# Preface

This thesis is devoted to the study of problems of automata theory from the point of view of descriptive set theory. The analyzed structures are $\omega$-words and infinite trees. Most of the results presented here have the form of an effective decision procedure that operates on representations of regular languages.

Special effort is put into providing effective characterizations of regular languages of infinite trees that are definable in weak monadic second-order logic (WMSO). Although no such characterization is known for all regular languages of infinite trees, the thesis provides characterizations in some special cases: for game automata, for languages of thin trees (i.e., trees with countably many branches), and for Büchi automata. Additionally, certain relations between WMSO-definable languages and Borel sets are proved.

Another problem studied in the thesis is the alternating index problem (also called the Rabin–Mostowski index problem). Again, the problem in its full generality seems to be out of the reach of currently known methods. However, a decision procedure for the class of game automata is proposed in the thesis. These automata form the widest class of automata for which the problem is currently known to be decidable.

The thesis also addresses the problem of providing an algebraic framework for regular languages of infinite trees. For this purpose, the notion of prophetic thin algebras is introduced. It is proved that finite prophetic thin algebras recognize exactly the bi-unambiguous languages — languages $L$ such that both $L$ and the complement $L^{\mathrm{c}}$ can be recognized by unambiguous automata. Additionally, a new conjecture about the definability of choice functions is stated. It is proved that this conjecture is strongly related to the class of prophetic thin algebras. In particular, the conjecture implies an effective characterization of the class of bi-unambiguous languages.

Finally, the thesis studies contemporary quantitative extensions of the class of regular languages. First, lower bounds (that match upper bounds) on the topological complexity of MSO+U-definable languages of $\omega$-words are given. These lower bounds can be used to prove that MSO+U logic is undecidable on infinite trees in a specific sense. It is also shown that languages of $\omega$-words recognizable by certain counter automata have a separation property with respect to $\omega$-regular languages. The proof relies on topological methods in the profinite monoid.

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