to refer to the *Enzyklopädie der mathematischen Wissenschaften*, which aims to present all the mathematics of the 19th century and to arrange all its publications.

F. Klein, *Elementary Mathematics from a Higher Standpoint*,
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If one wants to extend the same programme to other branches of mathematics, this would particularly concern the issues of the mathematical explanation of nature (mechanics, etc.). A series of pertinent ideas are contained in the Report elaborated by our Faculty of Philosophy in the year 1901 on the occasion of the celebration of the Beneke-Stiftung[8]. For the rest, I will refer to a paper of my own, which I published in the *Erlanger Berichten* in 1873 with the title "Über den allgemeinen Funktionsbegriff und dessen Darstellung durch eine willkürliche Kurve"[9]. I can also refer to the sixth lecture of my Evanston Colloquium "Lectures on Mathematics", New York 1894[10], translated into French by Léonce Laugel, Paris 1898. The title of that lecture is "On the mathematical character of space intuition and the relation of pure mathematics to the applied sciences".

Now I immediately start with the first part of my lecture course:

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8 [Printed in *F. Klein*: Gesammelte mathematische Abhandlungen, Vol. II, pp. 241–246]. Translator's note: *Felix Klein*, Auszug aus dem Gutachten der Göttinger philosophischen Fakultät betreffend die Beneke-Preisaufgabe für 1901, *Mathematische Annalen*, Vol. 55 (1901, 143–146). *Friedrich Eduard Beneke* (1798–1854) was a philosopher and psychologist at Berlin and Göttingen universities. He established a price award which became regulated by the Göttingen Königliche Gesellschaft der Wissenschaften in 1867, demanding the Philosophical Faculty of Göttingen University to publish yearly a subject for a price essay. The subject of 1898 for 1901 was to question the principles of continuity and the use of unrestrictedly differentiable functions for the study of the natural phenomena and to unravel in particular preconditions having remained implicit, in connection with limitations of sensory perception.

9 Reprinted in *Mathematische Annalen*, Vol. 22 (1883) [and in *F. Klein*, *ibid.*, pp. 214–224]

10 [New edition, obtained from the American Mathematical Society. New York 1911. The mentioned talk can be found also in *F. Klein, ibid.,* pp. 225–231]

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