

Contents

Lecture 1: Wigner Functions. Coherent States. Gabor Transform.	
Semiclassical Correlation Functions	1
1 Coherent States	5
2 Husimi Distribution	7
3 Semiclassical Limit Using Wigner Functions	11
4 Gabor Transform	14
5 Semiclassical Limit of Joint Distribution Function	15
6 Semiclassical Limit Using Coherent States	17
7 Convergence of Quantum Solutions to Classical Solutions	20
References.	25
Lecture 2: Pseudo-differential Operators. Berezin, Kohn–Nirenberg,	
Born–Jordan Quantizations.	27
1 Weyl Symbols	28
2 Pseudo-differential Operators	29
3 Calderon–Vaillantcourt Theorem.	32
4 Classes of Pseudo-differential Operators. Regularity Properties	36
5 Product of Operator Versus Products of Symbols	39
6 Correspondence Between Commutators and Poisson Brackets; Time Evolution	41
7 Berezin Quantization.	44
8 Toeplitz Operators	46
9 Kohn–Nirenberg Quantization	47
10 Shubin Quantization	48
11 Born–Jordarn Quantization.	49
References.	50
Lecture 3: Compact and Schatten Class Operators. Compactness	
Criteria. Bouquet of Inequalities	51
1 Schatten Classes	55
2 General Traces	57
3 General L^p Spaces	58

4	Carleman Operators	60
5	Criteria for Compactness	61
6	Appendix: Inequalities.	66
6.1	Lebesgue Decomposition Theorem	68
6.2	Further Inequalities.	69
6.3	Interpolation Inequalities	71
6.4	Young Inequalities	75
6.5	Sobolev-Type Inequalities	77
	References.	81

Lecture 4: Periodic Potentials. Wigner–Seitz Cell

	and Brillouin Zone. Bloch and Wannier Functions	83
1	Fermi Surface, Fermi Energy	84
2	Periodic Potentials. Wigner-Satz Cell. Brillouin Zone. The Theory of Bloch-Floquet-Zak.	87
3	Decompositions	88
4	One Particle in a Periodic Potential.	91
5	The Mathieu Equation.	95
6	The Case $d \geq 2$. Fibration in Momentum Space	96
7	Direct Integral Decomposition	98
8	Wannier Functions	103
9	Chern Class	106
	References.	108

Lecture 5: Connection with the Properties of a Crystal.

Born–Oppenheimer Approximation. Edge States and Role

	of Topology	111
1	Crystal in a Magnetic Field	113
2	Slowly Varying Electric Field	114
3	Heisenberg Representation.	118
4	Pseudo-differential Point of View	119
5	Topology Induced by a Magnetic Field	121
6	Algebraic-Geometric Formulation	123
7	Determination of a Topological Index	125
8	Gauge Transformation, Relative Index and Quantum Pumps	129
	References.	131

Lecture 6: Lie–Trotter Formula, Wiener Process,

	Feynman–Kac Formula.	133
1	The Feynman Formula	138
2	Stationary Action; The Fujiwara’s Approach	140
3	Generalizations of Fresnel Integral	141
4	Relation with Stochastic Processes	142
5	Random Variables. Independence	144
6	Stochastic Processes, Markov Processes.	145
7	Construction of Markov Processes	146

8	Measurability	148
9	Wiener Measure	151
10	The Feynman–Kac Formula I: Bounded Continuous Potentials.	152
11	The Feynman–Kac Formula II: More General Potentials	153
	References.	155

Lecture 7: Elements of Probability Theory. Construction

of Brownian Motion. Diffusions	157
1 Inequalities	158
2 Independent Random Variables	160
3 Criteria of Convergence.	161
4 Laws of Large Numbers; Kolmogorov Theorems	162
5 Central Limit Theorem	164
6 Construction of Probability Spaces	166
7 Construction of Brownian Motion (Wiener Measure).	167
8 Brownian Motion as Limit of Random Walks	169
9 Relative Compactness	170
10 Modification of Wiener Paths. Martingales.	172
11 Ito Integral.	175
References.	177

Lecture 8: Ornstein–Uhlenbeck Process. Markov Structure.

Semigroup Property. Paths Over Function Spaces	179
1 Mehler Kernel	179
2 Ornstein–Uhlenbeck Measure.	181
3 Markov Processes on Function Spaces	184
4 Processes with (Continuous) Paths on Space of Distributions. The Free-Field Process	186
5 Osterwalder Path Spaces	189
6 Strong Markov Property	190
7 Positive Semigroup Structure	191
8 Markov Fields. Euclidian Invariance. Local Markov Property.	194
9 Quantum Field	196
10 Euclidian Free Field	198
11 Connection with a Local Field in Minkowski Space	200
12 Modifications of the O.U. Process. Modification of Euclidian Fields	201
References.	202

Lecture 9: Modular Operator. Tomita–Takesaki Theory

Non-commutative Integration	203
1 The Trace. Regular Measure (Gage) Spaces	204
2 Brief Review of the K-M-S. Condition	206
3 The Tomita–Takesaki Theory.	208
4 Modular Structure, Modular Operator, Modular Group	212
5 Intertwining Properties	215

6	Modular Condition. Non-commutative Radon–Nikodym Derivative	219
7	Positive Cones	223
	References.	225
Lecture 10: Scattering Theory. Time-Dependent Formalism.		
	Wave Operators	227
1	Scattering Theory	228
2	Wave Operator, Scattering Operator	230
3	Cook–Kuroda Theorem	232
4	Existence of the Wave Operators. Chain Rule	234
5	Completeness	236
6	Generalizations. Invariance Principle	241
	References.	245
Lecture 11: Time Independent Formalisms. Flux-Across Surfaces. Enss Method. Inverse Scattering		
1	Functional Equations.	251
2	Friedrich’s Approach. Comparison of Generalized Eigenfunctions.	253
3	Scattering Amplitude.	254
4	Total and Differential Cross Sections; Flux Across Surfaces.	255
5	The Approach of Enss.	259
6	Geometrical Scattering Theory	260
7	Inverse Scattering Problem.	263
	References.	268
Lecture 12: The Method of Enss. Propagation Estimates. Mourre Method. Kato Smoothness, Elements of Algebraic Scattering Theory		
1	Enss’ Method.	271
2	Estimates.	273
3	Asymptotic Completeness	274
4	Time-Dependent Decomposition.	276
5	The Method of Mourre	279
6	Propagation Estimates	280
7	Conjugate Operator; Kato-Smooth Perturbations	285
8	Limit Absorption Principle.	288
9	Algebraic Scattering Theory.	289
	References.	292
Lecture 13: The N-Body Quantum System: Spectral Structure and Scattering		
1	Partition in Channels.	294
2	Asymptotic Analysis	296
3	Assumptions on the Potential	297
4	Zhislin’s Theorem.	298

5	Structure of the Continuous Spectrum	301
6	Thresholds	302
7	Mourre's Theorem	304
8	Absence of Positive Eigenvalues.	308
9	Asymptotic Operator, Asymptotic Completeness.	311
	References.	313

Lecture 14: Positivity Preserving Maps. Markov Semigroups.

	Contractive Dirichlet Forms	315
1	Positive Cones	315
2	Doubly Markov	317
3	Existence of the Ground State	319
4	Hypercontractivity	320
5	Uniqueness of the Ground State	324
6	Contractions.	328
7	Positive Distributions	330
	References.	332

Lecture 15: Hypercontractivity. Logarithmic Sobolev Inequalities.

	Harmonic Group	333
1	Logarithmic Sobolev Inequalities	334
2	Relation with the Entropy	337
3	Estimates of Quadratic Forms.	339
4	Spectral Properties	340
5	Logarithmic Sobolev Inequalities and Hypercontractivity	342
6	An Example: Gauss–Dirichlet Operator	344
7	Other Examples	346
	References.	350

Lecture 16: Measure (Gage) Spaces. Clifford Algebra, C.A.R.

	Relations. Fermi Field	351
1	Gage Spaces	351
2	Interpolation Theorem	354
3	Perturbation Theory for Gauge Spaces.	355
4	Non-commutative Integration Theory for Fermions	356
5	Clifford Algebra	357
6	Free Fermi Field.	359
7	Construction of a Non-commutative Integration	360
8	Dual System	361
9	Alternative Definition of Fermi Field.	362
10	Integration on a Regular Gage Space	364
11	Construction of Fock Space	367
	References.	372

	Index	373
--	------------------------	-----

Lectures on the Mathematics of Quantum Mechanics II:

Selected Topics

DellAntonio, G.

2016, XIX, 381 p., Hardcover

ISBN: 978-94-6239-114-7

A product of Atlantis Press