

Preface to the 2016 Edition

In general, for the preface for this volume II, I can refer to the preface for volume I – especially regarding the necessary correction of „advanced“ to „higher“, regarding the notion of *elementary mathematics* and of *elementarisation*, and regarding the need for a revised translation.

I should like to highlight the special features of this volume. While the first volume is dedicated to the analytic side of mathematics, this volume complements it by exposing geometry. But the second volume is complementary to the first one in two more respects. It is characteristic for the first volume that Klein – in treating arithmetic, algebra, and analysis – had always emphasised a geometric approach to the concepts: to provide a geometric interpretation and to reveal the key function of *Anschauung* in developing and in understanding the analytic concepts. Here, for geometry, Klein carefully elaborates the analytic side of the geometric concepts. His major aim is to show the ultimate unity of mathematics.

The second complimentary function is Klein's masterful manner of elaborating the process of elementarisation for the whole of geometry. In fact, it was his key scientific achievement to have changed the character of geometry: until his times, there existed side by side a number of – one might say – different geometries, having been established by the one or the other mathematician, for some specific objective and continuing in a rather dispersed manner and without seeking mutual relations or developments. Klein's *Erlanger Programm* of 1872 is emblematic for the radical change of this situation, for having succeeded in a new, "higher" unity of geometry. All the enormous variety of geometrical theories, approaches and subdisciplines had become unified – thanks to rebuilt foundations and, consequently, new "elements". Particularly illuminating is to see how the development of non-Euclidean geometries became a part of the processes leading to this unified architecture. As Klein has put it, based on Cayley: "Projective geometry is all geometry". Evidently, this elementarisation provided an exemplary methodology for realising a form of mathematics teacher training, which provided future teachers with a proper standpoint for their action later in their profession.

The structure of the second volume is different from the first one. While issues of teaching were there always integrated into the various conceptual topics, Klein

decided to give the conceptual exposition as a coherent presentation and to discuss questions of teaching in a separate chapter. In this manner, Klein was able to realise this illuminating way of revealing the essential unity of geometry and its restructured elements. Evidently, this last chapter on teaching was very dear to Klein and an integral part of his conception. It remains therefore absolutely incomprehensible why the two American translators, Earle Raymond Hedrick and Charles Albert Noble, decided in 1939 to omit this chapter – and without any notice.

This chapter analyses the teaching of geometry in various European countries, first at the time of the first international reform movement, and then for the inter-war period, which is otherwise not well explored. One gets revealing insight in the characteristic differences of methodology and organisation of school mathematics and of geometry teaching. Moreover, a number of issues of teaching geometry are discussed.

Regarding the first translation, their second volume reveals analogous problems mentioned in my preface to the first volume: wrong or inconsistent mathematical terminology and misunderstandings of the German text. To give just one example: on p. [4], the German text speaks, in a geometric context, of “Inhaltsbestimmung”. Well, “Bestimmung” is “determination”, but “Inhalt”? When no context is given, there are two meanings for this German term: either “area” (or “volume”) or “content”. Although the context is clearly geometric, they translated with “content”. Even more strangely, they continue at first with “area” – but after a few pages, they again use “content”.

In this volume, too, the reader will find, again in square brackets and in bold, *the page numbering of the original edition*. Cross-references in notes and in the text refer to this numbering, as well as *the name index and the subject index* (that is, the original text has not been changed in this respect).

In the present translation I have added, when possible, the first names of the persons mentioned. In the German edition, as it was customary at that time, the first names were indicated only with the initials. The bibliographic references in the notes have also been completed, when needed.

In the notes of English version of 1939, Hedrick and Noble had sometimes added references for recent pertinent American publications; these have been maintained. Several additional notes have been introduced; they are marked by square brackets.

As in volume I, the German names of the nine grades of secondary schools have been maintained, for greater exactness: *Sexta, Quinta, Quarta, Unter-Tertia, Ober-Tertia, Unter-Sekunda, Ober-Sekunda, Unter-Prima, Ober-Prima*.

I am thanking Leo Rogers for his careful re-reading of the book, and the various colleagues whom I asked advice, in particular Geoffrey Howson.

We are grateful to Dover Publication to have authorised the use of their book “Elementary Mathematics from an Advanced Standpoint”, translated by E. R. Hedrick and C. A. Noble, for a revised new edition.

Gert Schubring

Preface to the First Edition

In the preface to Part I of these lecture notes (Arithmetic, Algebra, Analysis) I expressed a doubt as to whether Part II, devoted to geometry, could appear soon. Nevertheless it has been possible to complete it, thanks to the diligence of Mr. Hellinger.

Concerning the origin and purpose of this series of lecture courses I have nothing especial to add to what was said in the foreword to Part I. However, a comment seems necessary concerning the new form, which this second part has assumed.

This form is, in fact, quite unlike that of Part I. I made up my mind to give, above all, a *comprehensive view* of the field of geometry, of such a range as I should wish every teacher in a secondary school would master; the discussions about geometry *teaching* were pushed into the background and were placed in connected form at the end, insofar as there was room left, but now in a connected form.

The choice of this new order was motivated partly by the desire to avoid a stereotyped form. There were, however, more important and deeper reasons. In geometry we possess no such homogeneous textbooks corresponding to the general level of the science, such as they exist in algebra and analysis, thanks to the prototype of the French *cours*. We find, rather, one aspect treated here, another particular aspect there, of this extensive subject, just as it has been developed by one or another group of researchers. In contrast to this, it seemed to be demanded by the pedagogic and the general scientific purposes, which I am intending that I attempt a more unified presentation.

I close with the wish that the two complementary parts of my *Elementary Mathematics from a Higher Standpoint* which are herewith completed may find the same friendly reception in the teaching world as the lectures on the organisation of mathematics teaching by Mr. Schimmack and myself, which appeared last year.

Göttingen, Christmas, 1908

Klein

Preface to the Third Edition

In virtue of the overall plan for the new edition of my lithographed lectures, which I explained in the preface to the third edition of the first volume, the text and presentation of the present second volume, have remained unaltered, except for small changes in detail and a few insertions.¹

The two supplements, which concern literature about scientific and pedagogic aspects, which was not considered in the original text, were prepared by Mr. Seyfarth, after repeated conferences with me. He assumed again the major portion of the burden entailed by the publication. Messrs. Ernst Hellinger, H. Vermeil, and Alwin Walther assisted him in the proof reading. Mr. Vermeil undertook the preparation of the two indexes. I am obliged to these gentlemen, and also to the publisher Julius Springer, who showed at all occasions much cooperation in realising my proposals.

Göttingen, May, 1925

Klein

¹ Newly added remarks are indicated by square brackets.

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