
Preface to the Third Edition

After the first edition of this book was published at the end of 2003, Springer requested a revision for the second edition around 2006, and that was published at the end of 2007. The original edition was written based on experience with the software design of KSGsoft (Xu et al. 1998) and KGsoft (Xu 1999), and related research and practice in Germany and Denmark. The second edition benefited from the design of the multi-functional GPS/Galileo software (MFGsoft, Xu 2004). The new model of solar radiation for satellite orbit determination discovered through GPS research led to an attempt to solve the equations of satellite motion affected by second-order disturbances, which resulted in the book *Orbits*, published at the end of 2008, with a second edition in 2013, combined with intensive research (cf. Xu et al. 2010a,b, 2011; Xu and Xu 2012, 2013a,b). During that time, the author was further involved in GPS activities through his role in the supervision of Ph.D. studies and software development (cf. Wang et al. 2010; He 2015). In 2011, the second edition of *GPS* was translated into Chinese by the Peking Institute of Satellite Controlling and Telecommunication and was published by Tsinghua University Press Peking, and was sold out around 2015. In 2014, *GPS* was translated into Persian, and that edition was published by Dhahran University. The preparation of the third edition of *GPS* was contracted with Springer several years ago, but was interrupted by work on the books *Orbits* (2008, 2013) and *Sciences of Geodesy* (2010, 2012), as well as other scientific endeavours. In 2014 the author moved from Germany to a position at Shandong University in Weihai to create a new area of study comprising navigation, remote sensing, and celestial mechanics. The rapid development of the GPS, GLONASS, Galileo, and BeiDou systems, and the realisation of net-based multi-system real-time civil GNSS applications at GFZ in Germany and at Shandong University, provided the impetus for this most recent revision of the book. Dr.-Ing. Yan Xu, who has played an important role, was asked to be the second author.

A fully new Chap. 12 has been added, which highlights the newly developed singularity-free theory of analytical satellite orbits. A brief historical review of the singularity problem is presented, and the solution is derived mathematically. The

study was conducted by the first author of this book with his friends from 2007 to 2015, and the most elegant formulas—the Lagrange-Xu and Gauss-Xu equations of motion—were derived rigorously and mathematically and are described here for the first time. Three singularity criteria with clear geometric meaning are defined. This theory opened up a new area of study in orbit determination using analytical theory and is believed to be extremely important with respect to GNSS applications for onboard orbit determination.

Chapters that have been intensively revised and supplemented are the first, “Introduction”; the fifth, “Physical Influences of GPS Surveying”; the eighth, “Cycle Slip Detection and Ambiguity Resolution”; and the ninth, “Parameterisation and Algorithms of GPS Data Processing”. The others are minorly revised. These are reviews of the works of scientists around the world. The supplemented content concerns the following 25 points.

1. Review of GPS modernisation
2. Review of the key developments of GLONASS
3. Review of the development of the Galileo system
4. Review of the development of the Compass (BeiDou) system
5. Review of progress in ionospheric studies
6. Review of tropospheric research
7. Research concerning the GPS clock
8. Review of the use of an external clock
9. Introduction of water vapour measurements
10. Review of activities involving GPS altimetry
11. Review of the study of GPS equivalence algorithms
12. The evaluation of ambiguity search criteria
13. Review of research progress in adaptive filtering
14. Research on GPS data processing by combined and uncombined methods
15. Research on reference satellite changes in the GPS difference algorithm
16. Research on tropospheric models in airborne kinematic positioning
17. Research on reference station changes in the GPS difference algorithm
18. Review of float ambiguity fixing
19. Outline of the progress in precise point positioning
20. GPS software introduction
21. Review of satellite orbital theory
22. Review of numerical orbit determination
23. Summary of GEO satellite orbit determination
24. Research on independent parameterisation
25. Summary of problems remaining in GPS research

With such intensive content supplementation, the authors hope that this new edition of *GPS* can better serve as a reference for GNSS research and study. The theoretical contributions and new findings in the original edition of this book can be summarised as follows:

1. The soft equivalence of differencing and undifferenced GPS algorithms
2. The unified GPS data processing algorithm

3. The general ambiguity search criterion
4. The equivalent ambiguity search criterion
5. The diagonalisation algorithm
6. Yang's filter—adaptive robust Kalman filtering
7. Theory of numerical orbit determination using GPS
8. The algebraic solution of the variation equation
9. The problem of the ambiguity function criterion

The new findings supplemented in the second edition are as follows:

1. The equivalence of combined and uncombined GPS algorithms
2. The independent parameterisation method
3. The equivalence theorem of GPS algorithms
4. The optimal differential GPS baseline network
5. The new model of solar radiation
6. The new model of atmospheric drag

New theoretical content in this third edition can be described as follows:

1. The equivalent equations of triple differences
2. The idea of an intelligent Kalman filter
3. Float ambiguity fixing in the case of ionosphere-free combination
4. The singularity-free Lagrange-Xu equations of motion
5. The singularity-free Gauss-Xu equations of motion
6. The criteria of singularity and their geometric meaning

The extended content is derived in part from published international papers and has been subjected to individual review. We thank Academician Yuanxi Yang of the Institute of Surveying and Mapping in Xian, Prof. Ta-Kang Yeh of Taibei University of Taiwan, Prof. Wu Chen of Hong Kong Polytech University, Prof. Yunzhong Shen of Tongji University, and Prof. Zhiping Lv of the Institute of Surveying and Mapping in Zhengzhou for their valuable review of portions of the supplemented content of this book.

The first author extends sincere thanks to Prof. Dr. D. Lelgemann of TU Berlin for supervision of the author's Ph.D. study and work many years ago. Thanks go to the directors Prof. Dr. Ch. Reigber, Prof. Dr. Markus Rothacher, and Prof. Dr. Harald Schuh of GFZ for their support and faith in the author during his approximately 20 years of research activities at GFZ. Prof. Ming Li of the China Academy of Space Technology (CAST) is thanked for the unwavering support since the author's involvement at CAST as Thousand Talents expert. Of course, Shandong University at Weihai is thanked for the opportunity to develop a new study area and form an international team, as well as obtaining internal and external funding support. The second author is thanked for taking over most of the text processing work for this new edition.

The navigation/remote sensing team members of Shandong University at Weihai are thanked for their warm support. Special thanks go to Thousand Talents expert Prof. Hermann Kaufmann of Germany, Guest Prof. Pierre Rochus of Belgium,

Guest Prof. Luisa Bastos of Portugal, Guest Prof. Anna Jenssen of Sweden, scientist Nina Boesche, and guest engineer Nan Jiang of Germany, senior engineers Wenlin Yan and Zhangzheng Sun, engineers Chunhua Jiang and Fangzhao Zhang, post-doctoral researchers Yujun Du and Fang Gao, and Ph.D. candidate Wenfeng Nie.

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Preface to the Second Edition

After the first edition of this book was published at the end of 2003, I was very happy to put the hard work of book writing behind me and concentrate with my small team on the development of multi-functional GPS/Galileo software (MFGsoft). The experiences from the practice and implementation of the theory and algorithms into high-standard software caused me to strongly feel that I would like to revise and to supplement the original book, to modify some of the content, and to report on the new developments and knowledge. Furthermore, with the EU Galileo system now being realised and the Russian GLONASS system under development, GPS theory and algorithms needed to be re-described so that they would be valid for the Galileo and GLONASS systems as well. Therefore, I am grateful to all of the readers of this book, whose interest led Springer to ask me to complete this second edition.

I remember that I was in a hurry during the last check of the layout of the first edition. The description of a numerical solution to the variation equation in Sect. 11.5.1 was added to the book at the last minute, and comprised exactly one page. Traditionally, variation equations in orbit determination (OD), geopotential mapping, and OD Kalman filtering, have been solved by integration, which is complicated and compute-intensive. This marks the first time in the history of OD that the variation equation is not integrated, but is solved by a linear algebraic equation system. However, this was mentioned neither in the preface nor at the beginning of the chapter. The high precision of this algebraic method is verified by a numerical test.

The problems discussed in Chap. 12 of the first edition are largely solved and are now described by the so-called independent parameterisation theory, which points out that in both undifferenced and differencing algorithms, the independent ambiguity vector is double-differencing. With the use of this parameterisation method, the GPS observation equations are regular equations, and can be solved without any a priori information. Many conclusions may be derived from this new knowledge. For example, synchronisation of the GPS clocks may not be realised by the carrier-phase observables because of the linear correlations between the clock error

parameters and the ambiguities. The equivalence principle is extended to show that the equivalences are valid not only between the undifferenced and differencing algorithms, but also between uncombined and combining algorithms and their mixtures. In other words, the GPS data processing algorithms are equivalent under the same parameterisation of the observation model. Different algorithms are beneficial for different data processing purposes. One consequence of the equivalence theory is that a so-called secondary data processing algorithm is created. Thus the complete GPS positioning problem may be separated into two steps (first to transform the data to the secondary observables and then to process the secondary data). Another consequence of the equivalence is that any GPS observation equation can be separated into two sub-equations, which is very advantageous in practice. Furthermore, it shows that the combinations under traditional parameterisation are inexact algorithms compared with those under independent parameterisation.

The additional content features a more detailed introduction, which includes not only GPS developments, but also those of the EU Galileo and Russian GLONASS systems, as well as the combination of the GPS, GLONASS, and Galileo systems. The book thus covers the theory, algorithms, and applications of the GPS, GLONASS, and Galileo systems. The equivalence of GPS data processing algorithms and the independent parameterisation of GPS observation models is discussed in detail. Other new content includes the concept of optimal network formation, application of the diagonalisation algorithm, and adjustment models of radiation pressure and atmospheric drag, as well as discussions and comments of what are currently, in the author's opinion, key research problems. Application of the theory and algorithms in the development of GPS/Galileo software is also outlined. The content concerning the ambiguity search is reduced, whereas the content regarding ionosphere-free ambiguity fixing is cancelled out, although it was reported by Lemmens (2004) as new. Some of the content of various sections has also been reordered. In this way, I hope this edition may better serve as a reference and handbook of GPS/Galileo research and applications.

The extended content is partly the result of the development of MFGsoft, and has been subjected to individual review. I thank Prof. Lelgemann of the TU Berlin, Prof. Yuanxi Yang of the Institute of Surveying and Mapping in Xian, Prof. Ta-Kang Yeh of ChingYun University of Taiwan, and Prof. Yunzhong Shen of Tongji University for their valuable reviews. I am grateful to Prof. Jiancheng Li and Dr. Zhengtao Wang of Wuhan University, as well as Mr. Tinghao Xiao of Potsdam University, for their cooperation during the development of the software from 2003 to 2004 at the GFZ.

I sincerely thank Prof. Dr. Markus Rothacher for his support and faith in me during my research activities at the GFZ. I also thank Dr. Jinghui Liu of the educational department of the Chinese Embassy in Berlin, Prof. Heping Sun and Jikun Ou of IGG in Wuhan, and Prof. Qin Zhang of ChangAn University for their warm support during my scientific activities in China. The Chinese Academy of Sciences is thanked for the Outstanding Overseas Chinese Scholars Fund. During this work, several interesting topics have been carefully studied by some of my

students. I am grateful to Ms. Daniela Morujao of Lisbon University, Ms. Jamila Bouaicha of TU Berlin, Dr. Jiangfeng Guo and Ms. Ying Hong of IGG in Wuhan, and Mr. Guanwen Huang of ChangAn University. I am also thankful for the valuable feedback from readers and from students through my professorships at ChangAn University and the IGG CAS.

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Preface to the First Edition

The contents of this book cover static, kinematic, and dynamic GPS theory, algorithms, and applications. Most of the content comes from the source code descriptions of the Kinematic/Static GPS Software (KSGsoft), which was developed at GFZ before and during the EU AGMASCO project. The principles described here have been largely applied in practice, and are carefully revised from a theoretical perspective. A portion of the content is dealt with on a theoretical basis and applied to the development of quasi-real-time GPS orbit determination software at GFZ.

The original purpose in writing this book was, indeed, to have it for myself as a GPS handbook and as a reference for a few of my friends and students who worked with me in Denmark. The desire to describe the theory in an exact manner comes from my mathematical education. My extensive geodetic research experience has led to a detailed treatment of most topics. The comprehensiveness of the content reflects my nature as a software designer.

Some of the results of research carried out in GFZ are published here for the first time. One example is the unified GPS data processing method using selectively eliminated equivalent observation equations. Methods including zero-, single-, double-, triple-, and user-defined differential GPS data processing are unified in a unique algorithm. This method has advantages of both un-differential and differential methods, in that the un-correlation property of the original observations is retained, and the unknown number may be greatly reduced. Another example is the general criterion and its equivalent criterion for integer ambiguity search. A search using this criterion can be carried out in ambiguity or coordinate domains, or both. The optimality and uniqueness properties of the criterion are proved. Further examples include the diagonalisation algorithm of the ambiguity search problem, the ambiguity-ionospheric equations for ambiguity and ionosphere determination, and the use of the differential Doppler equation as system equation in the Kalman filter.

The book includes 12 chapters. After a brief introduction, the coordinate and time systems are described in the second chapter. Because orbit determination is also an important topic of the book, the third chapter is dedicated to Keplerian

satellite orbits. The fourth chapter deals with the GPS observables, including code range, carrier phase, and Doppler measurements.

The fifth chapter covers all physical influences on GPS observations, including ionospheric, tropospheric, and relativistic effects, earth tide and ocean loading tide effects, clock errors, antenna mass centre and phase centre corrections, multipath effects, anti-spoofing, and historical selective availability, as well as instrumental biases. Theories, models, and algorithms are discussed in detail.

The sixth chapter first covers GPS observation equations, including their formation, linearisation, related partial derivatives, and linear transformation and error propagation. Useful data combinations are then discussed, particularly with respect to the introduction of the concept of ambiguity-ionospheric equations and related weight matrix. The equations include only ambiguity, ionospheric, and instrumental error parameters and can also be solved independently in kinematic applications. Traditional differential GPS observation equations, including the differential Doppler equations, are also discussed in detail. The method of selectively eliminated equivalent observation equations is proposed to unify the un-differential and differential GPS data processing methods.

The seventh chapter covers all adjustment and filtering methods that are suitable and necessary for GPS data processing. The main adjustment methods described are classical, sequential, block-wise, and conditional least squares. The key filtering methods include classical and robust filtering, as well as adaptively robust Kalman filters. In addition, a priori constraints, a priori datum, and quasi-stable datum methods are discussed for dealing with rank-deficient problems. The theoretical basis of equivalently eliminated equations is derived in detail.

The eighth chapter is dedicated to cycle slip detection and ambiguity resolution. Several cycle slip detection methods are outlined, with emphasis on deriving a general criterion for integer ambiguity search in ambiguity or coordinate domains, or both. Although the criterion is derived from conditional adjustment, in the end, it has nothing to do with any condition. An equivalent criterion is also derived, and shows that the well-known least squares ambiguity search criterion is just one of the terms of the equivalent criterion. A diagonalisation algorithm is proposed for use with ambiguity search, which can be done within one second after the normal equation is diagonalised. The ambiguity function and float ambiguity fixing methods are also outlined.

The ninth chapter describes GPS data processing in static and kinematic applications, and data pre-processing is outlined. Emphasis is given to solving the ambiguity-ionospheric equations and single point positioning, relative positioning, and velocity determination using code, phase, and combined data. The equivalent un-differential and differential data processing methods are discussed, and a method of Kalman filtering using velocity information is described. The accuracy of the observational geometry is outlined at the end of the chapter.

The tenth chapter covers the concepts of kinematic positioning and flight-state monitoring. The use of the IGS station, multiple static references, airport height information, kinematic tropospheric modelling, and the known distances of the

multiple antennas on aircraft are discussed in detail. Numerical examples are also given.

The 11th chapter deals with the topic of perturbed orbit determination. Perturbed equations of satellite motion are derived, and perturbation forces of satellite motion are discussed in detail, including the earth's gravitational field, earth tide and ocean tide, the sun, moon, and planets, solar radiation pressure, atmospheric drag, and coordinate perturbation. Orbit correction is outlined based on the analytical solution of C_{20} perturbation. Precise orbit determination is also discussed, including its principle and related derivatives, as well as numerical integration and interpolation algorithms.

The final chapter is a brief discussion regarding the future of GPS and comments on some remaining problems.

The book was subjected to an individual review of chapters and sections or according to content. I am grateful to reviewers Prof. Lelgemann of the Technical University (TU) Berlin, Prof. Leick of the University of Maine, Prof. Rizos of the University of New South Wales (UNSW), Prof. Grejner-Brzezinska of Ohio State University, Prof. Yuanxi Yang of the Institute of Surveying and Mapping in Xian, Prof. Jikun Ou of the Institute of Geodesy and Geophysics (IGG) in Wuhan, Prof. Wu Chen of Hong Kong Polytechnic University, Prof. Jiancheng Li of Wuhan University, Dr. Chunfang Cui of TU Berlin, Dr. Zhigui Kang of the University of Texas at Austin, Dr. Jinling Wang of UNSW, Dr. Yanxiong Liu of GFZ, Mr. Shfaqat Khan of KMS of Denmark, Mr. Zhengtao Wang of Wuhan University, and Dr. Wenyi Chen of the Max-Planck Institute of Mathematics in Sciences (Leipzig, Germany). The book was subjected to a general review by Prof. Lelgemann of TU Berlin. A grammatical check of technical English writing was performed by Springer-Verlag Heidelberg.

I offer my sincere thanks to Prof. Dr. Ch. Reigber for his support and faith in me throughout my scientific research activities at GFZ. Dr. Niels Andersen, Dr. Per Knudsen, and Dr. Rene Forsberg at KMS of Denmark are thanked for their support for starting work on this book, and Prof. Lelgemann of TU Berlin for his encouragement and help. During this work, many valuable discussions were held with many specialists. My thanks go to Prof. Grafarend of the University Stuttgart, Prof. Tscherning of Copenhagen University, Dr. Peter Schwintzer of GFZ, Dr. Luisa Bastos of the Astronomical Observatory of University Porto, Dr. Oscar Colombo of Maryland University, Dr. Detlef Angermann of German Geodetic Research Institute Munich, Dr. Shengyuan Zhu of GFZ, Dr. Peiliang Xu of the University Kyoto, Prof. Guanyun Wang of IGG in Wuhan, Dr. Ludger Timmen of the University of Hannover, and Ms. Daniela Morujao of Coimbra University. I thank Dr. Jürgen Neumeyer of GFZ and Dr. Heping Sun of IGG in Wuhan for their support. Dipl.-Ing. Horst Scholz of TU Berlin is thanked for redrawing a portion of the graphics. I am also grateful to Dr. Engel of Springer-Verlag Heidelberg for his advice.

I thank my wife Liping, son Jia, and daughters Yuxi and Pan for their loving support and understanding, as well as for their help on part of the text processing and graphing.

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GPS

Theory, Algorithms and Applications

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