

Aim and Form of this Lecture Course

Gentlemen! The lecture course, which I now begin, will be an immediate continuation of, and a supplement to, my course of last winter.² My purpose now, as it was then, is to summarise all the mathematics that you studied during your student years, insofar as this could be of interest for the future teacher, and, in particular, to show its importance for the practice of school teaching. I carried out this plan, during the winter semester, for *Arithmetic*, *Algebra*, and *Analysis*. During the current semester, attention will be given to *geometry*, which was then left aside. In this lecture course, comprehension of our considerations will be independent of knowledge of the preceding lecture course. Moreover, I shall give the whole a somewhat different tone: In the foreground I shall place, let me say, the *encyclopaedic approach* – you will be offered a *survey of the entire field of geometry* into which you can arrange, as into a rigid frame, all the separate items of knowledge which you have acquired in the course of your study, in order to have them at hand when occasion to apply them arises. Only afterward shall that *interest in mathematics teaching* appear by itself, which was always my emphasis last winter.

I should like to refer to a *vacation course* for teachers of mathematics and physics, which was given here in Göttingen during the Easter vacation in 1908. In it I gave an account of my winter lecture course. In connection with this, and also with the talk by Professor *Otto Behrendsen* of the local Gymnasium, there arose an interesting and stimulating discussion concerning the reorganisation of teaching arithmetic, algebra, and analysis, and more particularly about the introduction of differential and integral calculus into the schools.³ The participants showed an extremely gratifying interest in these questions and, in general, in our efforts to bring the university into living touch with the schools. I hope that my present lecture [2] course also may exert an influence in this direction. May they contribute their part toward the elimination of the old complaint, which we have had to hear continu-

² [Appeared as Volume I of this series of lecture notes on *Elementary Mathematics from a Higher Standpoint*, Berlin, 1924, 3rd edition. The quotation “Part I” refers to the third edition.]

³ See the report by Rudolf Schimmack, *Ueber die Gestaltung des mathematischen Unterrichts im Sinne der neueren Reformideen*, *Zeitschrift für mathematischen und naturwissenschaftlichen Unterricht*, vol. 39 (1908), pp. 513–527, (also printed separately, Leipzig, 1908).

ally – and often justly – from the schools: higher education provides, indeed, much of a special nature, but it leaves the beginning teacher entirely without orientation as to many important general things which he could really use later.

Concerning now the *topics of this lecture course*, let me say that, as in the preceding course, I shall now and then have to presuppose knowledge of important theorems from all of the fields of mathematics, which you have studied, in order to lay emphasis upon a *general survey of the whole*. To be sure, I shall always try to assist your memory by brief statements, so that you can easily orient yourself in the literature. On the other hand, I shall draw attention, more than is usually done, and as I did in Part I, to the *historical development of the science*, to the accomplishments of its great pioneers. I hope, by discussions of this sort, to further, as I like to say, your *general mathematical culture*: alongside of knowledge of details, as these are supplied by the special lecture courses, there should be a grasp of subject-matter and of historical contexts.

The Efforts for “Fusion”

Allow me to make a last general remark, in order to avoid a misunderstanding, which might arise from the nominal separation of this “geometric” part of my lectures from the first arithmetic part. In spite of this separation, I advocate here, as always in such general lecture courses, a tendency which I like best to designate by the catchphrase “*fusion of arithmetic and geometry*” – meaning by arithmetic, as is usual in the schools, the field which includes not merely the theory of integers, but also the whole of algebra and analysis. Some are inclined, especially in Italy, to use the word “*fusion*” as a catchphrase for efforts, which are restricted to geometry. In fact, it has long been the custom in secondary as well as in higher education, first to study geometry of the plane and then, entirely separated from it, the geometry of space. On this account, space geometry is unfortunately often slighted, and the noble faculty of space intuition, which we possess originally, is stunted. In contrast to this, the “fusionists” wish to treat the plane and space together, in order not to restrict our thinking artificially to two dimensions. This endeavour also meets my approval, but I am thinking, at the same time, of a still more far-reaching fusion. Last semester I endeavoured always to enliven the abstract discussions of arithmetic, algebra, and analysis by means of figures and graphic methods, which

[3] bring the things nearer to the individual and often only thus succeed in making him understand, for the first time, why he should be interested in them. Similarly, I shall now, from the very beginning, accompany space intuition, which, of course, will hold first place, with analytic formulas, which facilitate in the highest degree the precise formulation of geometric facts.

You will most easily see what I am meaning when I turn now to our subject; at first a series of simple geometric fundamental forms will be considered.

Elementary Mathematics from a Higher Standpoint

Volume II: Geometry

Klein, F.

2016, XVI, 315 p. 157 illus., Softcover

ISBN: 978-3-662-49443-1