

Preface

The aim of this monograph is to present a general method of proving continuity of the Lyapunov exponents (LE) of linear cocycles.

The method consists of an inductive procedure that establishes continuity of relevant quantities for finite, larger and larger number of iterates of the system. This leads to continuity of the limit quantities, the LE. The inductive procedure is based upon a deterministic result on the composition of a long chain of linear maps called the Avalanche Principle (AP). A geometric approach is used to derive a general version of this principle.

The main assumption required by this method is the availability of appropriate large deviation type (LDT) estimates for quantities related to the iterates of the base and fiber dynamics associated with the linear cocycle. Crucial for our approach is the uniformity in the data of these estimates.

We derive such LDT estimates for various models of random cocycles (over Bernoulli and Markov systems) and quasi-periodic cocycles (defined by one or multivariable torus translations). The random model, treated under an irreducibility assumption, uses an existing functional analytic approach which we adapt so that it provides the required uniformity of the estimates. The quasi-periodic model uses harmonic analysis and it involves the study of (pluri) subharmonic functions.

This method has its origins in a paper of M. Goldstein and W. Schlag which proves continuity of the Lyapunov exponent for the one-parameter family of quasi-periodic Schrödinger cocycles, assuming a uniform lower bound on the exponent. This is where the first version of the Avalanche Principle appeared, along with the use and proof of the relevant LDT estimate.

The present work expands upon their approach in both depth and breadth. Moreover, it reduces the general problem of proving continuity of the LE to one of a different nature—proving LDT estimates. This may be treated independently and by means specific to the underlying base dynamic of the cocycle.

Our geometric approach to the AP also gives rise to a mechanism for studying the most expanding singular direction of the composition of a long chain of linear maps. This allows us to obtain a new proof of the Multiplicative Ergodic

Theorem of Oseledets. Moreover, assuming the availability of the same LDT estimates, this extension of the AP leads to continuity properties of the Oseledets filtration and decomposition.

Most of the results presented in this research monograph are new. We assume the reader to have a certain degree of familiarity with basic dynamical systems and ergodic theory notions. The relevant concepts and definitions needed for the formulation of the main results are introduced in Chap. 1. While each subsequent chapter is to some extent self-contained and it may be read independently of the rest, all the arguments in this work are based upon the results in Chaps. 2 and 3. Besides the formulation and the proof of the AP, Chap. 2 contains Lipschitz estimates on certain Grassmann geometrical quantities that are crucial in Chap. 4, where we study the Oseledets filtration and decomposition and their continuity properties. In Chap. 3 we establish the abstract continuity theorem (ACT) of the LE and some other related technical results. In Chaps. 5 and 6, under appropriate assumptions, we derive the relevant LDT estimates for random and respectively quasi-periodic cocycles. The general results in Chaps. 3 and 4 are then applicable to these models, and they imply continuity properties of the LE and of the Oseledets filtration and decomposition for the corresponding spaces of cocycles.

Our work concludes in Chap. 7 with a list of related open problems, some of which may be treated using the methods described in this monograph.

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