

Preface

Microwave dielectric resonator antenna (DRA) materials or ceramics were demonstrated by Richtmeyer in 1939. Richtmeyer showed that these dielectric ceramics can resonate. Theory of DRA was expanded by Okaya and Brash in 1960. More experimental work on DRAs, done by Long in 1980, proved that DRAs can become efficient radiators and can be used as antennas. S.A. Long experimentally implemented DRAs of different shapes and sizes as a low-profile antenna.

Analysis and studies on characteristic equation, radiation patterns, and excitation methodology made DRAs popular by providing a new avenue compared to traditional patch antennas suffering from low gain and low bandwidth. Aldo Petosa made DRAs a very successful candidate as functional antennas. Both the limitations of low gain and low bandwidth in patch antennas can be eliminated by the use of a rectangular dielectric resonator antenna (RDRA) operating in higher modes and hybrid modes.

The modes theory of RDRA gives an important analysis on current distribution, impedance, and radiation patterns of an antenna. Modes form a real, orthogonal basis function for currents on the antenna. These are defined by boundary value problems using eigenvalues and eigenvectors. The scope of this book has been restricted to RDRAs, however, the concept can be extended to other geometries. In RDRAs, once the excitation is given, the total distributed current on the antenna structure becomes a weighted sum of eigen currents or a superposition of various modes at any instant of time.

Resonant modes in RDRAs can be classified as dominant and higher modes. Dominant modes correspond to lowest resonant frequency. These are called as TE, TM, and HEM modes. E and H field formats inside the RDRA at any instant of time at a known frequency are termed as resonant modes. Modes excitation is directly related to the surface current densities of the structure due to applied RF current. This current gets converted into modal fields based on Maxwell's equations. These fields are restricted by RDRA boundary conditions. Reflection and refraction of electromagnetic waves takes place because of dielectric interface at the boundary.

The generation of higher modes generally depends on RF excitation, device dimensions, permittivity of dielectric material and coupling techniques used in

design of the antenna. The higher-order modes and hybrid modes have much flexibility and design space in RDRA for different applications, but the excitation techniques are complex. Rectangular DRA has a high degree of design flexibility due to two aspect ratios (a/d and b/d), low cost, simplicity, and ease of fabrication. It can retrofit to the existing patch antenna technology for gain improvements.

Researchers have long felt the need for a rigorous theoretical analysis on resonant modes of RDRA, and resonators have become a demanding field for industry and academia. This is because knowledge of resonant modes gives physical insight to the antenna designer, based on which input impedance and radiation characteristics can be predicted. We hope that this book will help to fill the gap.

The investigations and theory developed are based on applying waveguide theory models. Propagation of electromagnetic fields has been taken along z -axis, i.e., $\exp(-\gamma z)$. Initially, these are exploited via the Maxwell's curl equations and then manipulating them to express the transverse components of the fields in terms of the partial derivatives of the longitudinal components of the fields w.r.t. x and y (i.e., the transverse coordinates).

Waveguide models of four different boundary conditions filled with homogeneous as well as inhomogeneous dielectric materials with linear and nonlinear permittivity, permeability, and conductivity have been developed to determine TE and TM propagating electromagnetic fields. These have resulted in different sine-cosine combinations. TE modes generation required H_z fields as longitudinal fields and E_x , E_y , H_x , and H_y fields as transverse fields.

If input excitation is applied along x -axis as partial fields, y -axis will have fixed variation and z -axis will have desired variation in propagating fields. For example, TE δ_{13} . Similar cases can be developed for TM modes so as to propagate E_z fields as longitudinal and E_x , E_y , H_x , and H_y as transverse fields. H_z field will get vanished because of boundary conditions.

An equivalent but computationally simpler way to pass on from waveguide physics to resonator physics is to just replace (γ) by $(-\frac{\partial}{\partial z})$ in all the waveguide formulae that express the tangential field components in terms of the longitudinal components. This is done after solving the full 3D Helmholtz equations using separation of variable as x, y, z .

$$\left(\nabla^2 + \frac{\omega^2}{c^2}\right) \begin{pmatrix} E_z \\ H_z \end{pmatrix} = 0$$

The discrete modes $\omega(mnp)$ enable us to visualize the resonator as collection of L, C oscillators with different L, C values. The outcome of all this analysis enables us to write down the \underline{E} and \underline{H} fields inside the resonator, as superposition of four and three vector-valued basis functions.

$$\begin{aligned}\underline{E}(x, y, z, t) = & \sum_{mnp=1}^{\infty} \text{Re} \left\{ C(mnp) e^{j\omega(mnp)t} \underline{\psi}_{mnp}^E(x, y, z) \right\} \\ & + \sum_{mnp=1}^{\infty} \text{Re} \left\{ D(mnp) e^{j\omega(mnp)t} \underline{\phi}_{mnp}^E(x, y, z) \right\}\end{aligned}$$

and

$$\begin{aligned}\underline{H}(x, y, z, t) = & \sum_{mnp=1}^{\infty} \text{Re} \left\{ C(mnp) e^{j\omega(mnp)t} \underline{\psi}_{mnp}^H(x, y, z) \right\} \\ & + \sum_{mnp=1}^{\infty} \text{Re} \left\{ D(mnp) e^{j\omega(mnp)t} \underline{\phi}_{mnp}^H(x, y, z) \right\}\end{aligned}$$

We note that there are only two sets $\{C(mnp)\}$ and $\{D(mnp)\}$ of linear combination of coefficients using from the E_z and H_z expansions. The vector-valued complex functions are as follows:

$$\underline{\psi}_{mnp}^E, \underline{\phi}_{mnp}^E, \underline{\psi}_{mnp}^H, \underline{\phi}_{mnp}^H \in R^3$$

where R is autocorrelation and contain components $\{\cos, \sin\} \otimes \{\cos, \sin\} \otimes \{\cos, \sin\}$ functions and hence for $(m', n', p') \neq (m, n, p)$, each function of the set, where m, n, p are integers.

$$\left\{ \underline{\psi}_{mnp}^E, \underline{\phi}_{mnp}^E, \underline{\psi}_{mnp}^H, \underline{\phi}_{mnp}^H \right\}$$

is orthogonal to each function of the set:

$$\left\{ \underline{\psi}_{m'n'p'}^E, \underline{\phi}_{m'n'p'}^E, \underline{\psi}_{mnp}^H, \underline{\phi}_{m'n'p'}^H \right\}$$

w.r.t. the measure of $dx \, dy \, dz$ over surface of RDRA $[0, a] \times [0, b] \times [0, d]$, where a, b , and d are RDRA dimensions. The exact form of the function $\underline{\phi}_{mnp}^E, \underline{\phi}_{mnp}^H, \underline{\psi}_{mnp}^E, \underline{\psi}_{mnp}^H$ depends on the nature of RDRA boundaries.

Excitation of RDRA plays very important role for modal analysis. To calculate the amplitude coefficients $\{C(mnp)\}$ and $\{D(mnp)\}$, we assume that at $z = 0$, an excitation $E_x^{(e)}(x, y, t)$ or $E_y^{(e)}(x, y, t)$ is applied for some time say $t \in [0, T]$ and then removed, as usually is done in L, C oscillators. Then, the Fourier components in this excitation corresponding to the frequencies $\{\omega_{mnp}\}$ are excited, and their solutions are the oscillations for $t > T$. The other Fourier components decay within the resonator.

$\{C_{mnp}, D_{mnp}\}$ magnitude components can be determined based on principle of orthonormality:

$$\sum_{mnp} \operatorname{Re} \left(C(mnp) e^{j\omega(mnp)t} \psi_{mnp X}^E(x, y, 0) \right) + \operatorname{Re} \left(D(mnp) e^{j\omega(mnp)t} \bar{\phi}_{mnp X}^E(x, y, 0) \right) = E_x^{(e)}(x, y, t)$$

and

$$\sum_{mnp} \operatorname{Re} \left(C(mnp) e^{j\omega(mnp)t} \psi_{mnp Y}^E(x, y, 0) \right) + \operatorname{Re} \left(D(mnp) e^{j\omega(mnp)t} \bar{\phi}_{mnp Y}^E(x, y, 0) \right) = E_y^{(e)}(x, y, t)$$

By using orthogonality of $\{\psi_{mnp X}^E(x, y, 0), \bar{\phi}_{mnp X}^E(x, y, 0)\}$; for different (m, n) , we write p fixed and likewise of $\{\psi_{mnp Y}^E(x, y, 0), \bar{\phi}_{mnp Y}^E(x, y, 0)\}$; in addition, we need to use Kolmogorov–Arnold–Moser (KAM) type of time averaging to yield:

$$C(mnp) \psi_{mnp X}^E(x, y, 0) + D(mnp) \bar{\phi}_{mnp X}^E(x, y, 0) = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T E_x^{(e)}(x, y, t) e^{-j\omega(mnp)t} dt$$

and likewise

$$C(mnp) \psi_{mnp Y}^E(x, y, 0) + D(mnp) \bar{\phi}_{mnp Y}^E(x, y, 0) = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T E_y^{(e)}(x, y, t) e^{j\omega(mnp)t} dt$$

In this book, RDRA resonant modes theoretical as wells as practical aspects have been investigated along with rigorous mathematical analysis for TE, TM, and HEM. Higher modes generation and control of resonant modes have been experimented. Shifting of dominant mode toward higher modes and vice versa is desired phenomenon for reconfigurability, merging of neighboring resonant modes have been exploited with simulation results. Use of higher modes for practical applications in antennas has been described. Merging of neighboring modes significantly increased antenna bandwidth. The device miniaturization using high-permittivity materials has been described. The devising control on modes has imparted reconfiguration of operating frequency, beam pattern, beam width, polarization, gain, and bandwidth. Higher modes radiation pattern, sensitivity analysis by changing dimensions, and permittivity analysis by changing permittivity have been mathematically modeled, and each is supported with simulated and experimental results. Selecting and cancelling a particular resonant mode has also been described. The concept of modes has been

supported with practically implemented case studies. Devising control on resonant modes in RDRA can be used for software-defined radios and military applications, where frequent change of antenna parameters is operational requirement. For automation on modes control, microcontrollers equipped with lookup table can be used.

The modes have been modeled by R , L , C networks. Antenna far fields patterns and impedance have been computed and measured. Analysis on hybrid modes in RDRA has been discussed. Hybrid modes are complex to determine. Their mathematical formulations have been described. These modes are diversified.

The excitation of hybrid modes is complex, and their effective control can revolutionize the antenna technology. Detailed study of mathematical modeling of hybrid modes has been described. Hybrid modes are more popular for azimuthally field variations. The transcendental equation and characteristic equation for RDRA modes are used for determining propagation constants and then resonant frequency.

The solution of resonant modes can be obtained using the following:

- (a) H_z and E_z fields are expressed as $u_{mnp}(x, y, z)$, $v_{mnp}(x, y, z)$ and ω_{mnp} based on solving Maxwell's equations with given boundary conditions.
- (b) At $z = 0$, surface (x, y) excitation with applied surface current density is

$$(J_{sx}(x, y, t), J_{sy}(x, y, t))$$

- (c) Surface current density is equated with generated magnetic fields

$$\{J_s(x, y, \delta) = (J_{sx}, J_{sy}) = (\hat{z} \times \underline{H}) = (-H_y, H_x)\};$$

at $z = 0$; amplitude coefficients (D_{mnp} and C_{mnp}) are obtained on expansion of H_z is terms D_{mnp} , and E_z terms as C_{mnp} .

- (d) Equate tangential component of E_z at boundary, i.e., $\underline{E}_y|_{z=0}$ to zero, and compute the coefficients D_{mnp} for H_z and C_{mnp} of E_z .
- (e) Excited by ω_{mnp} and arbitrary feed position in xy plane $(x_0, y_0)(\phi_0, \theta_0)$

$$\begin{aligned} H_{\perp} = & \left[\sum_{mnp} \text{Re} \left\{ \tilde{D}_{mnp} e^{j\omega(mnp)t} \right\} \nabla_{\perp} \tilde{u}_{mnp}(x, y, z) \right] \\ & - \sum_{mnp} \text{Re} \left\{ \tilde{C}_{mnp} e^{j\omega(mnp)t} \right\} \nabla_{\perp} \tilde{u}_{mnp}(x, y, z) \end{aligned}$$

and similarly E_{\perp} .

Depending on the boundary conditions, four cases have been developed. In RDRA, these four walls are assumed as perfect magnetic conductors and top and bottom walls are taken as perfect electric conductors.

$$u_{mnp} = \sin \sin \sin = E_z$$

$$v_{mnp} = \cos \cos \cos = H_z$$

Sidewalls and top walls all are perfect electric conductors

$$u_{mnp} = \sin \sin \cos = E_z$$

$$v_{mnp} = \sin \sin \sin = H_z$$

Sidewalls and top walls all are perfect magnetic conductors

$$u_{mnp} = \cos \cos \sin = E_z$$

$$v_{mnp} = \sin \sin \cos = H_z$$

Top and bottom walls are perfect magnetic conductors, and all four sidewalls are PEC

$$u_{mnp} = \cos \cos \cos = E_z$$

$$v_{mnp} = \sin \sin \sin = H_z$$

Transcendental equation is used to solve propagation constants, i.e., k_x , k_y , and k_z . The propagation constant gives rise to resonant frequency with the help of characteristic equation. These wave numbers k_x , k_y , and k_z are in x , y , and z -directions, respectively. The free space wave number is k_0 . The resonant frequency can be determined from combined solution of transcendental equation and characteristic equation of rectangular DRA. Time-averaged electric energy = time-averaged magnetic energy

$$\begin{aligned} \epsilon_r k_0^2 &= k_x^2 + k_y^2 + k_z^2 \\ \epsilon_0 k_0^2 &= k_x^2 + k_y^2 + k_z'^2 \\ k_z' &\neq p\pi/d \\ \tan(k_z d) &= \frac{k_z}{\sqrt{k_0^2(\epsilon_r - 1) - k_z^2}}; \end{aligned}$$

the final result of transcendental equation is thus achieved.

The contents of this book are the outcome of our research work on RDRA higher-order resonant modes. In this book, analyses have been restricted to rectangular resonators higher modes, however, the concept can be extended to other geometry resonators, such as cylindrical, conical, and hemispherical. With this book, we hope to fill the gap for rigorous theoretical analysis on RDRA resonant modes. The work is supported with live projects data and their case studies. This book should be very useful for antenna designers, both in research and development and for practical implementations. This book is written in a simple and reader friendly manner and can be easily understood with an initial knowledge of basic

electromagnetic theory. All the chapters are self-reliant, and no initial specialization is required to understand the contents. We hope that this book will help open the design space for a new class of antenna implementations.

This book is organized into 12 chapters including rigorous theoretical analysis of modes along with case studies and design data annexure. Introduction along with history of RDRA is given in Chap. 1. Introduction of resonant modes is explained in Chap. 2. Mathematical derivations for modes and the generation of TE/TM modes have been discussed in Chap. 3. Chapter 4 presents the derivation of RDRA transcendental equations. In Chap. 5, mathematical description of amplitude coefficients of even and odd modes is presented. Chapter 6 contains radiation parameters and mathematical explanations of RDRA. Chapter 7 describes derivations of higher-order resonant modes and their applications for high-gain antenna designs. Chapter 8 explains the effect of angular variation on excitation to produce various types of radiation patterns to meet military requirements. Chapter 9 discusses sensitivity analysis and mathematical modeling of radiation pattern solutions in RDRA. Chapter 10 presents the excitation of hybrid modes in RDRA and their possible applications. Chapter 11 covers inhomogeneous solution along with measurements. Basic RDRA resonant frequency formulations, materials required, and their sources are given in the annexures. Complete and detailed solutions of RDRA have been explained in case studies. Design data are provided in the annexures. Chapter 12 discusses case studies.

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Theory and Design

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