

Chapter 2

Reaction-Diffusion of the Number of Children

Abstract When we admit the background independence of fertility decline and aspects of diffusion, we can construct a reaction-diffusion model to describe fertility decline as a stochastic process independent of its background. Once we postulate a reaction-diffusion process for this phenomenon, we can estimate the velocity of a progressive wave of diffusion. By means of the estimated velocity, we can estimate where a singularity of fertility decline was and when it appeared. The singularity existed in a French district *Aquitaine* basin. From *Lot-et-Garonne* in *Aquitaine* the reaction-diffusion of fertility decline began to diffuse to all Europe maintaining independence of socio-economic conditions.

Keywords Fertility decline · Reaction-diffusion · Singularity · Progressive wave · Stochastic limit · *Aquitaine* · *Lot-et-Garonne*

It is the most suitable solution for the predictions falsifiable to make Reaction-Diffusion System as background independent. It is essential and crucial to know the stochastic causality of the spatial number of children for the theory of fertility decline. Motivations of birth control are trifling even though they seem primal for our subjectivity.

2.1 What Is the Reaction-Diffusion System?

I introduce the “reaction-diffusion system” briefly. Reaction-diffusion systems are used to explain the development of various patterns or structures in space such as black hole, organs of organic body, and geographic undulations of fertility. A.M. Turing [65] insisted that the differential equation such as Eq. (2.1) or Eq. (2.7) consists of diffusion term and nonlinear development term describes various patterns of processes by means of their interactions.

$$\frac{\partial f(x, t)}{\partial t} = \frac{\partial^2 f(x, t)}{\partial x^2} + g(f(x, t)) \quad (2.1)$$

The first term is a normal diffusion term. x is a parameter of space. t is a time. In reaction-diffusion system, “reaction” correspond to nonlinear development term $g(f(x, t))$. Reaction is rather an acceleration effect to a process. Reaction-diffusion system describes the development of f in a space by time.

Let us compare reaction-diffusion equation to McKendrick-von-Foerster equation well-known in Demography.

$$\frac{\partial p(t, a)}{\partial t} + \frac{\partial p(t, a)}{\partial a} = -\mu(a)p(t, a) \quad (2.2)$$

McKendrick equation describes the dynamics of the age structure $p(t, a)$ by mortality by age $\mu(a)$ and itself. In contrast with this, reaction-diffusion system pays no attention to the age structure of a given population. Containing a space parameter x in second order in the reaction-diffusion equation, the density-gradient plays a main role. Reaction-diffusion equation of the number of children describes solely how low fertility spreads and forms some pattern in a given space. In the first place R.A. Fisher [25] proposed RDE (Reaction–Diffusion Equation) in 1937 to describe how the advantageous genes spread in the space. Namely RDE congenitally equips the ability to describe spacial fertility decline.

2.2 Reaction-Diffusion of the Number of Children

The first proposition that I introduce is that parents decide stochastically the number of children they will have under the influence of parents in their neighbourhood.

Proposition 2.1 NDISD (Neighbourhood Dependent Isotropic Stochastic Decision)
The number of children of each set of parents is isotropically and stochastically dependent on neighbouring numbers of children.

We have good reason to suppose that the number of children is a stochastic variable. The indirect empirical evidence is that the completed marital fertility is almost proportional to the age when she married. L. Henry [28] observed this phenomena. When we assume that conceptions occur randomly, this proportionality is easily derived from this assumption.¹ So parents’ decisions have fluctuations and a variance caused by them. If we calculate the number of children of parents under constant socio-economic conditions, we must observe fluctuations and a variance caused by them.

¹Let p be a probability of a conception at a moment. $\int_0^t p dt$ is the number of children for t time. $\int_0^t p dt \equiv p(t + C)$.

As for the neighbourhood dependency, it is difficult to show empirical evidences here. When we admit this proposition, we can describe how high fertility changes to lower fertility, namely how relative lower fertility in a spot diffuses to other more fertile areas. After all, the validity of the neighbourhood dependency should be evaluated by this article itself. The discovery of a singularity of fertility decline can never be done without this assumption.

No one can deny that we receive influences form neighbouring others. I think NDISD is much more reliable than the doubtful revealed preference hypothesis.

2.2.1 Diffusion of Low Fertility

Consider parents distributed in one dimensional space (a linear habitat) divided into segments. Each segment has a value c which represents a level of fertility of parents resident in its segment. Normally we think c is a mean of the number of children of all sets of parents in a segment.

In the latter parts of this article, I consider higher dimensional spaces, so each segment will be rectangle in two or three dimensional space.

See Fig. 2.1. Let us conduct a “thought experiment” about what interaction takes place between a high fertility segment $c(x_0)$ and a neighbouring low fertility segment $c(x_{-1})$ in time. A segment $c(x_0)$ whose fertility is higher and a segment $c(x_{-1})$ whose fertility is lower affect each other, so that high fertility of $c(x_0)$ shifts to lower than before and low fertility of $c(x_{-1})$ shifts to higher. These alterations occur because parents in each segment stochastically affect each other.

Let us think, in Fig. 2.1, precisely and iteratively by time t_k about segments $c(x_{i-2}, t_0)$, $c(x_{i-1}, t_0)$ whose fertility is lower and about segments $c(x_{i+0}, t_0)$, $c(x_{i+1}, t_0)$ whose fertility is higher at time $t = 0$.

At $t = 1$, $c(x_{i-1}, t_1)$ rises, $c(x_{i+0}, t_1)$ descends. Next at $t = 2$, $c(x_{i-1}, t_2)$ rises and $c(x_{i+0}, t_2)$ descends exceedingly, along with these $c(x_{i-2}, t_2)$ rises and $c(x_{i+1}, t_2)$ descends a little. Between time t_0 to t_2 (for two unit of time), low fertility progressed from x_{i-2} to x_{i+1} . Thus low fertility have progressed three segments. These processes will be continued until there are no undulations to be smoothed.

In this simple “thought experiment”, we can recognise three characteristics of historical fertility decline that is : *irreversibility, stability, and smoothing*. In addition

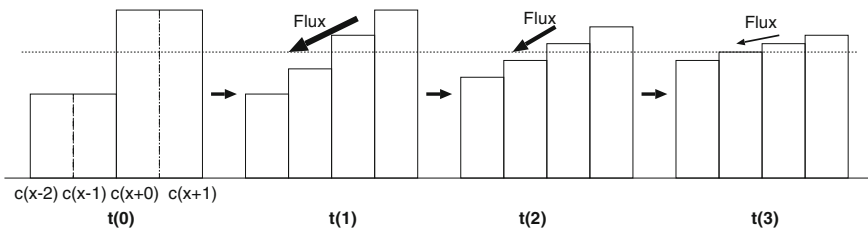


Fig. 2.1 When high fertility group contact to low fertility group

to them, we can see how a “progressive wave” moves. That is to say stochastic fluctuations of the number of children of parents convey lower fertility.

2.2.1.1 Not Imitation nor a Meme

Someone may consider this conveyance of lower fertility by stochastic fluctuations as “Imitation”. This understanding is not precise. In the process of fertility decline, rarely some parents imitate the number of neighbouring parents consciously. However an overwhelmingly majority of parents did not have such affection. They consciously and originally decide their number of children taking into account neighbouring number of children. After deliberation they coincidentally select same number.

Fertility decline is not the result of spread of a meme (R. Dawkins [17])—an idea, behavior, or style—from parents to parents within a given space. This realisation is not correct because it is difficult for RDE to spread the meme when we assume the meme of fertility decline. When an old meme which caused high fertility was the majority in a space, how can a new meme spread? Consequently we cannot help postulating the advantage or benefit of a new meme. Were there any benefit from the birth control in France around the end of eighteen century? A meme hypothesis cannot resolve these problems.

Another reason is that a meme is not well-defined so that we cannot verify its existence.

The final and simple consequence is that we act receiving the influence of neighbouring others. The most significant conception is that the way of receiving the influence of neighbouring others is universal.

2.2.2 Reaction as Disintegration of the Balance of Stochastic Fluctuations

The process of historical fertility decline suggests that there are a very few parents who acceleratingly diminish their number of children in reaction to the number of children in their space.

The reason of this reaction is explained by “disintegration of the balance of stochastic fluctuations”. We may postulate following corollary to explain “Reaction”.

Corollary 2.1 (Reaction as Outlier in Neighbourhood) *Parents rarely bear much different number of children from neighbourhood.*

Proposition 2.1 naturally includes this corollary. Also nowadays these rare numbers of children occur in lower and higher directions.² Before fertility decline, these rare

²These occurrences are also supported empirically. Even Hutterites have a maximum, a minimum, a mean, and a variances of their number of children.

occurrences balanced with each other. Statistically the average of the number of children is relatively stable then.

Corollary 2.2 (Reaction to One-way as declining fertility) *Child bearing is a product of behaviours more than two persons (a set of parents), whereas family limitation is not. The balance of each other inclines to disintegrated to lower directions.*

So the number of children of parents inclines to decline, if there are significant declines in neighbourhood. Let β be a highest fertility. $\beta - c(x)$ is a intensity of disintegration of balance. Let γ be a lowest fertility. $\gamma - c(x)$ is a room of decline.

2.2.3 Fertility Decline Difference Equation

When $c(x_0, t_0)$ significantly declines, what takes place? As a due course of our postulation, some decline of $c(x_0, t_0)$ affects other neighbouring segments' $c(x_i)$ recursively. Parents in each segment refer to each other, so at the next time period fertility in neighbouring segments' $c(x_{0-1}, t_1)$, $c(x_{0+1}, t_1)$ begins to decline. We can express this process in the following difference equation about $c(x_i, t_{k+1})$.

$$\begin{aligned} c(x_i, t_{k+1}) &= \mu c(x_{i-1}, t_k) + \mu c(x_{i+1}, t_k) + \\ &\quad (1 - 2\mu) [(c(x_i, t_k) + \alpha(\beta - c(x_i, t_k))(\gamma - c(x_i, t_k)))] \end{aligned} \quad (2.3)$$

μ is a coefficient of influence from adjoining segments. We can interpret it to be a coefficient of diffusion. We assume $0.0 \leq \mu \leq 1/2$. α is a coefficient of reaction within this $c(x_i)$ segment. Constant β is a highest fertility (=maximum \bar{c}). Constant γ is a lowest fertility (=minimum \bar{c}). Here after call Eq. (2.3) the “**Difference Equation of Spacial Distribution of Children (DESDC)**”. This difference equation is a reaction-diffusion system of the number of children in space.

When every $c(x_i, t_k) = \beta$, Eq. (2.3) is an equality (identity mapping). This state corresponds to maximum marital fertility. When every $c(x_i, t_k) = \gamma$, Eq. (2.3) is also an equality (identity mapping). This state corresponds to minimum marital fertility.

2.2.3.1 An Extreme Case of DESDC

We can think an undifferentiated space lacks neighbouring rectangles. The Eq. (2.3) degenerates to an equation of only the Reaction term.

$$c(x_i, t_{k+1}) = c(x_i, t_k) + \alpha(\beta - c(x_i, t_k))(\gamma - c(x_i, t_k)) \quad (2.4)$$

This equation describes fertility decline in a macro scale population such as a nation-state. I name this equation MFDDE (Macro Fertility Decline Equation). I can obtain

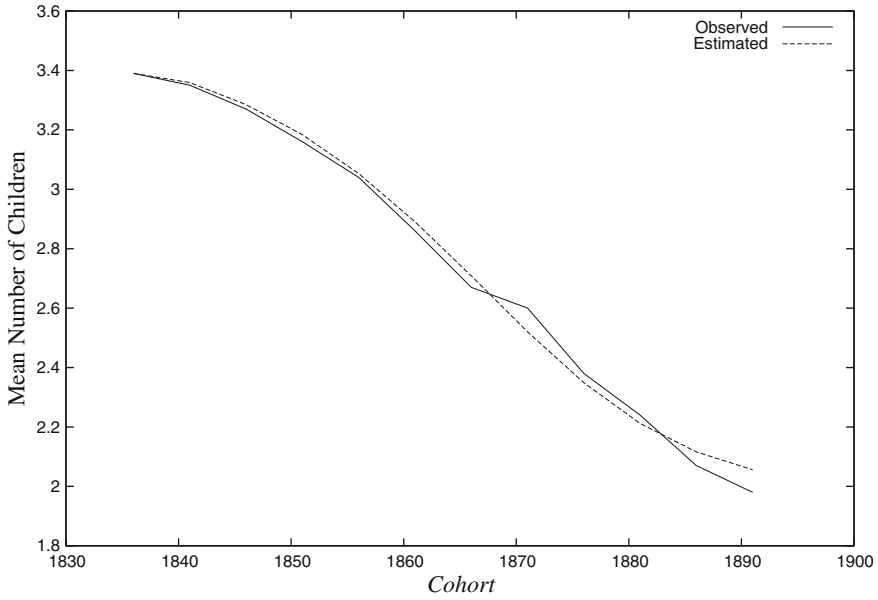


Fig. 2.2 Fertility decline of France and MFDDE. *Source* P. Festy [24], p. 48

fine fits of fertility decline for several European countries and Japan (Figs. 2.2, 2.3, 2.4, 2.5, 2.6).

These fine fits suggest that reaction-diffusion is a valid system to describe the fertility decline process of Europe and Japan (Table 2.1).

2.2.4 The Application of Fertility Decline Difference Equation

Next, I estimate coefficients μ , α from Japanese fertility decline processes, first with a space that is one dimensional and second with a two dimensional space.

2.2.4.1 One Dimensional Space

When we cut our geographical space from centre to periphery, we will observe one dimensional decline of fertility. By means of this linearisation, we can deal with our world as quasi-one dimensional one. It facilitates estimating coefficients μ , α for one dimensional space.

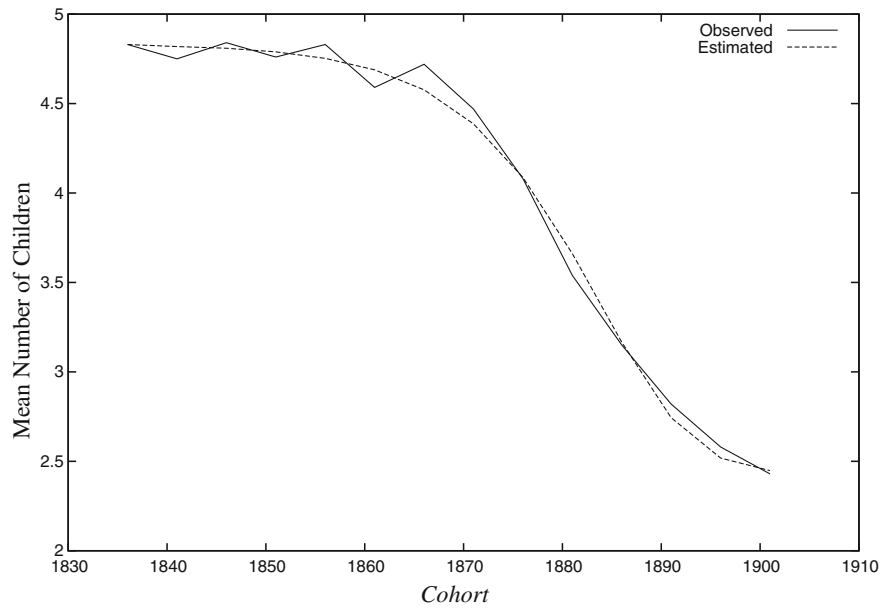


Fig. 2.3 Fertility decline of Finland and MFDDE. *Source* P. Festy [24], p. 48

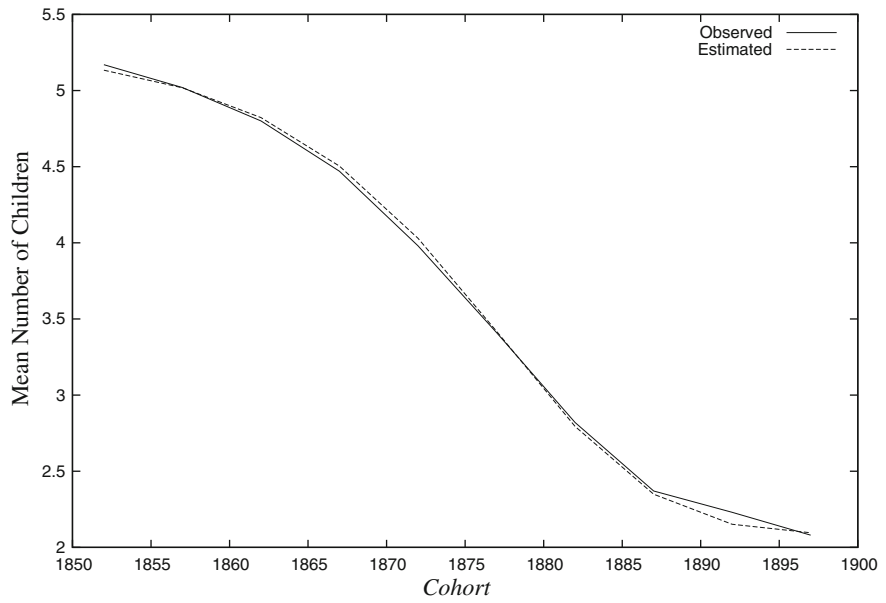


Fig. 2.4 Fertility decline of Germany and MFDDE. *Source* P. Festy [24], p. 49

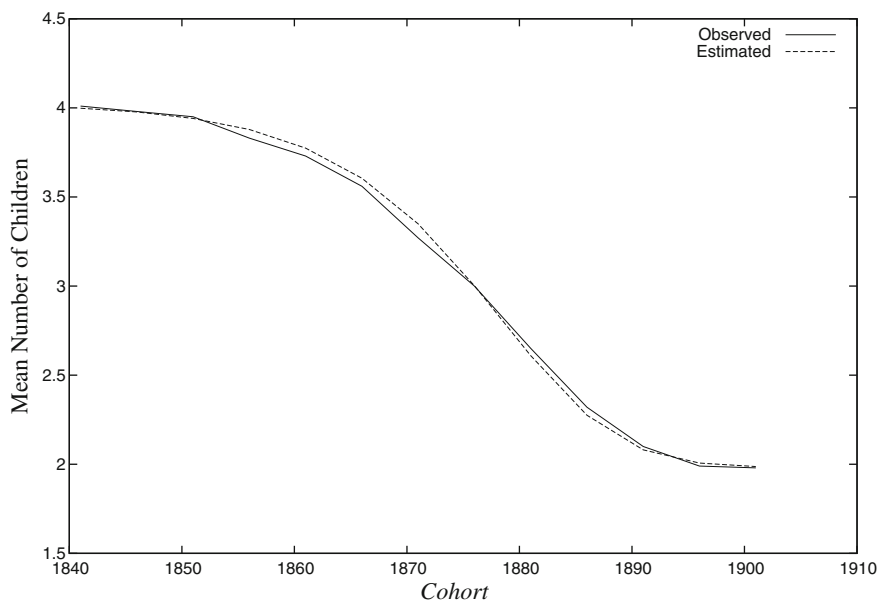


Fig. 2.5 Fertility decline of Swiss and MFDDE. *Source* P. Festy [24], p. 49

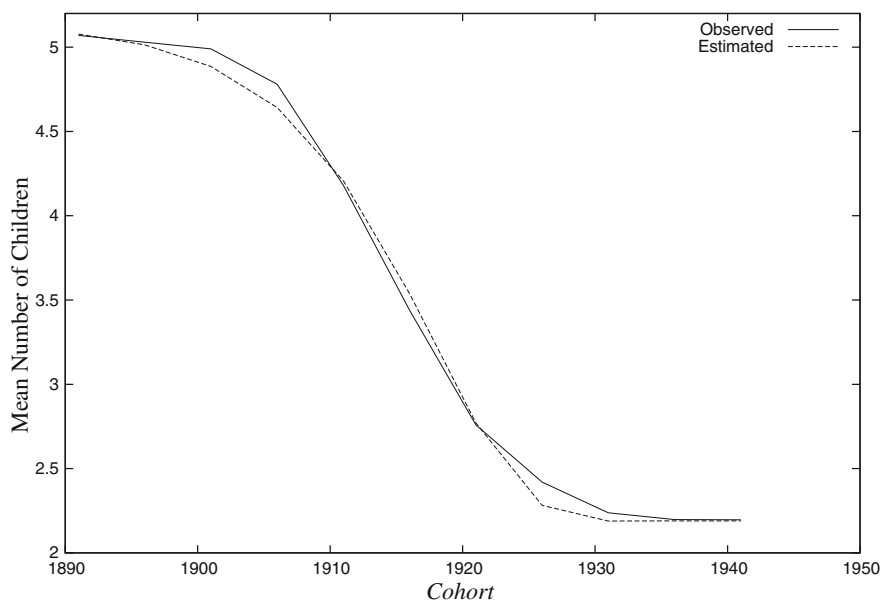
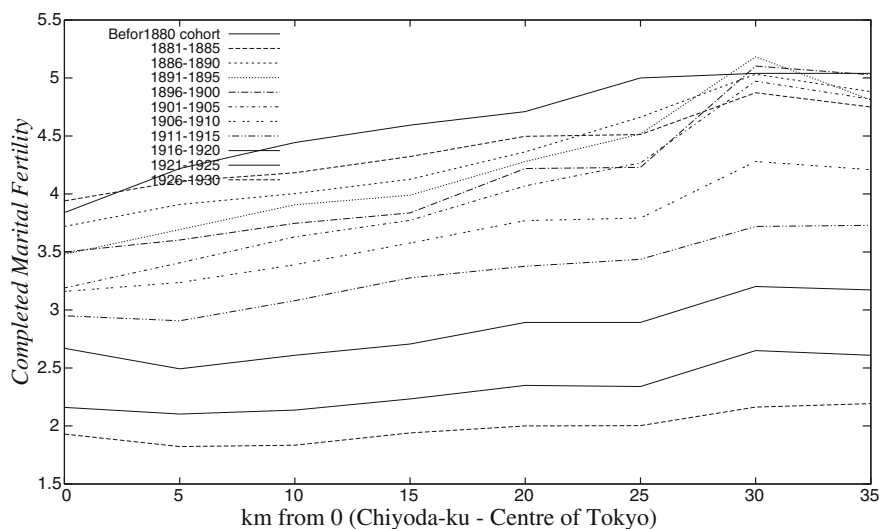


Fig. 2.6 Fertility decline of Japan and MFDDE. *Source* S. Ike [30], p. 38

Table 2.1 Estimated α' , β , γ in MFDDE

	α'	β	γ	$\sum error^2$	Period
Japan	0.33376	5.146	2.11	0.066	1890–1940
France	0.31376	3.531	1.98	0.020	1836–1891
Finland	0.34414	4.833	2.43	0.075	1836–1901
Germany	0.25000	5.285	2.08	0.013	1852–1897
Swiss	0.37500	4.026	1.98	0.017	1841–1901

**Fig. 2.7** Geographical marital fertility decline diffusion for Tokyo area. (observed). *Source* Sourihu Toukei Kyoku [62]

Thus I draw the following figures of geographical marital fertility decline diffusion for Tokyo area from Japanese census data (Fig. 2.7).³

The Method to Estimate μ , α

I use “direct search methods” [56] to find the estimations of μ , α . The direct search methods are those to find a minimizer (or maximizer) only by the values of functions, so it does not depend on Taylor expansion but on convex analysis. “Nelder-Mead

³Actually I selected Tokyo (the capital of Japan) area and Osaka (the centre of west Japan) and their surroundings. Japanese census recoded the means of number of children ever born of ever married women at 1960 and at 1970 by five-year cohorts for city, town and village. I draw circles with each radius (5, 10, 15, 20, 25, 30, 35 km) from the centre where fertility decline began. And I select some cities or towns or villages nearest to each circumference. I set an average of these \bar{c} for a representative \bar{c}_i at each radius. From 1896–1900 (born) cohort to 1921–1925 cohort, $c(x_i, t_k)$ was recoded for each cities.

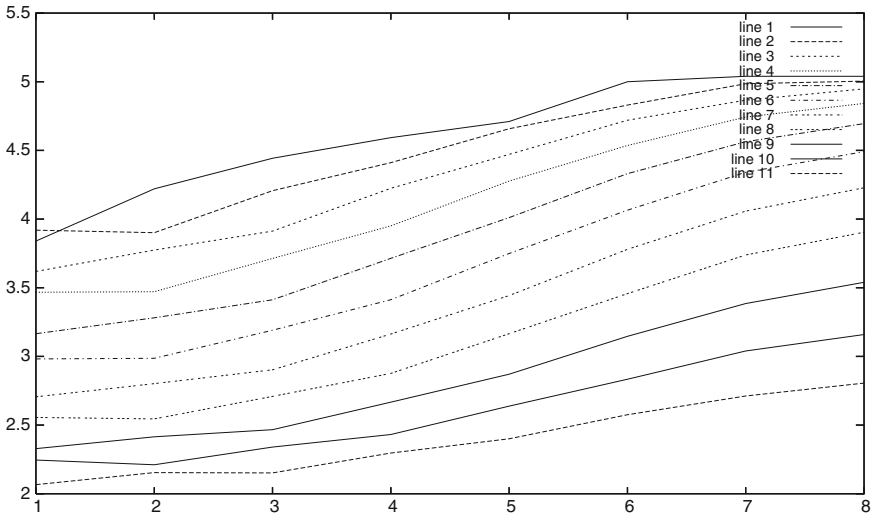


Fig. 2.8 Geographical marital fertility decline diffusion for Tokyo area. (estimated) $\mu = 0.4777, \alpha = 2.747, \beta = 5.11, \gamma = 2.11$. *line 1 = Before 1880 cohort, line 2 = 1881–1885, line 3 = 1886–1890, line 4 = 1891–1895, line 5 = 1896–1900, line 6 = 1901–1905, line 7 = 1906–1910, line 8 = 1911–1915, line 9 = 1916–1920, line 10 = 1921–1925, line 11 = 1926–1930

Simplex method” [18, 67] is the most frequently applied direct search method. This simplex method is robust against noises in function values and useful for problems of low dimensional variables. However it may not converge to a true minimizer and there is no adequate analysis of convergence.

I apply this Nelder-Mead Simplex method and subsidiary original algorithm (simulated annealing method [56]) to search for the minimizer by procedures to branch separated local space and search narrower area iteratively. This algorithm lacks the analytic bases. Unfortunately Nelder-Mead Simplex method depends upon the choice of the initial simplex, so several local minimizers are found with very slight difference of the sum of squares of residuals.

Nonetheless the simplex method can find a plausible minimiser for DESDC. Simplex method and simulated annealing method can find very approximate minimizers for μ, α (observed values are set for β, γ).

Next I draw the estimated figure of geographical marital fertility decline diffusion for Tokyo area (Fig. 2.8).

The DESDC can describe the geographical fertility reaction-diffusion process of Tokyo area in Japan fairly well (Table 2.2).

Table 2.2 Estimated μ, α , of Tokyo area

Method of estimation	μ	α	$\sum e^2$
Nelder-Mead simplex method (2)	0.47815	2.80000	3.73783
Simulated annealing method	0.47773	2.74658	3.74058

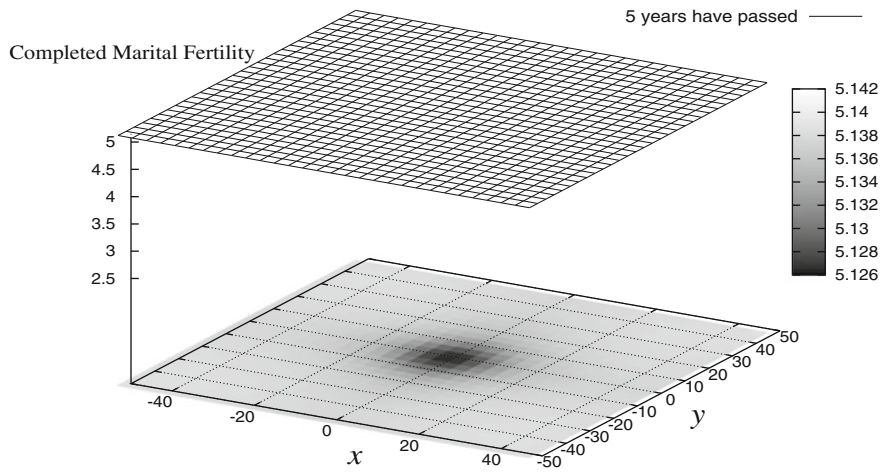


Fig. 2.9 Reaction-diffusion of fertility decline on two dimensional space (5 years have passed)

2.2.4.2 Two Dimensional Space

We can further extend DESDC to the equation for two dimensional space. Assume three dimensional surface composed by each rectangle $c(x_i, y_j)$. Let c be a mean number of children of each rectangle. A rectangle (x_i, y_j) correspond to geographical surface. c is imaged as a height of rectangle in three dimensional space. c is a function of x, y, t namely of space-time.

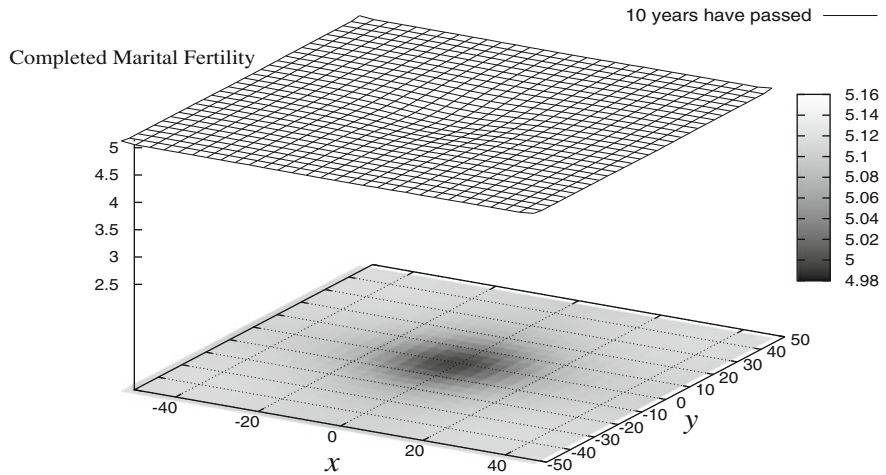


Fig. 2.10 Reaction-diffusion of fertility decline on two dimensional space (10 years have passed)

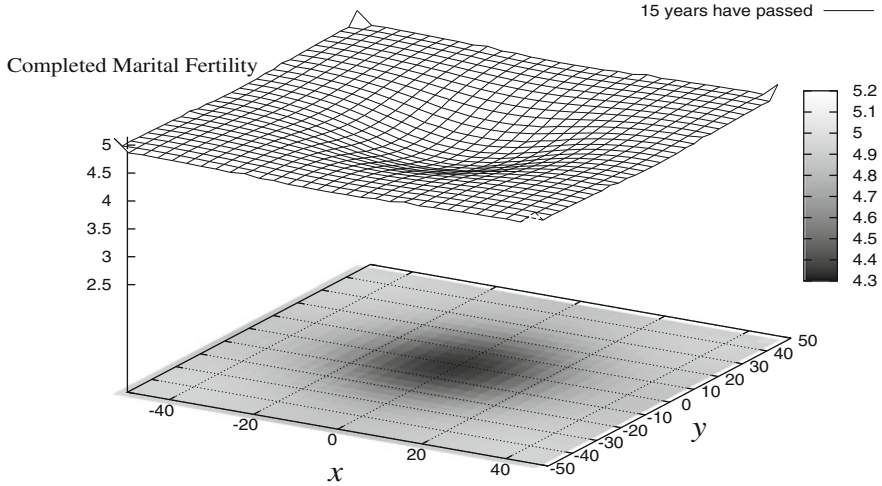


Fig. 2.11 Reaction-diffusion of fertility decline on two dimensional space (15 years have passed)

Consider a population distributed in a two dimensional space with c surface. Let $c(x_i, y_j, t_k)$ be a mean of the number of children of rectangle (x_i, y_j) at time k . Let $c(x_{i-1}, y_j, t_k)$, $c(x_{i+1}, y_j, t_k)$, $c(x_i, y_{j-1}, t_k)$, $c(x_i, y_{j+1}, t_k)$ be a mean of the number of children of neighbouring rectangles at time t_k .

From isotropy of Proposition 2.1, neighbouring rectangles $c(x_i, y_j, t_k)$ equally affect each other. Therefore $c(x_i, y_j, t_{k+1})$ is expressed from $c(x_{i-1}, y_j, t_k)$, $c(x_{i+1}, y_j, t_k)$, $c(x_i, y_{j-1}, t_k)$, $c(x_i, y_{j+1}, t_k)$ as the following difference equation (Figs. 2.9 and 2.10).

$$\begin{aligned}
 c(x_i, y_j, t_{k+1}) &= \mu c(x_{i-1}, y_j, t_k) + \mu c(x_{i+1}, y_j, t_k) + \\
 &\quad \mu c(x_i, y_{j-1}, t_k) + \mu c(x_i, y_{j+1}, t_k) + \\
 &\quad (1 - 4\mu) [c(x_i, y_j, t_k) + \alpha(\beta - c(x_i, y_j, t_k))(\gamma - c(x_i, y_j, t_k))]
 \end{aligned} \tag{2.5}$$

If at an origin, at $t = 0$, $c(x_0, y_0, t_0)$ declines significantly, neighbouring $c(x_i, y_j, t_1)$ begins to decline recursively by the Difference Equation DESDC (2.5) from origin to peripheral rectangles. We image a process of fertility decline as a developing hole in 3D surface like Fig. 2.11. For two dimensional space, we can also numerically simulate a reaction-diffusion system.

2.2.5 Differential Equation of Fertility Decline

By Taylor expansion, difference equation DESDC (2.5) is transposed to a differential equation. This conversion allows examining the velocity of the diffusion wave from the differential equation of spacial distribution of children (SDC) (Fig. 2.12).

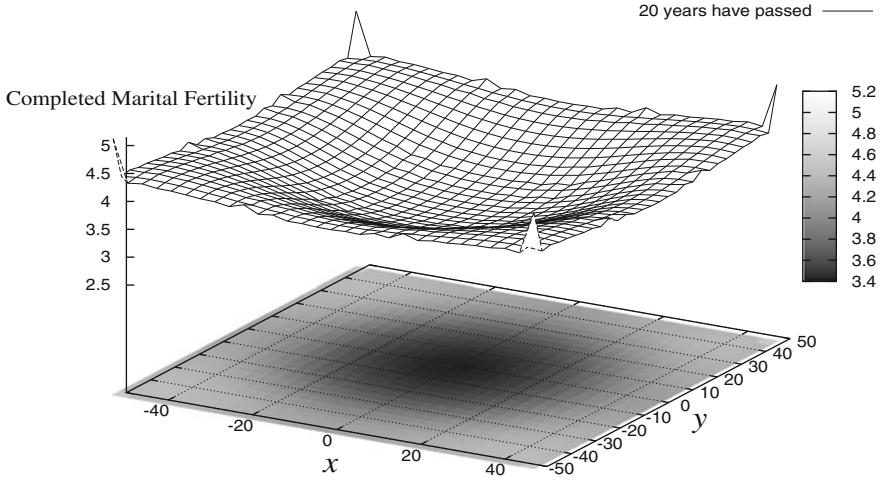


Fig. 2.12 Reaction-diffusion of fertility decline on two dimensional space (20 years have passed)

$$\begin{aligned} \frac{\partial c(x, y, t)}{\partial t} = & \mu \frac{\partial^2 c(x, y, t)}{\partial x^2} \frac{\Delta a^2}{\Delta t} + \mu \frac{\partial^2 c(x, y, t)}{\partial y^2} \frac{\Delta a^2}{\Delta t} \\ & + \frac{(1 - 4\mu)}{\Delta t} \alpha (\beta - c(x, y, t)) (\gamma - c(x, y, t)) \end{aligned} \quad (2.6)$$

When we accept a relation $\Delta a^2 = \Delta t = (1 - 4\mu)$ in an infinite transportation, differential equation (2.6)

$$\frac{\partial c(x, y, t)}{\partial t} = \mu \frac{\partial^2 c(x, y, t)}{\partial x^2} + \mu \frac{\partial^2 c(x, y, t)}{\partial y^2} + \alpha (\beta - c(x, y, t)) (\gamma - c(x, y, t)) \quad (2.7)$$

turn to the equation above with the same coefficients as the difference equation DESDC. This procedure means that we minimise rectangles to a limit of space-time.

We can write this differential equation using two dimensional value c and Laplacian Δ in another style as following.

$$\frac{\partial c}{\partial t} = \mu \Delta c + \alpha (\beta - c) (\gamma - c) \quad (2.8)$$

2.2.5.1 Variances of Fertility Decline Are Proportional to Time in Its Initial Stage

As long as the effects of Reaction-term are still little, we can recognise the differential equation (2.7) as a normal diffusion equation. We obtain an analytical basic solution as the Eq. (2.9).

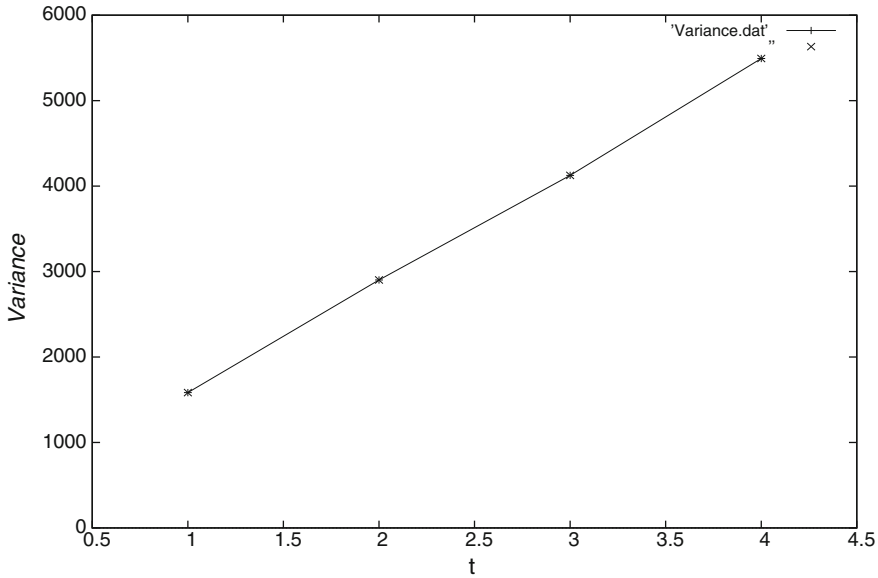


Fig. 2.13 Variances of fertility decline with time t in Hokkaido

$$f(x, t) = \frac{1}{\sqrt{4\pi t}} \exp\left(-\frac{x^2}{4t}\right) \quad (2.9)$$

It is just a normal distribution. We can find a relation $2t = \sigma^2$ in this equation. When fertility decline is described by the differential equation (2.7), variances of the amount of fertility decline are proportional to time t .

I have already observed fertility decline processes of three areas in Japan. All three areas Hokkaido, Tokyo, and Osaka have been observed over 50 years and their data of variances of fertility decline in initial 20 years analyzed. All these three areas showed a very similar upward tendency. However, the proportionality in Tokyo and Osaka cases seemed a bit imperfect. The linearity in Hokkaido was almost perfect (Fig. 2.13).^{4, 5}

⁴I believe this is because of the size of the area that data cover, with Tokyo and Osaka being much smaller regions than Hokkaido. In order to demonstrate the perfect proportionality, in line with the argument by J.G. Skellam I have resorted to use Hokkaido case, roughly 9 times as large as those in Tokyo, Osaka.

⁵The famous article in mathematical ecology by J.G. Skellam [61] reported the same linearity of diffusion of muskrats from Bohemia to mid-Europe. In his article, the space is widened from Munich to Breslau—in a circle with 300 km radius.

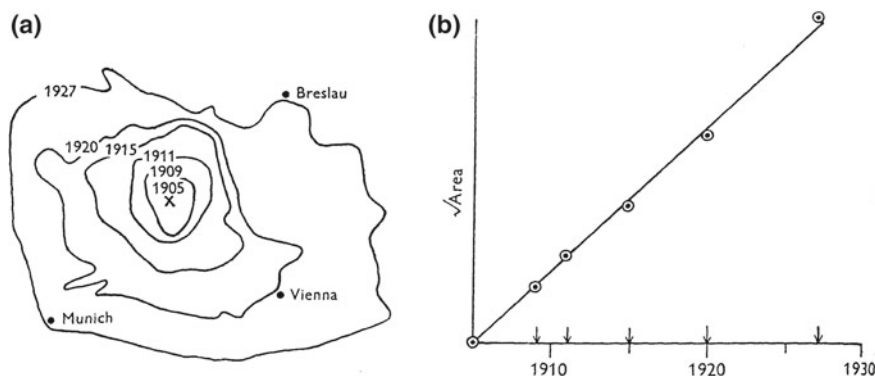


Fig. 2.14 The Spread of the muskrat—**a** The apparent boundaries, **b** The relation between time and $\sqrt{\text{area}}$. Source J. G. Skellam [61], p. 200

2.2.5.2 Properties of Reaction-Diffusion Equation of Fertility Decline

A nontrivial solution of the differential equation SDC (2.7) has some properties that are specifically observed in the phenomena of fertility decline. A solution of reaction-diffusion equation has a remarkable effect of smoothing (even though it depends on the balance of coefficients μ and α).

This smoothing effect explains the irreversibility, stability, and smoothing of fertility decline. After “demographic transition”, except for baby boom, fertility in developed countries have not risen up again. Baby boom was terminated at last and marital fertility have been converging to the minimum level of fertility in the developed countries. Marital fertility of them has been rather stable. Since smoothing effects have governed each parents in space-time, differences of fertility could had been brought about by socio-economic status have been diminishing. In other words, neighbouring dependency of the number of children of parents is an essential element. Differential fertility observed in the past was a temporal phenomenon caused by a partial geographical distribution of socio-economic status.

2.2.6 Progressive Wave

A nontrivial solution of the differential equation SDC (2.7) is called a “Kolmogoroff-Petrovsky-Piscounoff Progressive Wave” [43] (Fig. 2.14).

It is drawn graphically in Fig. 2.15. Progressive wave spread from a singularity to peripherals. In time the points where minimum fertility decline occurred had advanced to peripherals. We call these points “front” or “edge” of Progressive Wave (See “Front” in Fig. 2.15). In other words, when front arrives, fertility decline begins.

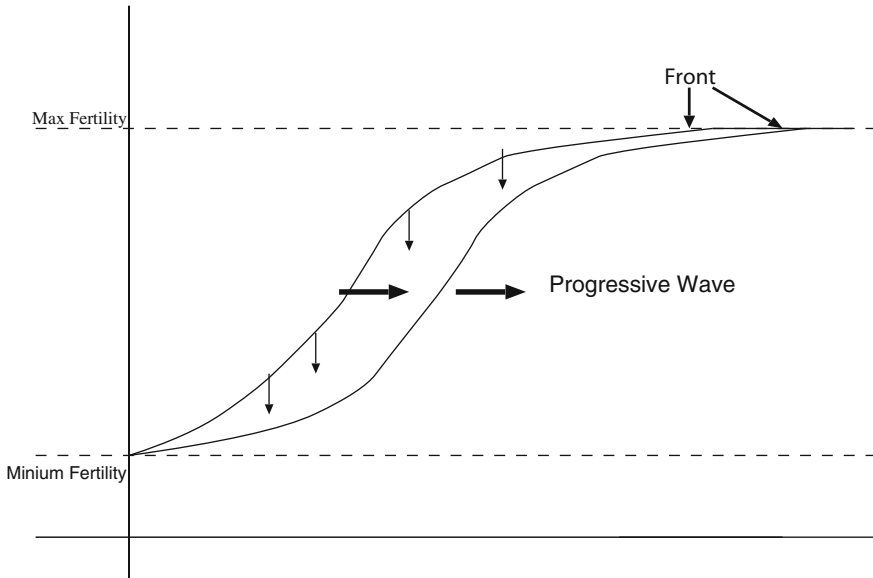


Fig. 2.15 Progressive wave of fertility decline

2.2.6.1 The Velocity for One Dimensional Space

R.A. Fisher [25] showed this progressive wave has a velocity. His mathematical model is designed for one dimensional space. This low dimensionality is rather convenient for our purpose of estimation.

For each geographical point, the times the front of wave takes to arrive differs proportional to the distances from a singularity. Thus by means of the difference in time when marital fertility decline began we are able to estimate the velocity of progressive wave.

For Japan, I estimate $10 \text{ km/year} \leq v \leq 14 \text{ km/year}$ as the velocity of the front of wave from differences of the occurrence of fertility decline in time recorded in census data is shown Table 2.3.

On the other hand, we previously obtained the estimations of μ, α by Simplex method. Fisher deduced that $v = 2\sqrt{\mu\alpha}$ in one dimensional space. In order to be accord the observations with a unit of velocity (km per year) in our data to, it is necessary to multiply the intervals of the ratio of the square of space (h^2) and to divide the ratio of time (k) to each coefficient. The data which I employ to estimate them are composed of means of children for segments (5 km long) by 5 years. So I multiply $\mu\alpha$ by $\frac{h^4}{k^2}$. Therefore the velocity of progressive wave of fertility decline (unit km per year) is calculated by the following expression.

Table 2.3 Distance and time progressive wave (front) took to arrive in Japan

From	Arrived area	Distance	Passed time	Estimated velocity
Tokyo	Marginal Kanto area	(max) 50 km	< 5 year	10 km/year >
Tokyo	Niigata	270 km	From 20 to 25 years	From 10.5 to 13.5 km/year
Sapporo	Hokaido	(max) 140 km	in 10 years	14 km/year

Kanto is the name of zone around Tokyo. Tokyo is one of the singularities of fertility decline in Japan. Niigata is the biggest of the cities front on the Japan Sea. Tokyo is the city fronts on Pacific Ocean, namely progressive wave ran cross the Japanese Island. Sapporo is the centre of Hokaido which is an isolated island of north Japan

$$2\sqrt{\mu\alpha} \frac{h^2}{k} \quad (2.10)$$

I substitute the previously obtained estimations (Table 2.2) for each μ, α in the expression (2.10). Thus I calculate an estimation of velocity (km per year) of progressive wave as a following expression.

$$11.45 < 2\sqrt{\mu\alpha} \frac{5^2}{5} < 11.68$$

These estimations resemble the values obtained by differences of time and distances between two points where fertility decline occurred. This resemblance supports the validity of a difference equation DESDC.

2.2.6.2 The Velocity for Two Dimensional Space

The velocity of wave in two dimensional space is $\sqrt{2\mu\alpha}$. Namely the front of wave moves with velocity $\sqrt{2\mu\alpha}$ in our surface. I obtained estimations of μ, α for two dimensional space composed of 1 km² rectangles by one month.

Therefore the velocity of progressive wave of fertility decline (unit km per year) for two dimensional space is calculated by the following expression.

$$\sqrt{2\mu\alpha} \frac{h^2}{k} \quad (2.11)$$

The estimations are somewhat smaller than those for one dimensional space.

$$10.04 < \sqrt{2\mu\alpha} \frac{1^2}{\frac{1}{12}} < 10.67$$

However this resemblance of estimations between one dimension and two dimensions suggests the validity of two difference equations.

2.3 A Search for a Singularity-Origin of Fertility Decline in Europe

In differential equation (2.8), when we go back to time $t = 0$,

lim $t \rightarrow 0$, $\frac{\partial c}{\partial t} = \mu \Delta c + \alpha(\beta - c)(\gamma - c) \rightarrow 0$ (2.12)

a partial differentiation is infinitely small. We must postulate a quantum of decline of c at time $t = 0$. This point is a singularity, in other words an origin of fertility decline.

By means of the estimated velocity of progressive wave of fertility decline, we are able to determine when and where a singularity of fertility decline was in Europe.

First of all, since France is the first country which began fertility decline, a singularity must exist in France. The question which should be solved is that in which district of France the singularity of fertility decline was.

2.3.1 The Date When a Singularity Appeared

The key is differences in time between countries when fertility decline began. From isomorphism of progressive wave, the differences between dates of decline by 10 % is nearly equal to the differences between dates of the beginning of fertility decline.

Then I obtain the differences in time by 10 % about some countries as in Table 2.4. These differences correspond to the differences in time when the minimum fertility decline began.

This means that when we know the date of the beginning of fertility decline in Germany and other places, we can estimate the time of the occurrence of a singularity in France. So I examine the dates when fertility decline began in European countries (Table 2.5).

In Germany 1852–1860 cohort began to decline its fertility. In Denmark 1855–1859. In Belgium 1836–1845. In Netherlands 1851–1860. We can calculate the date when a singularity occur by simple subtractions. For example $1852 - 90 = 1762$ for Germany. I suppose a singularity appeared in cohort born between 1754 and 1772.

Table 2.4 Date of decline in marital fertility by 10 %

France	Germany	Denmark	Belgium	Netherlands
ca. 1800	1890	1900	1882	1897

Source J. Knodel and Etienne van de Walle [36], pp. 221–222

Table 2.5 Initial fertility decline of Germany, Denmark, Belgium, and Netherlands

Germany		Denmark		Belgium		Netherlands	
Cohort	CTFR	Cohort	CTFR	Cohort	CTFR	Cohort	CTFR
1852–1860	5.17	1840–1844	4.39	1836–1845	4.50	1836–1845	5.06
1857–1865	5.02	1845–1849	4.41	1841–1850	4.43	1841–1850	5.13
1862–1870	4.80	1850–1854	4.44	1846–1855	4.30	1846–1855	5.16
		1855–1859	4.38			1851–1860	4.98
		1860–1864	4.16			1856–1865	4.64

Source *Fertility in Western Countries from 1870 to 1970*, pp. 48–49, [24].

It is plausible that a singularity appeared in a last half of 1750s cohort, so it occurred about in 1770s in France as real period date.

2.3.2 Where Is a Singularity?

If we make an assumption that a velocity of progressive wave is universally equal about 10 km/year, we are able to determine the location of the singularity. It took almost 90 years for progressive wave to arrive to Germany from a singularity. This means that the singularity is about 900 km away from Germany. It took 82 years for progressive wave to arrive to Belgium so the singularity is about 820 km away from Belgium.

I let Frankfurt am Main be a representative point for Germany (Frankfurt was near the then centre of population gravity of Germany.) and Brussels for Belgium. I drew a circle which has a centre of Frankfurt and a radius of 900 km.⁶ I also drew a circle which has a centre of Brussels and a radius of 820 km. One of the intersections of the circles is the point that a singularity of fertility decline appeared. Namely the estimated point is around *Aquitaine* district (Fig. 2.16).

2.3.2.1 Lot-et-Garonne

After I estimated *Aquitaine* as a singularity, I noticed somehow incidentally the critical fact. A proceeding study recorded that Lot-et-Garonne in *Aquitaine* had the lowest fertility in Europe. A.J. Coale and R.Treadway [13] pointed out that the French *département* of Lot-et-Garonne was the bottom of marital fertility in Europe.

⁶If we set Hamburg as a point to draw a circle, as Hamburg started fertility decline around 1880 (from *Coale and Treadway* [13]), it took about 110 years to arrive, so we drew a radius of 1100 km. The intersection is still on *Aquitaine*.

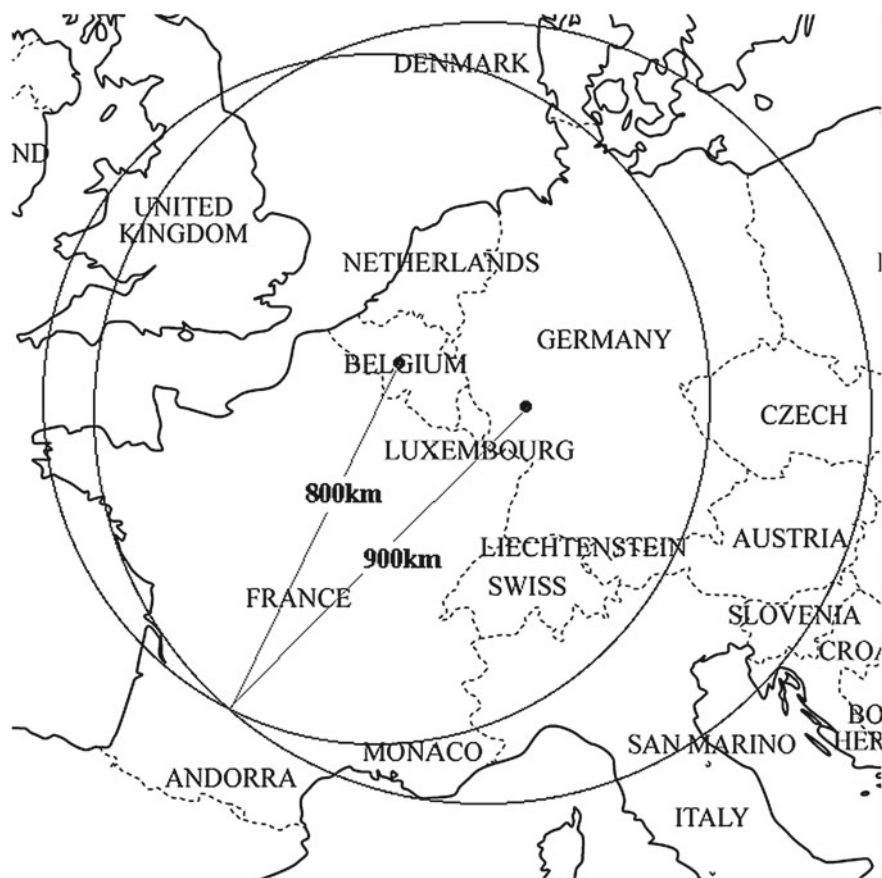


Fig. 2.16 The point 900 km from Frankfurt am Main and 800 km from Brussels

A sense of the meaning of these indexes can be gained by considering some examples. Three provinces at the bottom of the range (the lowest 10% in the distribution) of overall fertility (I_f) in Europe in 1900 were **Lot-et-Garonne** (a *département* of France), Geneva, and County Tipperary in Ireland. The overall rate of childbearing in the first two areas was only about one-sixth of the Hutterites (I_f of 0.168 and 0.164), while in Tipperary I_f was a little higher at 0.220.

Ansley J. Coale and Roy Treadway [13], p. 34 (boldfaced by author)

The French *département* of Lot-et-Garonne had especially low fertility from a very early date, and the age distribution of women in 1900 from 15 to 50 was nearly uniform. I_m in Lot-et-Garonne (0.700) was the highest in Western Europe; with the age distribution of Russia it would have been 0.699. Without the special age distribution generated by its history of especially low fertility, the I_m of Lot-et-Garonne would still have been highest (or nearly so) in Western Europe.

Ansley J. Coale and Roy Treadway [13], p. 159

Fig. 2.17 Geographical location of Lot-et-Garonne.

Source <http://fr.wikipedia.org/wiki/Lot-et-Garonne>
April, 30, 2015



There is a relation between I_f and I_m , I_g that $I_f \simeq I_m \cdot I_g$. So the marital fertility of Lot-et-Garonne is the lowest in western Europe.⁷ In 1831 $I_g = 0.351$ is estimated for Lot-et-Garonne. These data means that Lot-et-Garonne was the lowest of marital fertility in western Europe.

Therefore I speculate that a singularity is *département* of Lot-et-Garonne in *Aquitaine*. This coincidence that the point of the lowest fertility existed in *Aquitaine* suggests that coefficients μ, α are universal constants and that reaction-diffusion exactly traces the fertility decline process in Europe.

2.3.2.2 Why Lot-et-Garonne Not Paris?

Economic growth definitely increases fertility as “Easterlin Hypothesis” [19–21]. If a singularity might have occurred near London, economic growth caused by industrialisation filled up the singularity before it developed. Economic growth is an interference with fertility decline (Fig. 2.17).

The confusion created by the French Revolution could have robbed a singularity of its diffusion mechanism. A singularity had to come into existence in the point far from “Industrialisation” and “Revolutions”. This coincidence was amazing and providential.

⁷Geneva’s I_g was 0.458 in 1860. This figure is far more than Lot-et-Garonne. Low fertility of Geneva was caused by the low level of I_m —low level of the rate of ever married. Swiss still remained in the medieval fertility control stage.

2.3.2.3 The Probability of Disintegration of the Balance

The questions why no singularities in centuries before 1770, and why no singularities in Sweden or Italy are natural and intelligent. I try to give an answer to them.

I think that the singularity is equal to the decisive disintegration of the balance of stochastic fluctuations of the number of children in the direction to declining the number of children. Even though the appearance of the singularity is a genuine coincidence, the probability that it appears is absolutely small. So it takes a long time for the decisive disintegration of the balance of stochastic fluctuations to occur.

I calculate its probability experimentally. Medieval villages had 400–450 population on average consisting of about 50 families. Suppose that the size of the singularity is a village, the occurrence of a singularity requires that completed marital fertility must continuously decline for ten or twenty years without any socio-economic disturbance.

The probability that all neighbouring 30 or 35 sets of parents (they are neighbouring in the time axis) simultaneously decline their completed fertility is roughly calculated at $10^{-3} \sim 3.64 \times 10^{-5}$ per year. This is the reason that the singularity had not occurred before 1770.

I cannot know the minimum size of singularity, therefore this probability could be smaller or larger. I think the occurrence is a lucky coincident which may take place once or not once for previous human history. This is an answer why the singularity had not occurred before.

I will also give an answer to the other question why the singularity occurred in France not in Sweden or Italy or others. I think that the answer is also a genuine coincidence. However the probability of disintegration of the balance of stochastic fluctuations was higher in *Lot-et-Garonne* than other places. Its marriage rate was known to be the highest of all Europe. In addition to it, political, economic and social conditions were quite stable. These higher uniformity of the major sets of parents in *Lot-et-Garonne* prompted the disintegration. The high uniformity inclines to break spontaneously.

2.4 Reaction-Diffusion from the Singularity of Lot-et-Garonne

The early diffusionists recorded the province-wise process of fertility decline in Europe. The reaction-diffusion model is constructed based on the number of children in the space, because the number of children in the space lattice width facilitates an examination of the model's its validity.⁸

⁸Strictly speaking, smaller is not better. There is a lower limit of the space lattice and time pitch widths when the numbers of children of each set of parents are treated in a stochastic manner. Of course these widths are much larger within statistical data.

The data collected for the Princeton European Fertility Project from the 1960s to the 1970s is invaluable. The findings of this project were published as a book entitled, *The Decline of Fertility in Europe*, edited by Coale and Watkins. Using these data, I will trace the reaction-diffusion process based on each province's I_g (index of marital fertility, for the definition, see p. 15.), as recorded in *The Decline of Fertility in Europe*.

Mainland France is currently composed of 95 départements. However, it should be noted that these départements are not the same as those investigated by Coale and the early diffusionists. Based on institutional transitions that occurred, I will first estimate a singularity (the origin of fertility decline). I will then examine peripheral areas of the singularity in central France and the départements that were last to experience fertility decline (Fig. 2.18).

Although early diffusionists such as Coale and Treadway frequently referred to Lot-et-Garonne as having the lowest fertility, they did not conceive of it as an origin



Fig. 2.18 Préfectures de France. Source http://ja.wikipedia.org/wiki/2eAu:Préfectures_de_France.svg



Fig. 2.19 The location of Agen in Lot-et-Garonne. Source <http://www.agen.fr/1-12516-Situation-acces.php>

point of the fertility decline in Europe. Because they did not use a mathematical model of diffusion to investigate the process of fertility decline, they could not perceive an origin point for its diffusion.

My examination of spatial variation of the number of children for each set of parents led me to seek the singularity (origin) of fertility decline. During a period of around 200 years, fertility decline diffused from the singularity at Lot-et-Garonne to all of Europe, North America, and even extending to East Asia.

I would like to note here that from the latter half of the nineteenth century, reaction-diffusion spread like wildfire. Immigrants in North America and students returned from abroad, as well as hired foreigners in Japan were the source of the infection. In addition to the normal crawling pace of diffusion on the ground, the active movement of people as a result of transportation development altered the pace of diffusion. Thus, from the latter half of the nineteenth century, we must postulate another precise mechanism of diffusion.

2.4.1 Features of Lot-et-Garonne

The commune of Agen is the capital of the Lot-et-Garonne département located in Aquitaine in southwestern France. It has a long history dating back to ancient times. It is and was a relaxed countryside area, and there is no mention of any accidents or significant events occurring in the eighteenth century (Fig. 2.19).

Lot-et-Garonne had no special features except that the ratio of married females for every age group is the highest, exceeding that of every other place. This homogeneity has been the trigger for the reaction-diffusion process. Against common sense, living in peace and quiet was the prerequisite for a truly novel change of fertility behavior. A decline by Zufall⁹ (fortuity), along with fluctuation caused the singularity. Uniformity in the initial conditions of the parents implies uniformity at the end of the period of fertility decline.

⁹“Kein Sieger glaubt an den Zufall.” (F. Nietzsche) I think fertility decline in the European modern period was the coincidence.

Table 2.6 Changes in I_g in Lot-et-Garonne and neighboring *départments*

	Lot-et-Garonne	Gironde	Dordogne	Landes	Pyrenees-Atlantiques*	Gers	Lot	Tarn-et-Garonne	Corse
1831	0.351	0.393	0.520	0.624	0.621	0.421	0.509	0.390	0.683
1836	0.339	0.375	0.496	0.592	0.595	0.395	0.493	0.364	0.701
1841	0.333	0.358	0.472	0.585	0.597	0.380	0.486	0.351	0.681
1846	0.315	0.333	0.461	0.556	0.576	0.352	0.465	0.337	0.676
1851	0.298	0.341	0.452	0.562	0.563	0.338	0.446	0.329	0.656
1856	0.290	0.338	0.447	0.528	0.577	0.328	0.446	0.330	0.646
1861	0.282	0.336	0.451	0.511	0.580	0.321	0.437	0.328	0.636
1866	0.279	0.338	0.468	0.513	0.566	0.337	0.435	0.328	0.656
1871	0.280	0.316	0.465	0.534	0.589	0.346	0.429	0.325	0.676
1876	0.271	0.308	0.453	0.530	0.614	0.333	0.418	0.316	0.685
1881	0.266	0.303	0.437	0.506	0.622	0.316	0.385	0.299	0.710
1886	0.246	0.287	0.398	0.475	0.620	0.290	0.331	0.283	0.727
1891	0.232	0.265	0.371	0.440	0.601	0.259	0.319	0.274	0.710
1896	0.230	0.256	0.361	0.403	0.591	0.247	0.329	0.273	0.632
1901	0.230	0.245	0.352	0.392	0.576	0.241	0.333	0.274	0.593
1911	0.207	0.212	0.294	0.294	0.444	0.219	0.271	0.242	0.510
1921	0.240	0.256	0.307	0.291	0.414	0.254	0.296	0.282	0.479
1931	0.238	0.219	0.273	0.252	0.343	0.257	0.262	0.265	0.331
1961	0.288	0.283	0.279	0.304	0.348	0.302	0.310	0.306	0.508

Source Ansley J. Coale and Roy Treadway [13].

The recovery period commenced from 1921. The baby boom affected the I_g for 1961.

*Only Pyrenees-Atlantiques is not a direct neighbor of Lot-et-Garonne. This is the reason why the fertility decline effect of the singularity was relatively small for Pyrenees-Atlantiques

According to E. van de Walle, the level of fertility in Lot-et-Garonne was below the population replacement level. Consequently, from 1836, its population monotonically declined, even without emigration.¹⁰ The reaction-diffusion process was propelled by its inertia and could not, therefore, stop until it reached its lower limit.

2.5 Areas Around the Singularity: Aquitaine and Parts of the Midi-Pyrénées

We can identify the I_f , I_g , and I_m in all of the French provinces from 1830 by referring to *The Decline of Fertility in Europe*. For details on the I_g , see Table 2.6. The progressive wave traveled at 10km per year from the singularity, and by 1831, the fertility of neighboring *départments* had already evidenced a natural decline.

¹⁰ The commune of Agen did not disappear. It remains a beautiful garden city, as portrayed on the website <http://www.agen.fr/>. The decline of fertility did not endure.

Applying the reaction-diffusion hypothesis, I estimated that the singularity of fertility decline occurred for the 1750–1760 birth cohort. Consequently, the decline of I_g should have started during the period from 1770–1780. About 60 years should have passed from the occurrence of singularity in 1830.

Fertility decline began in 1836 in the island of Corsica (Corse Département). This date is reasonable given that the progressive wave from Cote D'Azur was obstructed by the Mediterranean Sea.

2.5.1 Background Independence of Fertility Decline

The French industrial revolution temporarily restrained the fertility decline. The temporariness of the ascent of fertility (I_g) strongly implies background independence of fertility decline.

2.5.1.1 Effects of Economic Development: Restraint Factors in Fertility Decline

Although by 1830, *départments* on the periphery of Lot-et-Garonne began to evident fertility decline, all of them had temporarily moved away from the monotonic locus of decline by around 1860. The rise in fertility at the singularity, namely, Lot-et-Garonne, and in neighboring Gironde was only slight. However, other *départments* recorded a significant ascent of I_g from 1860 to 1880.

This significant ascent was caused by the economic development that took place during the period of the Second French Empire under the reign of Napoleon III. During this period French industries grew at an extraordinary pace, resulting in the provision of ample employment. Construction of the railways and the remodeling of Paris during the reign of Napoleon III required a huge amount of manpower, and contributed to a rise of fertility. The evolving industrialization at that time demanded many workers. This same mechanism resulted in population growth in England and Wales during the early nineteenth century.

However this ascent was impermanent, because, as is always the case, economic development did not sustain. Even if this ascent of fertility by economic development was temporary, should inverse arising of fertility have occurred in the decline process under the constant dynamics of the decline caused by the reaction-diffusion system? We should note that the index I_g is the periodical index of fertility, and not for cohorts. When there was an inclination toward early marriage and early births, without any change in the completed fertility of cohorts, I_g increased, as reflected in numerical values. This estimation is supported by the phenomenon of an accelerated decline after the ascent (Fig. 2.20).

Of course at some points, the increase of completed marital fertility of each set of parents may occur. Indeed the French industrial revolution brought about a small baby boom, but as long as fertility was relatively low at the singularity, the fertility

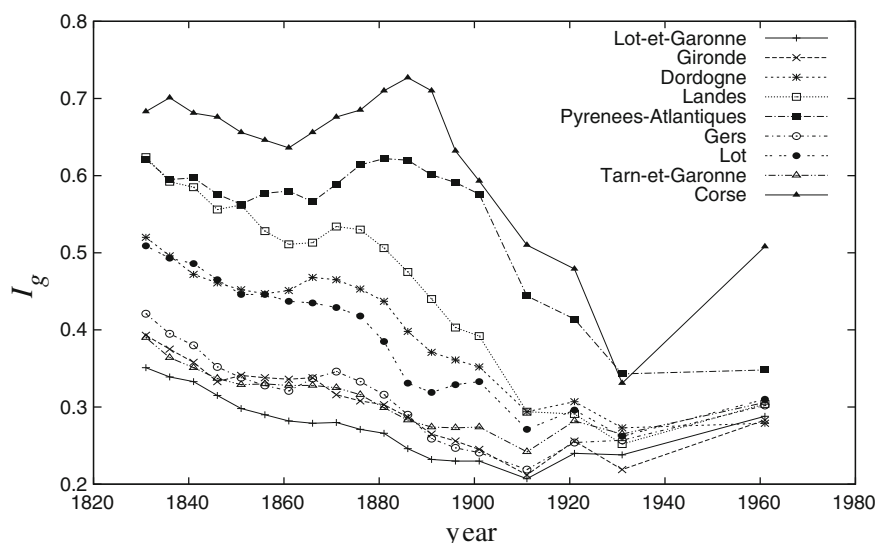


Fig. 2.20 The decline of I_g in Aquitaine and parts of Midi-Pyrénées. *Source* Ansley J. Coale and Roy Treadway [13]. The data shown in Table 2.6 are depicted here as a *line graph*

at other points should have reverted to the locus of the reaction-diffusion process. This occurrence suggests the potent and unvarying strength of the reaction-diffusion. While countermeasures against the falling birth rate may temporarily increase the mean number of children of each set of parents, their effects are soon exhausted in the reaction-diffusion process. It is impossible to expect such extensive and continuous economic development, which occurred during the nineteenth century, to be repeated in present day developed countries.

2.6 Districts Situated 600–500 km Away from the Singularity

It is essential to note that the onset date of fertility decline within a particular location in France, depended almost exclusively on its distance from the singularity. These phenomena provide us with decisive evidence of the background independence of fertility decline.

As I have already noted, the data of Coale and Treadway are from 1831. The singularity occurred in around 1770, so a period of 60 years had passed by 1831. The progressive wave had traveled a distance of approximately 600 km in this period. This meant that I_g should have monotonically declined in départements within a 600 km radius of Lot-et-Garonne, and that the decline of I_g should not be observed beyond that radius.

Bourgogne is located at a distance of about 500 km from Lot-et-Garonne. In this district, fertility decline resulting from a process of reaction-diffusion had started on time. I_g had declined in all of the départements, for example, Côte-d'Or, which is famous for its Bourgogne wine, Nièvre, Saône-et-Loire, and Yonne, which produces “Chablis”, a reputed white wine.

J. Fourier was born in Auxerre in the Yonne département in 1768. As the progressive wave had not reached Yonne, it is not at all surprising that he was the nineteenth child of a tailor.

Aube (Aube), Haute-Marne, Haute-Saone, Doubs are all located at a distance of about 600 km from Lot-et-Garonne. We can confirm that I_g declined in each of these départements.

The location of Haute-Normandie region is at the furthest point within the range of access of the progressive wave. This wave had reached Eure, which is relatively close to the singularity in 1831. However, we cannot confirm whether it had already reached Seine-Maritime, which is further away from the singularity than Eure.

The progressive wave reached Finistère (Bretagne) in 1831. However fertility in this area was originally high, leading to the belief among the early diffusionists that fertility decline was delayed here compared with other French districts.

2.7 Districts Located Further than 600 km Away from the Singularity

The progressive wave did not reach the départements that were situated beyond a 600 km radius of Lot-et-Garonne.

Pas-de-Calais is located at a distance of 700 km from the singularity. Nord is near the border with Belgium. These areas had not yet encountered the progressive wave. We can observe fluctuations of I_g from 1831 up to the 1850s. Subsequently, their fertility increased as a result of the French industrial revolution.

I would like my readers to recall from chapter one that in 1890, Charles de Gaulle was born in Lille (Nord département) as the third out of five children. Since the occurrence of the singularity in 1770, 120 years had passed. Lille is located at a distance of about 740 km from Lot-et-Garonne. Thus, the progressive wave reached Lille after 74 years. In 1890, Lille was still in the midst of the reaction-diffusion process. Because of the effect of the French industrial revolution, fertility in Nord remained relatively high. Thus, de Gaulle's parents merely behaved in conformity with their neighbors.

2.7.1 *Advanced Industrial Development in Nord-Pas-de-Calais*

The region of Nord-Pas-de-Calais, which has favorable geographical conditions, was one of the relatively advanced industrialized areas in 1831. This region provided high quality steel ore and coal and was a port for relaying these items to England. With its favorable climate conditions, it was industrialized, like Lorraine, at an early stage of the French industrialization process.

The fact that fertility decline did not commence in this region in 1830, and that agricultural regions were the predecessors regarding fertility decline, suggests the fallacy of the industrialization hypothesis. Moreover, it indicates the background independence of fertility decline.

In 1831, the progressive wave had not reached the Lorraine region. It had not arrived in the Muse département located near the border with Belgium. The lack of data for Moselle, Bas-Rhine prevented me from making any observations for this area.

2.7.2 *Low Fertility in Northern France*

Fairly low fertility was indicated for Paris and its peripheral départements, for example, Oise, Somme, and Eure in the 1830s. In particular, Eure's I_g was lower than that of Lot-et-Garonne. However it is almost certain that a singularity did not occur in northern France, judging by the arrival date of the progressive wave. In 1861, during the reign of Napoleon III, I_g increased again.¹¹ Thus, a singularity evidently did not occur in northern France.

The map included at the end of *The Decline of Fertility in Europe* (Fig. 1.4) indicates low I_g in 1870 in Paris and its peripheral areas, and in departments situated in areas along the national border. However, the low fertility in these areas can be attributed first to the high infant mortality caused by the custom of no breast feeding second to the riots and disturbances caused by the French Revolution, by Napoleon I, the July Revolution, and the French Revolution of 1848.

2.7.3 *The Reaction-Diffusion Process in Belgium*

It is apparent that fertility decline in Belgium was caused by the reaction-diffusion process. We can provide a fairly good description of the phases of fertility decline for each of the cities in Belgium from 1880 to 1900, based on the reaction-diffusion process.

¹¹ See Fig. 2.20 and Table 2.6. I_g did not increase in the Lot-et-Garonne singularity. The singularity had maintained the lowest marital fertility during the process of reaction-diffusion.

Table 2.7 Decline of I_g for provinces in Belgium

	Mons	Chalerol	Dinant	Namur	Bruxelles	Liege	Arlon
1880	0.526	0.596	0.663	0.646	0.673	0.688	0.730
1890	0.472	0.478	0.573	0.571	0.570	0.577	0.650
1900	0.382	0.390	0.511	0.485	0.470	0.470	0.582
	Antwerpen	Leuven	Bastogne	Hasselt	Kortrijk	Ostende	
1880	0.810	0.849	0.854	0.868	0.894	0.899	
1890	0.713	0.806	0.804	0.818	0.853	0.905	
1900	0.584	0.751	0.756	0.865	0.812	0.800	

Source Ansley J. Coale and Roy Treadway [13]. Data after 1910 are omitted

Suppose that the progressive wave traveled from the direction of Lot-et-Garonne in southwestern France. This assumption leads to the belief that fertility decline was delayed in the northwestern provinces of Belgium.

We can definitely verify this belief (see Table 2.7 and Fig. 2.21. In the national border areas between France and Belgium, the southern areas experienced fertility decline earlier than the northern areas. Mons, which is at a distance of 740 km from Lot-et-Garonne demonstrated a decline of I_g earlier than Kortrijk, which is at a distance of 760 km from Lot-et-Garonne. Fertility decline in northern Belgian cities such as Ostende, Antwerpen, and Hasselt was delayed compared with southern cities.

Although Bruxelles, located in central Belgium, is the national capital and an industrial, economic, and political center, fertility decline in this city was delayed compared with that of southern Mons, Chalerol, and Namur. This fact is further evidence of the background independence of fertility decline.

Readers may perceive the relatively late decline of I_g in southeastern Belgium, namely, Arlon and Bastogne to be an inconsistency. One of the properties of a progressive wave is that it stagnates in highlands and plateaus where the population is sparse. Consequently, it slowed down in Ardennes. This accounted for the delay of fertility decline in southeastern Belgium.

The following figure depicts the declining I_g of each Belgian city. Figure 2.20 on p. 63 resembles Fig. 2.22. The question, then, is why did the fertility decline process in southwestern France (Aquitaine) that occurred 60 years earlier in what was still an agricultural area resemble that which occurred in Belgium, where industry was considerably developed? This coincidence suggests that an identical reaction-diffusion process occurred in both areas. It implies that the reaction-diffusion process proceeds in a way that does not depend on the socioeconomic background.



Fig. 2.21 Map of Belgium. Source <http://www.freeworldmaps.net/europe/belgium/political.html>. I have added some cities to the map, as well as an arrow indicating the direction of the progressive wave

2.7.3.1 The Solution of Industrial and Cultural Differences in Fertility Decline in Belgium

To examine fertility decline of Belgium from the perspective of a reaction-diffusion process may also enable us to solve a question that remains unsettled. This refers to the contradiction relating to the industrialization hypothesis that was raised by the initial diffusionists.

I will request my readers to return to Table 1.1 on p. 3. This indicates from the “Date of Decline in Marital Fertility by 10%” that fertility decline occurred earlier in Belgium (1882) than in the Netherlands (1897). However other socioeconomic indices such as infant mortality and percentages of rural areas and cities with populations over 20,000 indicate greater advancement of the Netherlands compared with Belgium, at least in terms of industrialization. If the industrialization hypothesis is true, then fertility decline in both countries occurred out of sequence.

The conception that the progressive wave of fertility decline had been diffusing from Lot-et-Garonne solves this contradiction. The progressive wave must have passed Belgium to reach the Netherlands in the reverse direction to that followed by advancing German armies during both WWI and WWII. It took ten years for the progressive wave to reach the Netherlands from the France-Belgium border, because it was traveling at a velocity of 10km/year. This solution to the contradiction is further evidence of background independence.

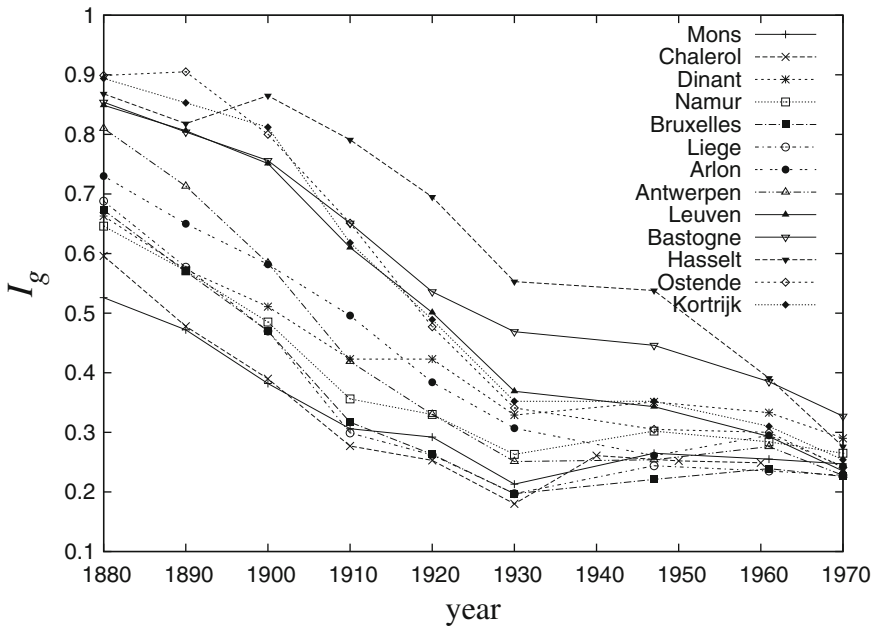


Fig. 2.22 Decline of I_g in each city in Belgium

This simple explanation provided by the reaction-diffusion hypothesis can solve another question concerning Belgium’s fertility decline. The early diffusionists believed that cultural differences between the Flemish and Walloon communities caused the difference in the tempo of fertility decline. The hypothesis that the linguistic difference between the Flemish community within a Dutch-speaking area and the Walloon community within a French-speaking area was the underlying factor causing the difference in the tempo of fertility decline was widely accepted.

In fact, this language distribution perfectly matches the geographical distribution of Belgium’s population (please refer again to Figs. 2.21 and 2.23). The progressive wave first touched the southern portion of Belgium, which is a French-speaking

Fig. 2.23 Belgian communities. The North is a Dutch-speaking area inhabited by the Flemish community; the South is a French-speaking area inhabited by the Walloon community, and the East is inhabited by the German-speaking community. Source <http://en.wikipedia.org/wiki/Belgium>



area and the home of the Walloon community. It did not reach the Flemish community/Dutch-speaking area until it had passed through the Walloon community. Thus, the geographical distribution of the linguistic population and the advancing route taken by the progressive wave did result in a cultural difference relating to fertility decline.

However, if we had instead divided Belgium into western and eastern communities, we would not have observed any differences in fertility decline related to community and language. Thus, the reaction-diffusion process is also independent of the cultural background. The background naturally entails cultural factors.

2.8 Conclusion

To postulate a reaction-diffusion process for fertility decline in Europe, Japan, and other countries results in novel findings. Accordingly, we can estimate the velocity of diffusion and discover the origin (singularity) of fertility decline. Consequently, I estimate that fertility decline originated in *Lot-et Garonne* around 1770.

By applying the reaction-diffusion model, we can explain the fertility decline process in Europe, both longitudinally and geographically. Even though we cannot exclude the possibility that socioeconomic conditions played an important role in the occurrence of singularity, they actually played a minor role in the middle of this process. Rapid economic growth during the reign of Napoleon III did indeed lead to a temporary increase in French fertility from the 1860s to 1870s. However, at the conclusion of this period of economic growth, French fertility declined again, as has been captured by the trajectory of decline. Fertility decline caused by Reaction-diffusion was independent of its background.

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2016, XXI, 97 p. 41 illus., 7 illus. in color., Softcover

ISBN: 978-4-431-55150-8