Preface to the 2016 Edition

The Notion of Elementary Mathematics

This is the first volume of the three-volume-series of Felix Klein's "Elementarmathematik vom höheren Standpunkte aus", the first two volumes now in revised and completed editions and the third volume the first time in English translation. This translation is based on the last German edition of volume I, the fourth of 1933; the English translation of 1931, which was used for this new revised version, had the third edition of 1924 as its master copy. The third and the fourth German edition are basically identical, except the three paragraphs on pp. 296–297, which replace pp. 297–303 of the 3rd edition.

The volumes are lectures notes of courses, which Klein offered often to future secondary school mathematics teachers at Göttingen university, and published between 1902 and 1908, proposing and realizing a new form of teacher training, which became a model for many mathematicians and which remained valid and effective until today. Jeremy Kilpatrick emphasized the importance of these volumes thus:

> In print for a century, the volumes of Klein's textbook have been used in countless courses for prospective and practicing teachers. They provide excellent early examples of what today is termed *mathematical knowledge for teaching*. Klein's courses for teachers were part of his reform efforts to improve secondary mathematics by improving the preparation of teachers. Despite the many setbacks he encountered, no mathematician has had a more profound influence on mathematics education as a field of scholarship and practice (Kilpatrick 2008, p. 27).[1]

It was Kilpatrick, too, who as the first called attention to the misleading translation of the term "höher" in that English translation of the 1930's. While all the

[1] I am quoting from Kilpatrick's lecture at ICME 11 in Mexico. Unfortunately, the Proceedings of this Congress were never published. The ICMI Executive Committee decided therefore, to make the lectures delivered there accessible online at the ICMI site.

other translations had given the "höher" correctly, it became "advanced" instead of "higher" – and was received thus over so many decades as the English version:

> When it came time for the American translators of Klein's *Elementarmathematik* to render the title in English, they chose to translate *vom höheren Standpunkte aus* as *from an advanced standpoint*. The term *higher* is not only a more literal translation of *höheren* than *advanced* is, but it also captures better the image Klein had for his work. *Advanced* can mean *higher,* but its connotation is more like 'more developed' or 'further along in space and time'. Klein wanted to emphasize that his courses would give prospective teachers a better, more panoramic view of the landscape of mathematics. As noted above, he wanted those teachers to 'stand above' their subject. (ibid., p. 40)

In fact, the term "advanced" implies a fundamental misunderstanding of Klein's notion of *elementary* and of *Elementarmathematik*. The term "advanced" implies that elementary mathematics is somewhat delayed, lagging behind, of another nature. It means exactly the contrary of what Klein was intending. By contrasting two poles, "elementary" versus "advanced", one would admit just that discontinuity between school mathematics and academic mathematics, which Klein wanted to eliminate.

For Klein, there was no separation between an elementary mathematics and an academic mathematics. His conception for training teachers in higher education departed from a holistic vision of mathematics: mathematics, steadily developing and reforming itself within this process, leading to ever new restructured elements, provides therefore new accesses to the elements. There is a widespread understanding of the term "elementary", meaning it something "simple" and not loaded with conceptual dimension – even somehow approaching "trivial". Connected, in contrast, with the notion of element, "elementary" means for Klein to unravel the fundamental conception. What is at stake, hence, is the notion of *elements*.

Beyond mere factual information, with his lecture notes Klein leads the students to gain a more comprehensive and methodological point of view on school mathematics. The three volumes thus enable us to understand Klein's far-reaching conception of *elementarisation*, of the "elementary from a higher standpoint", in its implementation for school mathematics: The elements are understood as the fundamental concepts of mathematics, related to the whole of mathematics – according to its restructured architecture.

This notion of elements corresponds neatly to the first reflections on the nature of elements undertaken in the wake of Enlightenment how to make knowledge teachable and how to disseminate knowledge thus in society to ensure its general understanding. One has to name in particular Jean le Rond d'Alembert (1717–1783) who conceptualized in a profound manner what he called to "elementarise" the sciences. It was his seminal and extensive entry "élémens des sciences" in the *Encyclopédie*, the key work of the Enlightenment, where he gave this analysis and reflection how to elementarise a science, that is how to connect the elements with the whole of that science. This procedure is to be able to identify the elements of a science, or in other words, have rebuilt it in a new coherent way all parts of a science that may have accumulated independently and not methodically:

> On appelle en général élémens d'un tout, les parties *primitives & originaires* dont on peut supposer que ce tout est formé (d'Alembert 1755, 491 l).

In this sense, there is no qualitative difference between the elementary parts and the higher parts. The elements are considered as the "germs" of the higher parts:

> Ces propositions réunies en un corps, formeront, à proprement parler, les élémens de la science, puisque ces *élémens* seront comme un germe qu'il suffiroit de développer pour connoitre les objets de la science fort en détail (d'Alembert 1755, 491 r).

An extensive part of the entry is dedicated to the reflection on elementary books, such as schoolbooks, which are essential, on the one hand, to disseminate the sciences and, second, to make progress in the sciences, that is, to obtain new truths. In his reflection on elementary books, d'Alembert emphasised another aspect of great importance regarding the relationship between the elementary and the higher: he underlined that the key issue for the composition of good elementary books consists in investigating the "metaphysics" of propositions – or in terms of today: the epistemology of science.

In fact, Klein's work can be understood exactly as providing such an epistemological, or methodological access to mathematics. It was not to provide factual knowledge:

> I shall by no means address myself to beginners, but I shall take for granted that you are all acquainted with the main features of the most important disciplines of mathematics (Klein, this volume, p. [1] et seq.).

Whereas he outlined as his goal:

> And it is precisely in such summarising lecture courses as I am about to deliver to you that I see one of the most important tools (ibid., p. [1]).

Indeed, Klein explicitly exposed the epistemological aspect of his work: explaining the connections, the connections between subdisciplines, which normally are treated separately and pointing out the links of particular mathematical issues and questions with a synthetic view of the whole of mathematics. Thus, future teachers would achieve to deepen their understanding of the basic concepts of mathematics and appreciate the nature of mathematical concepts:

> My task will always be to show you the *mutual connection between problems in the various disciplines*, these connections use not to be sufficiently considered in the specialised lecture courses, and I want more especially to emphasize the relation of these problems to those of school mathematics. In this way I hope to make it easier for you to acquire that ability which I look upon as the real goal of your academic study: the ability to draw (in ample

> measure) from the great body of knowledge taught to you here vivid stimuli for your teaching (ibid., p. [2]).

There is a decisive difference between d'Alembert's and Klein's notion of elementarisation. D'Alembert's notion basically was not a historical one; he did not reflect the effect of scientific progress on the elements. But this was exactly Klein's notion. He emphasised:

> The normal process of development [...] of a science is the following: higher and more complicated parts become gradually more elementary, due to the increase in the capacity to understand the concepts and to the simplification of their exposition ("law of historical shifting"). It constitutes the task of the school to verify, in view of the requirements of general education, whether the introduction of elementarised concepts into the syllabus is necessary or not (Klein & Schimmack 1907, p. 90).

The historical evolution of mathematics entails therefore a process od restructuration of mathematics where new theories, which at first might have ranged somewhat isolated and not well integrated, turn well connected to other branches of mathematics and effect a new architecture of mathematics, based on re-conceived elements, thus on a new set of elementarised concepts.

Set theory was a case for Klein where this theoretical development was too fresh and even not yet accomplished and even more far from having matured in a manner to having induced an intra-disciplinary process of integration and restructuration. The concepts of set theory did not (yet) provide new elements for mathematics – therefore Klein's polemic against Friedrich Meyer's schoolbook of 1885 who's intention had been, in fact, to use set theory as new elements for teaching arithmetic and algebra (see the note on p. [289]). In Klein's times, mathematics had not achieved the level of the architecture established by Bourbaki – and hence not of "modern math".

This volume I is devoted to what Klein calls the three big "A's": arithmetic, algebra and analysis. They are presented and discussed always together with a dimension of geometric interpretation and visualisation – given his epistemological viewpoint of mathematics being based in space intuition. A particularly revealing example for elementarisation is his chapter on the transcendence of e and π, where he succeeds in giving a concise, well accessible proof for the transcendence of these two numbers.

The Use of History of Mathematics

A particularly characteristic feature of Klein's lecture courses and of his approach of "Elementary Mathematics from a Higher Standpoint" is the important role attributed to the history of mathematics. Klein explains various times his conviction that exposing key features of the historical development of concepts will support his methodological orientation to lead to a deeper understanding of the fundaments of mathematics. Klein thus reveals himself as a probably first and staunch supporter of the use of history for the teaching of mathematics – what thus became since 1976 the international movement, well known as HPM.[2]

Yet, one has to admit that at his time there was only one conception available for the use of mathematics history in teaching: the so-called biogenetic law, affirming a recapitulation of the historical development by the individual. There was a widespread conviction of the validity of this "law" for biology; and its applicability for education belonged to the dominant mentalities of the epoch, though with some more reservations. As quickly as the biogenetic law had spread after Haeckel's propagation, as quickly it disappeared from public discourses in education, in the inter-war period, and seemingly completely. It was mentioned for the first time, after this falling into oblivion, in 1962, in the memorandum of 65 mathematicians against "new math". What is the most astonishing, however, is the revival of this conception – reputed to be dead, and this exactly with more work done on the use of the history of mathematics in teaching: since about the 1990s. It seems that there are still no other well-developed or known conceptions how to relate history with teaching (see Schubring 2006). Therefore, one cannot blame Klein to have referred often to this conception.

A further issue in this regard is which historiography of mathematics is adapted to be used for the context of teaching. Felix Klein was not only highly interested in the history of mathematics; he promoted strongly research into the history. For instance, he initiated research into the manuscripts of Gauß and he organised the publication of Collected Works of several mathematicians. Thus, he was completely aware of the results of historical research into the history of mathematics as achieved until his times. And this knowledge was the basis for his historical annotations and affirmations in the three volumes. Clearly, as historiographical research has progressed since then, not all his information is today still the state of the art.

Why a Revised Translation of Volume I?

This edition is the first complete English translation of Klein's first volume of the *Elementarmathematik*. In fact, the original volume contains, at its end, two appendices, of 14 pages: one on the efforts to reform mathematics teaching, while the other gives complementary information on mathematical and pedagogical literature – thus revealing sections for complementing to understand Klein's views on teaching mathematics. The translators were aware of these two appendices: they are mentioned in their version of 1931, in the footnote 1 on p. 1, added by themselves. However, they omitted these two sections without any comment or justification.

[2] The International Study Group on the relations between the HISTORY and PEDAGOGY of MATHEMATICS, founded in 1976, an affiliated Group of ICMI.

Moreover, the misleading translation of the title for the entire series had already to be outlined. One is therefore led to ask who were the two American translators. Their biography shows them to have been well-qualified mathematicians. Both translators, Earle Raymond Hedrick and Charles Albert Noble studied in Göttingen, then the internationally leading centre of mathematics, with Klein and with Hilbert. Both obtained their PhD as doctoral students of Hilbert, in 1901. After their return to the States, they played an important role in building up the country's mathematical institutions. Hedrick, first mathematics professor at the University of Missouri, was called in 1924 to the University of California at Los Angeles. He served as first President of the Mathematical Association of America in 1916, and later as President of the American Mathematical Society (1929–1930). Noble, at first a mathematics teacher at colleges, became a mathematics professor at the University of California at Berkeley (Parshall & Rowe 1994, p. 410 and 440 et seq.). One is therefore struck to remark their translation being marred by numerous terminological and textual faults.

What catches the eye immediately, besides the "advanced" issue, is their translation for *Mengenlehre*. Although *Mengenlehre* was a major issue of discussion during the time of their studies in Göttingen, they are not familiar with the English term. In the first part, they use persistently "theory of point sets". In the special section on set theory, they give as title "Theory of assemblages" and use here "assemblage" for set, but not consistently – they also use "aggregate". One might infer that set theory had not yet really arrived in the States by the 1930s.

Actually, the two translators have qualified, in their preface, their work as "a rather free translation". That would be admissible, but this is not the case. At too many places, the text gives not the intended meaning, but rather erroneous and misleading translations, in particular with regard to mathematics, not only with regard to the general style.

Surely, a problem might have arisen by the character of Klein's text as lecture notes – taken during his courses by students. Thus, the text represents oral teaching, and not a text intentionally composed for printing. The lecture notes are therefore written quite often in a rather colloquial style – and the specifity of translating consists in rendering well the meaning of such idioms. Not being aware of this textual style is already a first reason for misleading translations. Surely, both translators did not acquire such an intimate knowledge of the German language during their stay in Germany. Noble, for instance, gave his presentations in Klein's seminar in English (Parshall & Rowe 1994, p. 257).

But even Klein's famous statement about the double discontinuity between secondary schools and university studies is rendered in a misleading way: while Klein is complaining about this discontinuity as a persistent problem, including his own times, the translators transformed this into past tense – as if the problem had already been overcome! This introductory passage of the book, on p. [1], contains no colloquial terms at all.

A problem pervading both volumes I and II consists in not being aware of one of the most basic concepts of Klein: of *Anschauung* – for him the fundament of conceptual development of mathematics. And, as it is well know, *Anschauung* presents since the times of Immanuel Kant, a key notion of German philosophy. Hedrick and Noble did not attempt to take the complexity of this notion into account. Thus, *Raum-Anschauung* becomes simply "space perception", and *anschaulich* is generally translated as "graphical", thus entirely misleading.

Of the countless cases of translation errors, I should just mention a few types:

- *missing care of mathematical statements*. On p. [55], discussing the polygons, which can be constructed with ruler and compass, the correct translation says: "It was known to the ancients, too, that this construction was possible for the number of vertices $n = 2^h, 3, 5$ (h an arbitrary integer), and likewise for the composite values $n = 2^h \cdot 3, n = 2^h \cdot 5, n = 2^h \cdot 3 \cdot 5$."
  Their text says, however: "...for the numbers $n = 2^h, 3, 5$ (h an arbitrary integer), and likewise for the composite values $n = 2^h \cdot 3 \cdot 5$."
- *changing mathematical statements in Klein's text*. In the section on transformation of fractions into decimal numbers, Klein had dealt with the "fraction $1/p$, where p is a prime number different from 2 and 5" and wanted to prove "*that* $1/p$ *is equal to an infinite periodic decimal fraction*", using the congruence: $10^\delta \equiv 1 \pmod p$, with δ being the number of places in the period. For his proof, Klein used Fermat's little theorem:

  > which states that for every prime number p and for every integer a not divisible by p:
  > $a^{p-1} \equiv 1 \pmod p$.

  Hedrick and Noble apparently thought Klein had confused a with 10 and replaced Klein's text by a "proper" Fermat theorem:

  > which states that for every prime number p except 2 and 5:
  > $10^{p-1} \equiv 1 \pmod p$.

- *inconsistent mathematical terminology*. When Riemann surfaces are discussed, they use in many parts as term "leaf/leaves/leaved". There is one section, however, where they use "sheet/sheets/sheeted".
- *wrong terms*. Discussing the pendulum law, Klein said on p. [202]: "Man geht hier aus von dem *konischen Pendel,* das ist ein *räumliches* Pendel". Thus "konisch" is clearly a mathematical term, more exactly a geometrical one. Yet, their translation was: "One begins with a *canonical* pendulum, i.e. a pendulum in space". The religious term "canonical" is even repeated on the next page.

One gets the impression that some parts were given to students for translation, and that the resulting various parts were not coordinated and checked. And there was no critical reading by a native German.

Information About this Edition

Klein participated, together with Fritz Seyfarth, in the whole project of re-editing the three volumes on *Elementary mathematics from a higher standpoint*. He succeeded, in fact, to finish the third edition of the first two volumes.

In the text the reader will find, in square brackets and in bold, *the page numbering of the original edition*. Cross references in notes and in the text *refer to this numbering*, as well as *the name index and the subject index* (that is, the original text has not been changed to this respect). Moreover, I left, as in the German edition, the comma to separate the integer part from the fractional part of a decimal number.

In the present translation I have added, when possible, the first names of the persons mentioned. In the German edition, as it was customary at that time, the first names were indicated only with the initials. The bibliographic references in the notes have also been completed, when needed.

In the English version of 1932, Hedrick and Noble had sometimes added in the notes references for recent pertinent American publications; these have been maintained; their notes are marked with asterisks. Several additional notes have been introduced; they are marked by square brackets.

I am thanking Leo Rogers for his careful re-reading of the book, and the many colleagues whom I asked advice, in particular Geoffrey Howson.

These three volumes will be produced by the same file for the print version and for the parallel ebook version. Since the present technology for ebooks does not allow the wrapping of figures within the text as it did in the traditional practice of printing, we have to live with this restriction for the type face.

We are grateful to Dover Publication to have authorised the use their book "Elementary Mathematics from an Advanced Standpoint", translated by E. R. Hedrick and C. A. Noble, for a revised new edition.

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Gert Schubring

Preface to the First Edition

The new lithographed volume which I herewith offer to the mathematical public, and especially to the teachers of mathematics in our secondary schools, is to be looked upon as a first continuation of the lectures *Über den mathematischen Unterricht an den höheren Schulen**, in particular, of those on *Die Organisation des mathematischen Unterrichts*** by *Schimmack* and me, which were published last year by Teubner. At that time our concern was with the different ways in which the problem of instruction can be presented to the mathematician. At present my concern is with developments in the subject matter of instruction. I shall endeavor to put before the teacher, as well as the maturing student, from the view-point of modern science, but in a manner as simple, stimulating, and convincing as possible, both the content and the foundations of the topics of instruction, with due regard for the current methods of teaching. I shall not follow a systematically ordered presentation, as do, for example, Weber and Wellstein, but I shall allow myself free excursions as the changing stimulus of surroundings may lead me to do in the course of the actual lectures.

The program thus indicated, which for the present is to be carried out only for the fields of *Arithmetic*, *Algebra*, and *Analysis*, was indicated in the preface to Klein-Schimmack (April 1907). I had hoped then that Mr. Schimmack, in spite of many obstacles, would still find the time to put my lectures into form suitable for printing. But I myself, in a way, prevented his doing this by continuously claiming his time for work in another direction upon pedagogical questions that interested us both. It soon became clear that the original plan could not be carried out, particularly if the work was to be finished in a short time, which seemed desirable if it was to have any real influence upon those problems of instruction which are just now in the foreground. As in previous years, then, I had recourse to the more convenient method of *lithographing* my lectures, especially since my present assistant, Dr. Ernst Hellinger, showed himself especially well qualified for this work. One should not underestimate the service which Dr. Hellinger rendered. For it is a far

* On the teaching of mathematics in the secondary schools.
** The organization of mathematical instruction.

cry from the spoken word of the teacher, influenced as it is by accidental conditions, to the subsequently polished and readable record.

In precision of statement and in uniformity of explanations, the lecturer stops short of what we are accustomed to consider necessary for a printed publication.

I hesitate to commit myself to still further publications on the teaching of mathematics, at least for the field of *geometry*. I prefer to close with the wish that the present lithographed volume may prove useful by inducing many of the teachers of our higher schools to renewed use of independent thought in determining the best way of presenting the material of instruction. This book is designed solely as such a mental spur, not as a detailed handbook. The preparation of the latter I leave to those actively engaged in the schools. It is an error to assume, as some appear to have done, that my activity has ever had any other purpose. In particular, the *Lehrplan der Unterrichtskommission der Gesellschaft Deutscher Naturforscher und Ärzte****
(the so-called "Meraner" *Lehrplan*) is not mine, but was prepared, merely with my cooperation, by distinguished representatives of school mathematics.

Finally, with regard to the method of presentation in what follows, it will suffice if I say that I have endeavored here, as always, to combine geometric intuition with the precision of arithmetic formulas, and that it has given me especial pleasure to follow the historical development of the various theories in order to understand the striking differences in methods of presentation which parallel each other in the instruction of today.

Göttingen, June, 1908

Klein.

*** Curriculum prepared by the commission on instruction of the Society of German Natural Scientists and Physicians.

Preface to the Third Edition

After the firm of Julius Springer had completed so creditably the publication of my collected scientific works, it offered, at the suggestion of Professor Courant, to bring out in book form those of my lecture courses which, from 1890 on, had appeared in lithographed form and which were out of print except for a small reserve stock.

These volumes, whose distribution had been taken over by Teubner, during the last decades were, in the main, the manuscript notes of my various assistants. It was clear to me, at the outset, that I could not undertake a new revision of them without again seeking the help of younger men. In fact I long ago expressed the belief that, beyond a certain age, one ought not publish independently. One is still qualified, perhaps, to direct in general the preparation of an edition, but is not able to put the details into the proper order and to take into proper account recent advances in the literature. Consequently I accepted the offer of Springer only after I was assured that liberal help in this respect would be provided.

These lithographed volumes of lectures fall into two series. The older ones are of special lectures which I gave from time to time, and were prepared solely in order that the students of the following semester might have at hand, the material which I had already treated and upon which I proposed to base further work. These are the volumes on *Non-Euclidean Geometry*, *Higher Geometry*, *Hyper geometric Functions, Linear Differential Equations, Riemann Surfaces*, and *Number Theory*. In contrast to these, I have published several lithographed volumes of lectures which were intended, from the first, for a larger circle of readers. These are:

a) The volume on *Applications of Differential and Integral Calculus to Geometry*, which was worked up from his manuscript notes by C. H. Müller. This was designed to bridge the gap between the needs of applied mathematics and the more recent investigations of pure mathematicians.

b) and c) Two volumes on *Elementary Mathematics from a Higher Standpoint*, prepared from his manuscript notes by Ernst Hellinger. These two were to bring to the attention of secondary school teachers of mathematics and science the significance for their professional work of their academic studies, especially their studies in pure mathematics.

A thoroughgoing revision of the volumes of the second series seemed unnecessary. A smoothing out, in places, together with the addition of supplementary notes, was thought sufficient. With their publication therefore, the initial step is taken. Volumes b), c), a) (in this order) will appear as Parts I, II, III of a single publication bearing the title *Elementary Mathematics from a Higher Standpoint*. The combining, in this way, of volume a) with volumes b) and c) will meet the approval of all who appreciate the growing significances of applied mathematics for modern school teaching.

Meantime the revision of the volumes of the first series has begun, starting with the volume on *Non-Euclidean Geometry*. But a more drastic recasting of the material will be necessary here if the book is to be a well-rounded presentation, and is to take account of the recent advances of science. So much as to the general plan. Now a few words as to the first part of the *Elementary Mathematics*.

I have reprinted the preface to the 1908 edition of b) because it shows most clearly how the volume came into existence[3]. The second edition (1911), also lithographed, contained no essential changes, and the minor notes which were appended to it are now incorporated into the text without special mention. The present edition retains[4], in the main, the text of the first edition, including such peculiarities as were incident to the time of its origin. Otherwise it would have been necessary to change the entire articulation, with a loss of homogeneity. But during the sixteen years which have elapsed since the first publication, science has advanced, and great changes have taken place in our school system, changes which are still in progress. This fact is provided for in the appendices which have been prepared, in collaboration with me, by Dr. Seyfarth (Studienrat at the local Oberrealschule). Dr. Seyfarth also made the necessary stylistic changes in the text, and has looked after the printing, including the illustrations, so that I feel sincerely grateful to him. My former co-workers, Messrs. Hellinger and Vermeil, as well as Mr. Alwin Walther of Göttingen, have made many useful suggestions during the proof reading. In particular, I am indebted to Messrs. Vermeil and C. Billig for preparing the list of names and the index. The publisher, Julius Springer has again given notable evidence of his readiness to print mathematical works in the face of great difficulties.

Göttingen, Easter, 1924

Klein.

[3] My co-worker, R. Schimmack, who is mentioned there, died in 1912 at the age of thirty-one years, from a heart attack with which he was seized suddenly, as he sat at his desk.
[4] New comments are placed in brackets.

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