Chapter 1
Nature of Plasma

Abstract Charge neutrality is one of fundamental property of plasma. Section 1.2 explains Debye length $\lambda_{\rm D}$ in (1.2), a measure of shielding distance of electrostatic potential, and electron plasma frequency $\Pi_{\rm e}$ in (1.4), a measure of inverse time scale of electron's motion to cancel the electric perturbation. Both parameters are related with $\lambda_{\rm D}\Pi_{\rm e} = v_{\rm Te}$, $v_{\rm Te}$ being electron thermal velocity. Section 1.3 describes the condition of fusion core plasma. The necessary condition for the density, ion temperature and energy confinement time is given by (1.9).

1.1 Introduction

As the temperature of a material is raised, its state changes from solid to liquid and then to gas. If the temperature is elevated further, an appreciable number of the gas atoms are ionized and become the high temperature gaseous state in which the charge numbers of ions and electrons are almost the same and charge neutrality is satisfied in a macroscopic scale.

When the ions and electrons move collectively, these charged particles interact with Coulomb force which is long range force and decays only in inverse square of the distance r between the charged particles. The resultant current flows due to the motion of the charged particles and Lorentz interaction takes place. Therefore many charged particles interact with each other by long range forces and various collective movements occur in the gaseous state. The typical cases are many kinds of instabilities and wave phenomena. The word "plasma" is used in physics to designate the high temperature ionized gaseous state with charge neutrality and collective interaction between the charged particles and waves.

When the temperature of a gas is T (K), the average velocity of the thermal motion, that is, thermal velocity $v_{\rm T}$ is given by

$$mv_{\rm T}^2/2 = \kappa T/2 \tag{1.1}$$

where κ is Boltzmann constant $\kappa = 1.380658(12) \times 10^{-23}$ J/K and κT indicates the thermal energy. Therefore the unit of κT is Joule (J) in SI unit. In many fields

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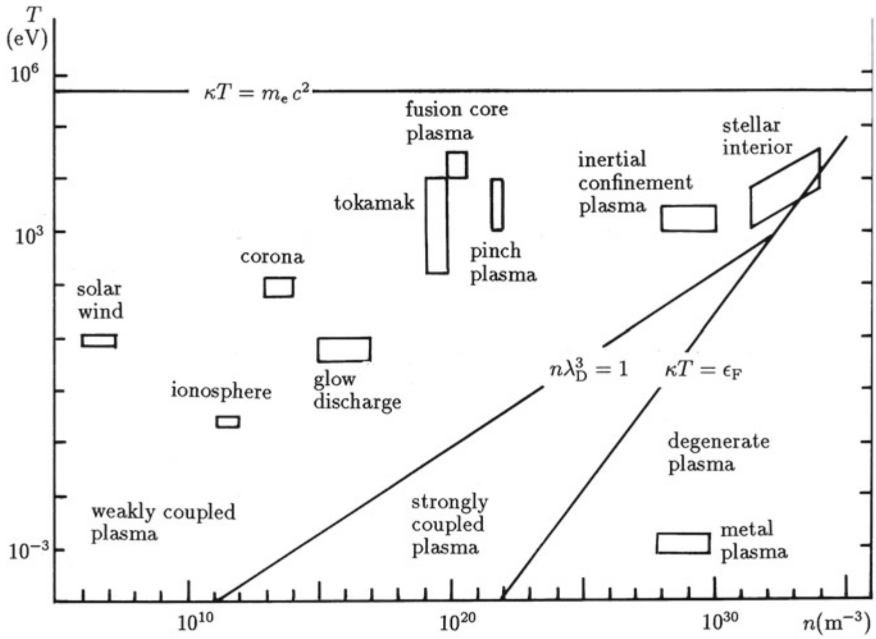

Fig. 1.1 Various plasma domain in n–T diagram

of physics, one electron volt (eV) is frequently used as a unit of energy. This is the energy necessary to move an electron, charge $e = 1.60217733(49) \times 10^{-19}$ Coulomb, against a potential difference of 1 volt:

$$1\mathrm{eV} = 1.60217733(49) \times 10^{-19}\,\mathrm{J}.$$

The temperature corresponding to the thermal energy of 1eV is 1.16×10^4 K ($=e/\kappa$). From now on the thermal energy κT is denoted by just T for simplicity and new T means the thermal energy. The ionization energy of hydrogen atom is 13.6 eV. Even if the thermal energy (average energy) of hydrogen gas is 1 eV, small amount of electrons with energy higher than 13.6 eV exist and ionize the gas to a hydrogen plasma. Plasmas are found in nature in various forms (see Fig. 1.1). There exits the ionosphere in the heights of 70–500 km (density $n \sim 10^{12}\,\mathrm{m}^{-3}$, $T \sim 0.2\,\mathrm{eV}$). Solar wind is the plasma flow originated from the sun with $n \sim 10^{6\sim7}\,\mathrm{m}^{-3}$, $T \sim 10\,\mathrm{eV}$. Corona extends around the sun and the density is $\sim 10^{14}\,\mathrm{m}^{-3}$ and the electron temperature is $\sim 100\,\mathrm{eV}$ although these values depend on the different positions. White dwarf, the final state of stellar evolution, has the electron density of $10^{35\sim36}\,\mathrm{m}^{-3}$. Various plasma domains in the diagram of electron density $n(\mathrm{m}^{-3})$ and electron temperature T (eV) are shown in Fig. 1.1. Active researches in plasma physics have been motivated by the aim to create and confine hot plasmas in fusion researches. Plasmas play important roles in the studies of pulsars radiating microwave or solar

X ray sources observed in space physics and astrophysics. The other application of plasma physics is the study of the earth's environment in space. Practical applications of plasma physics are MHD (magnetohydrodynamic) energy conversion for electric power generation, ion rocket engines for space crafts, and plasma processing which attracts much attention recently.

1.2 Charge Neutrality and Landau Damping

One of the fundamental property of plasma is the shielding of the electric potential applied to the plasma. When a probe is inserted into a plasma and positive (negative) potential is applied, the probe attracts (repulses) electrons and the plasma tends to shield the electric disturbance. Let us estimate the shielding length. Assume that the ions are in uniform density ($n_{\rm i} = n_0$) and there is small perturbation in electron density $n_{\rm e}$ or potential ϕ. Since the electrons are in Boltzmann distribution usually, the electron density $n_{\rm e}$ becomes

$$n_{\rm e} = n_0 \ \exp(e\phi/T_{\rm e}) \simeq n_0(1 + e\phi/T_{\rm e}).$$

Poisson's equation is

$$\boldsymbol{E} = -\nabla\phi, \qquad \nabla(\epsilon_0 \boldsymbol{E}) = -\epsilon_0\nabla^2\phi = \rho = -e(n_{\rm e} - n_0) = -\frac{e^2 n_0}{T_{\rm e}}\phi$$

and

$$\nabla^2\phi = \frac{\phi}{\lambda_{\rm D}^2}, \qquad \lambda_{\rm D} = \left(\frac{\epsilon_0 T_{\rm e}}{n_{\rm e}e^2}\right)^{1/2} = 7.45 \times 10^3 \left(\frac{1}{n_{\rm e}}\frac{T_{\rm e}}{e}\right)^{1/2} \ (\mathrm{m}) \tag{1.2}$$

where $n_{\rm e}$ is in m^{-3} and $T_{\rm e}/e$ is in eV. When $n_{\rm e} \sim 10^{20}\,\mathrm{cm}^{-3}$, $T_{\rm e}/e \sim 10\,\mathrm{keV}$, then $\lambda_{\rm D} \sim 75\,\mu\mathrm{m}$.

In spherically symmetric case, Laplacian ∇^2 becomes $\nabla^2\phi = (1/r^2)(\partial/\partial r)(r^2\partial\phi/\partial r)$ and the solution is

$$\phi = \frac{q}{4\pi\epsilon_0}\frac{\exp(-r/\lambda_{\rm D})}{r}.$$

It is clear from the foregoing formula that Coulomb potential $q/4\pi\epsilon_0 r$ of point charge is shielded out to a distance $\lambda_{\rm D}$. This distance $\lambda_{\rm D}$ is called the *Debye length*. When the plasma size is a and $a \gg \lambda_{\rm D}$ is satisfied, then plasma is considered neutral in charge. If $a < \lambda_{\rm D}$ in contrary, individual particle is not shielded electrostatically and this state is no longer plasma but an assembly of independent charged particles. The number of electrons included in the sphere of radius $\lambda_{\rm D}$ is called *plasma parameter* and is given by

$$n\lambda_{\mathrm{D}}^3 = \left(\frac{\epsilon_0}{e}\frac{T_{\mathrm{e}}}{e}\right)^{3/2}\frac{1}{n_{\mathrm{e}}^{1/2}}. \tag{1.3}$$

When the density is increased while keeping the temperature constant, this value becomes small. If the plasma parameter is less than say ~ 1, the concept of Debye shielding is not applicable since the continuity of charge density breaks down in the scale of Debye length. Plasmas in the region of $n\lambda^3 > 1$ are called classical plasma or *weakly coupled plasma*, since the ratio of electron thermal energy T_{e} and coulomb energy between electrons $E_{\mathrm{coulomb}} = e^2/4\pi\epsilon_0 d$ ($d \simeq n^{-1/3}$ is the average distance between electrons with the density n) is given by

$$\frac{T_{\mathrm{e}}}{E_{\mathrm{coulomb}}} = 4\pi(n\lambda_{\mathrm{D}}^3)^{2/3}$$

and $n\lambda^3 > 1$ means that coulomb energy is smaller than the thermal energy. The case of $n\lambda_{\mathrm{D}}^3 < 1$ is called *strongly coupled plasma* (see Fig. 1.1). Fermi energy of degenerated electron gas is given by $\epsilon_{\mathrm{F}} = (h^2/2m_{\mathrm{e}})(3\pi^2 n)^{2/3}$. When the density becomes very high, it is possible to become $\epsilon_{\mathrm{F}} \geq T_{\mathrm{e}}$. In this case quantum effect is more dominant than thermal effect. This case is called *degenerated electron plasma*. One of this example is the electron plasma in metal. Most of plasmas in experiments are classical weakly coupled plasma except the plasma compressed by inertial confinement.

Let us consider the case where a small perturbation occurs in a uniform plasma and the electrons in the plasma move by the perturbation. It is assumed that ions do not move because the ion's mass is much heavier than electron's. Due to the displacement of electrons, electric charges appear and an electric field is induced. The electric field is given by Poisson's equation:

$$\epsilon_0 \nabla \cdot \boldsymbol{E} = -e(n_{\mathrm{e}} - n_0).$$

Electrons are accelerated by the electric field:

$$m_{\mathrm{e}}\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = -e\boldsymbol{E}.$$

Due to the movement of electrons, the electron density changes:

$$\frac{\partial n_{\mathrm{e}}}{\partial t} + \nabla \cdot (n_{\mathrm{e}}\boldsymbol{v}) = 0.$$

Denote $n_{\mathrm{e}} - n_0 = n_1$ and assume $|n_1| \ll n_0$, then we find

$$\epsilon_0 \nabla \cdot \boldsymbol{E} = -en_1, \quad m_{\mathrm{e}}\frac{\partial \boldsymbol{v}}{\partial t} = -e\boldsymbol{E}, \quad \frac{\partial n_1}{\partial t} + n_0 \nabla \cdot \boldsymbol{v} = 0.$$

For simplicity the displacement is assumed only in the x direction and is sinusoidal:

$$n_1(x,t) = n_1 \exp(ikx - i\omega t).$$

Time differential $\partial/\partial t$ is replaced by $-i\omega$ and $\partial/\partial x$ is replaced by ik, then

$$ik\epsilon_0 E = -en_1, \qquad -i\omega m_e v = -eE, \qquad -i\omega n_1 = -ikn_0 v$$

so that we find

$$\omega^2 = \frac{n_0 e^2}{\epsilon_0 m_e}. \tag{1.4}$$

This wave is called *electron plasma wave* or *Langmuir wave* and its frequency is called *electron plasma frequency* Π_e:

$$\Pi_e = \left(\frac{n_e e^2}{\epsilon_0 m_e}\right)^{1/2} = 5.64 \times 10^{11} \left(\frac{n_e}{10^{20}}\right)^{1/2} \quad \text{rad/s}.$$

There is following relation between the plasma frequency and Debye length λ_D:

$$\lambda_D \Pi_e = \left(\frac{T_e}{m_e}\right)^{1/2} = v_{Te}.$$

The other fundamental process of plasma is collective phenomena of charged particles. Waves are associated with coherent motions of charged particles. When the phase velocity v_{ph} of wave or perturbation is much larger than the thermal velocity v_T of charged particles, the wave propagates through the plasma media without damping or amplification. However when the refractive index N of plasma media becomes large and plasma becomes hot, the phase velocity $v_{ph} = c/N$ (c is light velocity) of the wave and the thermal velocity v_T become comparable ($v_{ph} = \omega/k = c/N \sim v_T$), then the exchange of energy between the wave and the thermal energy of plasma is possible. The existence of a damping mechanism of wave was found by L.D. Landau. The process of Landau damping involves a direct wave-particle interaction in collisionless plasma without necessity of randomizing collision. This process is fundamental mechanism in wave heatings of plasma (wave damping) and instabilities (inverse damping of perturbations). Landau damping will be described in Chaps. 10 and 11.

1.3 Fusion Core Plasma

Progress in plasma physics has been motivated by how to realize fusion core plasma. Necessary condition for fusion core plasma is discussed in this section. Nuclear fusion reactions are the fused reactions of light nuclides to heavier one. When the sum of the masses of nuclides after a nuclear fusion is smaller than the sum before the reaction by Δm, we call it mass defect. According to theory of relativity, amount of energy $(\Delta m)c^2$ (c is light speed) is released by the nuclear fusion.

Nuclear reactions of interest for fusion reactors are as follows (D; deuteron, T; triton, He^3; helium-3, Li; lithium):

(1) $D + D \rightarrow T(1.01\,MeV) + p(3.03\,MeV)$
(2) $D + D \rightarrow He^3(0.82\,MeV) + n(2.45\,MeV)$
(3) $T + D \rightarrow He^4(3.52\,MeV) + n(14.06\,MeV)$
(4) $D + He^3 \rightarrow He^4(3.67\,MeV) + p(14.67\,MeV)$
(5) $Li^6 + n \rightarrow T + He^4 + 4.8\,MeV$
(6) $Li^7 + n(2.5\,MeV) \rightarrow T + He^4 + n$

where p and n are proton (hydrogen ion) and neutron respectively ($1\,MV = 10^6\,eV$). Since the energy released by chemical reaction of $H_2 + (1/2)O_2 \rightarrow H_2O$ is 2.96 eV, fusion energy released is about million times as large as chemical one. A binding energy per nucleon is smaller in very light or very heavy nuclides and largest in the nuclides with atomic mass numbers around 60. Therefore, large amount of the energy can be released when the light nuclides are fused. Deuterium exists abundantly in nature; for example, it comprises 0.015 atom percent of the hydrogen in sea water with the volume of about $1.35 \times 10^9\,km^3$.

Although fusion energy was released in an explosive manner by the hydrogen bomb in 1951, controlled fusion is still in the stage of research development. Nuclear fusion reactions were found in 1920s. When proton or deuteron beams collide with target of light nuclide, beam loses its energy by the ionization or elastic collisions with target nuclides and the probability of nuclear fusion is negligible. Nuclear fusion researches have been most actively pursued by use of hot plasma. In fully ionized hydrogen, deuterium and tritium plasmas, the process of ionization does not occur. If the plasma is confined in some specified region adiabatically, the average energy does not decrease by the processes of elastic collisions. Therefore if the very hot D–T plasmas or D–D plasmas are confined, the ions have velocities large enough to overcome their mutual coulomb repulsion, so that collision and fusion take place.

Let us consider the nuclear reaction that D collides with T. The effective *cross section* of T nucleus is denoted by σ. This cross section is a function of the kinetic energy E of D. The cross section of D–T reaction at $E = 100\,keV$ is $5 \times 10^{-24}\,cm^2$. The cross sections σ of D–T, D–D, D–He^3 reaction versus the kinetic energy of colliding nucleus are shown in Fig. 1.2a [1, 2]. The probability of fusion reaction per unit time in the case that a D ion with the velocity v collides with T ions with the density of n_T is given by $n_T \sigma v$ (we will discuss the collision probability in more details in Sect. 2.5). When a plasma is Maxwellian with the ion temperature of T_i, it is necessary to calculate the average value $\langle \sigma v \rangle$ of σv over the velocity space. The dependence of $\langle \sigma v \rangle$ on ion temperature T_i is shown in Fig. 1.2b [3]. A fitting equation of $\langle \sigma v \rangle$ of D–T reaction as a function of T in unit of keV is [4]

$$\langle \sigma v \rangle (m^3/s) = \frac{3.7 \times 10^{-18}}{H(T) \times T^{2/3}} \exp\left(-\frac{20}{T^{1/3}}\right),$$

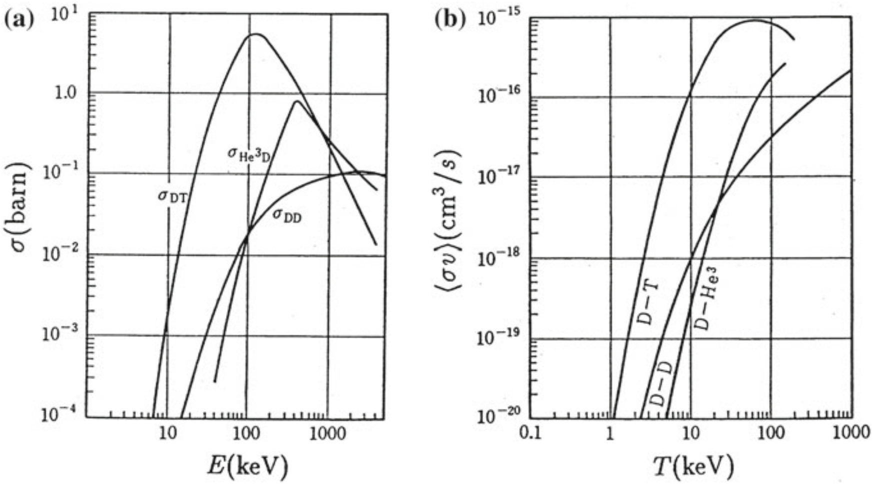

Fig. 1.2 **a** The dependence of fusion cross section σ on the kinetic energy E of colliding nucleus. σ_{DD} is the sum of the cross sections of D–D reactions (1) (2). 1 barn $= 10^{-24}$ cm^2. **b** The dependence of fusion rate $\langle\sigma v\rangle$ on the ion temperature T_i

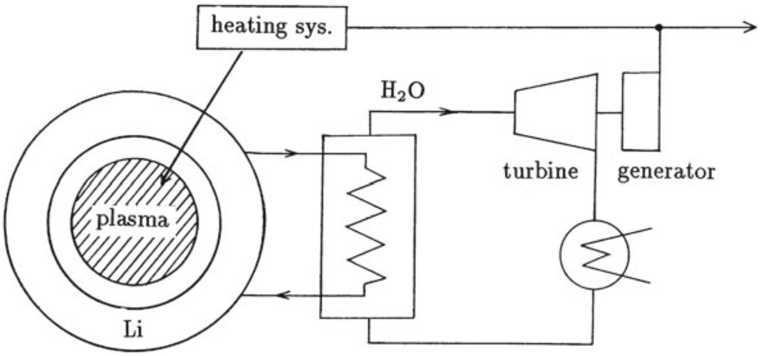

Fig. 1.3 An electric power plant based on a D–T fusion reactor

$$H(T) \equiv \frac{T}{37} + \frac{5.45}{3 + T(1 + T/37.5)^{2.8}} \tag{1.5}$$

Figure 1.3 shows an example of electric power plant based on D–T fusion reactor. Fast neutrons produced in fusion core plasma penetrate the first wall and a lithium blanket surrounding the plasma moderates the fast neutrons, converting their kinetic energy to heat. Furthermore the lithium blanket breeds tritium due to reaction (5), (6). Lithium blanket gives up its heat to generate the steam by a heat exchanger; steam turbine generates electric power. A part of the generated electric power is used to operate heating system of plasma to compensate the energy losses from the plasma to keep the plasma hot. The fusion output power must be larger than the necessary heating input power taking account the conversion efficiency. Since the necessary heating input power is equal to the energy loss rate of fusion core plasma, good energy confinement of hot plasma is key issue.

The thermal energy of plasma per unit volume is given by $(3/2)n(T_\mathrm{i} + T_\mathrm{e})$. This thermal energy is lost by thermal conduction and convective losses. The notation P_L denotes these energy losses of the plasma per unit volume per unit time (power loss per unit volume). There is radiation loss R due to bremsstrahlung of electrons and impurity ion radiation in addition to P_L. The total energy confinement time τ_E is defined by

$$\tau_\mathrm{E} \equiv \frac{(3/2)n(T_\mathrm{e} + T_\mathrm{i})}{P_\mathrm{L} + R} \simeq \frac{3nT}{P_\mathrm{L} + R}. \tag{1.6}$$

The necessary heating input power P_heat is equal to $P_\mathrm{L} + R$. In the case of D–T reaction, the sum of kinetic energies $Q_\alpha = 3.52\,\mathrm{MeV}$ of α particle (He^4 ion) and $Q_\mathrm{n} = 14.06\,\mathrm{MeV}$ of neutron is Q_fus=17.58 MeV per 1 reaction. Since the densities of D ions and T ions of equally mixed plasma are $n/2$, number of D–T reaction per unit time per unit volume is $(n/2)(n/2)\langle\sigma v\rangle$, so that fusion output power per unit volume P_fus is given by

$$P_\mathrm{fus} = (n/2)(n/2)\langle\sigma v\rangle Q_\mathrm{fus}. \tag{1.7}$$

Denote the thermal-to-electric conversion efficiency by η_el and heating efficiency (ratio of the deposit power into the plasma to the electric input power of heating device) by η_heat. Then the condition of power generation is

$$P_\mathrm{heat} = P_\mathrm{L} + R = \frac{3nT}{\tau_\mathrm{E}} < (\eta_\mathrm{el})(\eta_\mathrm{heat})P_\mathrm{fus} \tag{1.8}$$

that is

$$\frac{3nT}{\tau_\mathrm{E}} < (\eta_\mathrm{heat})(\eta_\mathrm{el})\frac{Q_\mathrm{fus}}{4}n^2\langle\sigma v\rangle,$$

$$n\tau_\mathrm{E} > \frac{12T}{\eta Q_\mathrm{fus}\langle\sigma v\rangle} \tag{1.9}$$

where η is the product of two efficiencies. The right-hand side of the last foregoing equation is the function of temperature T only. When $T = 10^4\,\mathrm{eV}$ and $\eta \sim 0.3$ ($\eta_\mathrm{el} \sim 0.4$, $\eta_\mathrm{heat} \sim 0.75$), the necessary condition is $n\tau_\mathrm{E} > 1.7 \times 10^{20}\,\mathrm{m}\mathrm{s}^{-3} \cdot \mathrm{s}$. The condition of D–T fusion plasma in the case of $\eta \sim 0.3$ is shown in Fig. 1.4. In reality the plasma is hot in the core and is cold in the edge. For the more accurate discussion, we must take account of the profile effect of temperature and density and will be analyzed in Sect. 15.9.

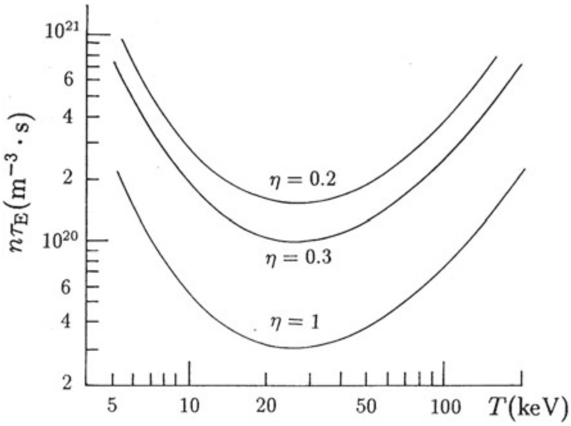

Fig. 1.4 Condition of D–T fusion core plasma in $n\tau_E$–T diagram in the case of $\eta = 0.3$, critical condition ($\eta = 1$) and ignition condition ($\eta = 0.2$)

The condition $P_{\text{heat}} = P_{\text{fus}}$ is called *break even condition*. This corresponds to the case of $\eta = 1$ in the condition of fusion core plasma. The ratio of the fusion output power due to α particles to the total is $Q_\alpha/Q_{\text{fus}} = 0.2$. Since α particles are charged particles, α particles can heat the plasma by coulomb collision (see Sect. 2.5). If the total kinetic energy (output energy) of α particles contributes to heat the plasma, the condition $P_{\text{heat}} = 0.2P_{\text{fus}}$ can sustain the necessary high temperature of the plasma without heating from outside. This condition is called *ignition condition*, which corresponds the case of $\eta = 0.2$.

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