

# Preface

In 2016, one decade after the publication of our *Dictionary of Distances*, the number of WWW entries offered by Google on the topics “distance” and “distance metric” has grown from 300 million (about 4 % of all entries) and 12 million to 1.35 billion (about 5.4 % of all entries) and 114 million.

This fourth edition is characterized by updated and rewritten sections on some items suggested by experts and readers, as well as a general streamlining of content and the addition of essential new topics.

Though the structure remains unchanged, the new edition also explores recent advances in the use of distances and metrics for, e.g., generalized distances, probability theory, graph theory, coding theory, data analysis.

New topics in the purely mathematical sections include, e.g., the Vitanyi multiset-metric, algebraic point-conic distance, triangular ratio metric, Rossi–Hamming metric, Taneja distance, spectral semimetric between graphs, channel metrization, and Maryland bridge distance, which are addressed in Chaps. 3, 4, 6, 10, 14, 15, 16, and 17, respectively.

The multidisciplinary sections have also been supplemented with new topics, including dynamic time wrapping distance, memory distances, allometry, atmospheric depth, elliptic orbit distance, VLBI distance measurements, the astronomical system of units, and walkability distances, which can be found in Chaps. 21, 22, 23, 24, 25, 26, 27, and 28, respectively.

We’d like to take this opportunity to once again thank the team at Springer for their very efficient and friendly assistance.

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May 2016

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# Preface to the Third Edition

Since the publication of the second edition in 2012, several people have again given us their valued feedback and have thus contributed to the publication of this third edition. We are thankful to them for their input.

In the latest edition, new items from very active research areas in the use of distances and metrics such as geometry, graph theory, probability theory, and analysis have been added. We have kept the structure but have revised many topics, simplifying, shortening, and updating them, especially in Chaps. 23–25 and 27–29.

Among the new topics included are, for example, polyhedral metric spaces, nearness matrix problems, distances between belief assignments, distance-related animal settings, diamond-cutting distances, natural units of length, Heidegger's de-severance distance, and brain distances in Chaps. 9, 12, 14, 23, 24, 27, 28, and 29, respectively.

We would also like to thank the team at Springer for their very efficient and friendly assistance.



## Preface to the Second Edition

The preparation of the second edition of *Encyclopedia of Distances* has presented a welcome opportunity to improve the first edition published in 2009 by updating and streamlining many sections and by adding new items (especially in Chaps. 1, 15, 18, 23, 25, 27–29), increasing the book’s size by about 70 pages. This new edition preserves, except for Chaps. 18, 23, 25, and 28, the structure of the first edition.

The first large conference with a scope matching that of this encyclopedia is MDA 2012, the International Conference “Mathematics of Distances and Applications,” held in July 2012 in Varna, Bulgaria (cf. [DPM12]).



# Preface to the First Edition

*Encyclopedia of Distances* is the result of rewriting and extending our *Dictionary of Distances*, published in 2006 (and put online at <http://www.sciencedirect.com/science/book/9780444520876>) by Elsevier. About a third of the definitions are new, and majority of the remaining ones are upgraded.

We were motivated by the growing intensity of research on metric spaces and, especially, in distance design for applications. Even if we do not address the practical questions arising during the selection of a “good” distance function, just a sheer listing of the main available distances should be useful for the distance design community.

This encyclopedia is the first one treating fully the general notion of distance. This broad scope is useful *per se*, but it also limited our options for referencing. We give an original reference for many definitions but only when it was not too difficult to do so. On the other hand, citing somebody who well developed the notion but was not the original author may induce problems. However, with our data (usually, author name(s) and year), a reader can easily search sources using the Internet.

We found many cases when authors developed very similar distances in different contexts and, clearly, were unaware of it. Such connections are indicated by a simple “cf.” in both definitions, without going into priority issues explicitly.

Concerning the style, we tried to make it a mixture of resource and coffee-table book, with maximal independence of its parts and many cross-references.



## Preface to *Dictionary of Distances*, 2006

The concept of *distance* is a basic one in the whole human experience. In everyday life, it usually means some degree of closeness of two physical objects or ideas, i.e., length, time interval, gap, rank difference, coolness, or remoteness, while the term *metric* is often used as a standard for a measurement.

But here we consider, except for the last two chapters, the mathematical meaning of those terms which is an abstraction of measurement. The mathematical notions of *distance metric* (i.e., a function  $d(x, y)$  from  $X \times X$  to the set of real numbers satisfying to  $d(x, y) \geq 0$  with equality only for  $x = y$ ,  $d(x, y) = d(y, x)$ , and  $d(x, y) \leq d(x, z) + d(z, y)$ ) and of *metric space*  $(X, d)$  were originated a century ago by M. Fréchet (1906) and F. Hausdorff (1914) as a special case of an infinite topological space. The *triangle inequality* above appears already in Euclid. The infinite metric spaces are usually seen as a generalization of the metric  $|x - y|$  on the real numbers. Their main classes are the measurable spaces (add measure) and Banach spaces (add norm and completeness).

However, starting from K. Menger (who, in 1928, introduced metric spaces in geometry) and L.M. Blumenthal (1953), an explosion of interest in both finite and infinite metric spaces occurred. Another trend is that many mathematical theories, in the process of their generalization, settled on the level of metric space. It is an ongoing process, for example, for Riemannian geometry, real analysis, and approximation theory.

Distance metrics and distances have become now an essential tool in many areas of mathematics and its applications including geometry, probability, statistics, coding/graph theory, clustering, data analysis, pattern recognition, networks, engineering, computer graphics/vision, astronomy, cosmology, molecular biology, and many other areas of science. Devising the most suitable distance metrics and similarities, in order to quantify the proximity between objects, has become a standard task for many researchers. Especially intense ongoing search for such distances occurs, for example, in computational biology, image analysis, speech recognition, and information retrieval.

Often the same distance metric appears independently in several different areas, for example, the edit distance between words, the evolutionary distance in biology,

the Levenshtein metric in coding theory, and the Hamming+Gap or shuffle-Hamming distance.

This body of knowledge has become too big and disparate to operate within. The number of worldwide web entries offered by Google on the topics “distance,” “metric space,” and “distance metric” is about 216, 3, and 9 million, respectively, not to mention all the printed information outside the web, or the vast “invisible web” of searchable databases. About 15,000 books on Amazon.com contain “distance” in their titles. However, this huge information on distances is too scattered: the works evaluating distance from some list usually treat very specific areas and are hardly accessible for nonexperts.

Therefore many researchers, including us, keep and cherish a collection of distances for use in their areas of science. In view of the growing general need for an accessible interdisciplinary source for a vast multitude of researchers, we have expanded our private collection into this dictionary. Some additional material was reworked from various encyclopedias, especially *Encyclopedia of Mathematics* ([EM98]), *MathWorld* ([Weis99]), *PlanetMath* ([PM]), and *Wikipedia* ([WFE]). However, the majority of distances are extracted directly from specialist literature.

Besides distances themselves, we collected here many distance-related notions (especially in Chap. 1) and paradigms, enabling people from applications to get those (arcane for nonspecialists) research tools, in ready-to-use fashion. This and the appearance of some distances in different contexts can be a source of new research.

In the time when overspecialization and terminology fences isolate researchers, this dictionary tries to be “centripetal” and “ecumenical,” providing some access and altitude of vision but without taking the route of scientific vulgarization. This attempted balance defined the structure and style of the dictionary.

This reference book is a specialized encyclopedic dictionary organized by subject area. It is divided into 29 chapters grouped into 7 parts of about the same length. The titles of the parts are purposely approximative: they just allow a reader to figure out her/his area of interest and competence. For example, Parts II, III, IV, and V require some culture in, respectively, pure and applied mathematics. Part VII can be read by a layman.

The chapters are thematic lists, by areas of mathematics or applications, which can be read independently. When necessary, a chapter or a section starts with a short introduction: a field trip with the main concepts. Besides these introductions, the main properties and uses of distances are given, within items, only exceptionally. We also tried, when it was easy, to trace distances to their originator(s), but the proposed extensive bibliography has a less general ambition: just to provide convenient sources for a quick search.

Each chapter consists of items ordered in a way that hints of connections between them. All item titles and (with majuscules only for proper nouns) selected key terms can be traced in the large subject index; they are boldfaced unless the meaning is clear from the context. So, the definitions are easy to locate, by subject, in chapters and/or, by alphabetic order, in the subject index.

The introductions and definitions are reader-friendly and maximally independent each from another; still they are interconnected, in the 3D HTML manner, by hyperlink-like boldfaced references to similar definitions.

Many nice curiosities appear in this “Who Is Who” of distances. Examples of such sundry terms are ubiquitous Euclidean distance (“as-the-crow-flies”), flower-shop metric (shortest way between two points, visiting a “flower-shop” point first), knight-move metric on a chessboard, Gordian distance of knots, Earth mover’s distance, biotope distance, Procrustes distance, lift metric, post office metric, Internet hop metric, WWW hyperlink quasi-metric, Moscow metric, and dogkeeper distance.

Besides abstract distances, the distances having physical meaning appear also (especially in Part VI); they range from  $1.6 \times 10^{-35}$  m (Planck length) to  $8.8 \times 10^{26}$  m (the estimated size of the observable Universe, about  $5.4 \times 10^{61}$  Planck lengths).

The number of distance metrics is infinite, and therefore our dictionary cannot enumerate all of them. But we were inspired by several successful thematic dictionaries on other infinite lists, for example, on numbers, integer sequences, inequalities, and random processes, and by atlases of functions, groups, fullerenes, etc. On the other hand, the large scope often forced us to switch to the mode of laconic tutorial.

The target audience consists of all researchers working on some measuring schemes and, to a certain degree, students and a part of the general public interested in science.

We tried to address all scientific uses of the notion of distance. But some distances did not make it to this dictionary due to space limitations (being too specific and/or complex) or our oversight. In general, the size/interdisciplinarity cut-off, i.e., decision where to stop, was our main headache. We would be grateful to the readers who will send us their favorite distances missed here.

Encyclopedia of Distances

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2016, XXII, 756 p. 2 illus., Hardcover

ISBN: 978-3-662-52843-3