# Chapter 2 From data to the MM1 meta-model and MM1MS1 models

Chapter 2 (From data to the MM1 meta-model and MM1MS1 models) discusses modeling. We discuss differences between inputs to models and outputs from models. We discuss the desirability of extending modeling practices to include models that produce as outputs information that traditional models use as inputs but do not produce as outputs. We point to some physics data that generally accepted physics models treat as inputs and generally do not produce as outputs. We develop a meta-model. We use the MM1 meta-model to produce MM1MS1 models. We correlate aspects of the models with aspects of traditional mathematical physics. We correlate aspects of the models with possible future mathematical physics.

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People might say that results in Chapter 2 (From data to the MM1 meta-model and MM1MS1 models) might correlate with or provide insight about the following aspects of physics. Spins of elementary particles. Similarities and differences between boson elementary particles and fermion elementary particles. Fields and particles. Number of generations, for elementary fermions. Number of color charges, for relevant elementary fermions. Aspects related to interactions that preserve generation for elementary fermions. A length characterizing the weak interaction. Conservation laws. Conservation of charge. Possible bases for dark-energy stuff. Possible bases for dark matter. Quantum aspects related to magnetic dipole moments of elementary particles. Symmetries related to QCD (or, quantum chromodynamics). Aspects of kaon CP-violation. Aspects of neutral B meson flavor oscillation.

## Section 2.1 Math-based models for quantum phenomena

Section 2.1 discusses modeling. We discuss differences between inputs to models and outputs from models. We discuss the desirability to extend modeling practices to include models that produce as outputs information that traditional models use as inputs but do not produce as outputs. We point to some physics data that generally accepted physics models treat as inputs and generally do not produce as outputs. We discuss goals for the MM1 meta-model and for MM1MS1 models. We note that trying



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to develop models that use quantum harmonic oscillator math may be useful. We introduce terminology regarding classes of physics theories, classes of models this monograph shows, and some physics concepts.

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This subsection provides perspective about this section.

One key to advancing science can be to pinpoint known data for which people do not think theories or models are adequate. We start this section by noting some such data.

Another key to advancing science can be to pinpoint models that seem to be more precise than are other models. Perhaps, such more-precise models provide promise for reuse. We note a seemingly precise model.

Another key can be to develop meta-models via which people can choose or develop models. This section discusses goals for such a meta-model and for models that correlate with the meta-model.

Another key can be to categorize models and theories into useful sets. This section discusses some categories we think can be useful for people's understanding, developing, and using some models and theories.

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This subsection provides perspective about modeling - in general and specifically as pertains to this monograph.

Math-based models provide key aspects of science and of applied science. People use outputs from models to shape thinking and action. People use outputs to design research efforts. People use outputs to do engineering and to develop products and services.

Models are not nature. People design models to reflect thoughts about nature. Models incorporate people's thinking. Models incorporate or use assumptions and other inputs.

Sometimes inputs to models reflect interpretations of experiments or observations for which people think that the totality of generally accepted models does not produce as outputs those interpretations.

People might say that this monograph reflects attempts to develop models that produce, as outputs, information that traditional models do not produce as outputs. Generally, some such information correlates with results of experiments or observations. People might say that other such output information constitutes possible predictions.

People might say that we try to develop models that reflect and extend a trend. Some traditional classical-physics modeling features concepts for which people might correlate the term continuous. Over time, concepts for which people might correlate the term discreet gained prominence regarding physics and physics models. For example, models relevant to chemistry changed with the discovery of atoms. People had previously discussed concepts for a discreet unit of stuff. People's thinking embraced the discovery - the atom. The advent of quantum mechanics led to more concepts that correlate with the notion of discreet. People might say that today's quantum mechanics features some discreet concepts that correlate with inputs to, but not outputs from, generally accepted models. People might say that we try to develop models that produce as outputs some of those discreet numbers.

People might say that we develop models that do not significantly contradict known data. Room for debate exists. Some of that data reflects not only results of experiments or observations but also interpretations of such results via traditional models. So, some such debate could be significantly about traditional models as well as about the extent to which new models dovetail with known data.

We think that not enough data exists to validate completely today models we present.

We adopt a practice common in numerical simulation. We provide, in effect, a meta-model though which people can develop or specify models. Within the meta-model, people can choose from among various parameters and various modeling assumptions. People might say that the MM1 meta-model is adequately specific that people can efficiently make such choices and can effectively use the resulting specific MM1MS1 models.

People might say that at least one choice of parameters and assumptions correlates with producing as outputs information adequately matching known data that generally accepted traditional models do not produce as outputs.

People might say that at least one choice of parameters and assumptions correlates adequately well with mainstream traditional modeling regarding aspects of elementary-particle physics, astrophysics, and cosmology.

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This subsection lists data for which the word discreet pertains and the word continuous does not pertain.

Table 2.1.1 features sets of discreet physics numbers. People use these numbers in physics models. People might say that, generally, people do not derive some of these numbers from physics models.

**Table 2.1.1** Types of physics observations that yield discreet numbers

1. For each known elementary particle, the expression $j|q_e|/3$, with j being an integer, describes the charge of the particle.
   1.1. $q_e$ denotes the charge of an electron.
2. For each elementary particle, the expression $S\hbar$, with $2S$ being a nonnegative integer, describes the spin of the particle.
   2.1. $\hbar$ denotes Planck's constant (reduced).

3. Harmonic-oscillator-math raising operators and lowering operators correlate with quantum descriptions of modes of the vector potential.
    3.1. For each photon, the spectrum of excitation states is discreet.
4. The number $S(S + D^* - 2)$, with $D^* = 3$, pertains for the radial-component of some harmonic oscillator math-based models correlating with particles having spin/ħ of S.
$$S(S + D^* - 2) = S(S + 1), \text{ for } D^* = 3 \tag{2.1}$$
    4.1. For each known elementary particle, such an $S(S + 1)$ is relevant to the physics of the particle.
5. For each known elementary particle, a nonnegative number, m', describes the rest mass of the particle.
    5.1. The spectrum of such rest masses is discreet.
6. The single speed c and the next expression pertain for all free-ranging elementary particles.
$$E^2 - c^2P^2 = (m')^2c^4 \tag{2.2}$$
    6.1. In the expression, E denotes energy, c denotes the speed of light (in a vacuum), P denotes momentum, and m' denotes the rest mass of the specific particle.
    6.2. Free-ranging elementary particles include electrons and photons.
    6.3. Free-ranging elementary particles do not include quarks.
7. For each known elementary fermion, the number of generations is 3.
    7.1. Examples of elementary fermions include the electron, neutrinos, and quarks.
    7.2. For neutrinos, people say that 3 flavors exist.

Table 2.1.2 points to a possible discreet physics number for which we use the term #ENS. Much traditional physics does not explore whether #ENS can be other than 1. Some MM1MS1 models correlate with #ENS = 48. For #ENS = 48, we discuss the possibility that 8 sets of ensembles exist in the universe. Each set includes 6 ensembles. Each set of 6 ensembles would have its own gravitons. One set includes one ensemble that correlates with ordinary matter and five ensembles that correlate with dark matter. Some MM1MS1 models correlate with the possibility that #ENS = 6. For #ENS = 6, we discuss the possibility that 1 set of 6 ensembles exists in the universe. The set includes one ensemble that correlates with ordinary matter and five ensembles that correlate with dark matter. Some MM1MS1 models correlate with the possibility that #ENS = 1.

**Table 2.1.2** A discreet number (#ENS) for which much traditional physics assumes one value and for which MM1MS1 models explore more than one value

1. Below (See, Section 2.13.), …
    1.1. We identify a symmetry we call INSSYM7.
    1.2. We correlate this symmetry with …
        1.2.1. The (mathematics) group SU(7).
        1.2.2. The number 48, which is the number of generators of SU(7).
    1.3. We posit that (mathematically) each generator correlates with an ensemble.
    1.4. An ensemble includes …
        1.4.1. A copy (or, instance) of the Standard Model (as of the year 2015) and, thereby, the following.
            1.4.1.1. A set of elementary particles including …
                1.4.1.1.1. Leptons (the electron, the muon, the tauon, and neutrinos).
                1.4.1.1.2. Quarks.
                1.4.1.1.3. The weak-interaction bosons (the Z and W bosons).
                1.4.1.1.4. The Higgs boson.
                1.4.1.1.5. The photon.
                1.4.1.1.6. Gluons.
        1.4.2. Possibly, some other elementary particles that would correlate with the MM1 meta-model.
            1.4.2.1. Some possible relatives of the Higgs, Z, and W bosons provide examples.
    1.5. We assume that one ensemble correlates with ordinary matter (or, baryonic matter).
        1.5.1. That ensemble correlates with aspects of nature with which people are familiar on a daily basis.
    1.6. We develop MM1MS1 models that correlate with the following possibilities.
        1.6.1. #ENS = 1. Here, …
            1.6.1.1. Models point to no forces that connect the ordinary-matter ensemble with other ensembles.
            1.6.1.2. People might say that …
                1.6.1.2.1. The universe features 1 ensemble.
                1.6.1.2.2. The other 47 mathematically possible ensembles do not pertain to nature.
        1.6.2. #ENS = 6. Here, …
            1.6.2.1. Models point to forces that connect the ordinary-matter ensemble with 5 other ensembles.
            1.6.2.2. People might say that …
                1.6.2.2.1. The universe features 6 ensembles.

1.6.2.2.2. The other 42 mathematically possible ensembles do not pertain to nature.
1.6.3. #ENS = 48. Here, …
1.6.3.1. Models point to forces that connect the ordinary-matter ensemble with 47 other ensembles.
1.6.3.2. People might say that …
1.6.3.2.1. The universe features 48 ensembles.
1.7. We mention the possibility for MM1MS1 models that correlate with the following possibility. This monograph does not further explore this possibility.
1.7.1. #ENS = 2.
2. #ENS denotes the number of ensembles that nature exhibits.
2.1. For #ENS ≠ 1, an ensemble does not include some elementary particles that would correlate with the MM1 meta-model.
2.1.1. The graviton provides an example.
3. We use the following acronyms.
3.1. The acronym ENS48 pertains to models for which #ENS = 48.
3.2. The acronym ENS6 pertains to models for which #ENS = 6.
3.3. The acronym ENS1 pertains to models for which #ENS = 1.

Table 2.1.3 defines some notation that we use.

**Table 2.1.3** Notation of the form #Z

1. Usually, …
1.1. A symbol of the form #Z (in which Z can be a combination of letters, numbers, and punctuation marks) …
1.1.1. Denotes a nonnegative integer.
2. Sometimes, …
2.1. #Z correlates with the termination of one of the following series of sequential nonnegative integers.
2.1.1. #Z, #Z − 1, …, 1, 0.
2.1.2. 0, 1, …, #Z − 1, #Z.
2.2. In such cases, we may use the statement #Z = 'Ø to correlate with the empty set of integers.
2.2.1. In mathematics, the symbol Ø to denotes the empty (or, null) set.
3. For example, …
3.1. In Table 2.3.1, we use the expression #'E = 'Ø.
3.1.1. This usage correlates with SIDE = Ø. (See Table 2.2.3.)

Table 2.1.4 attempts to categorize concepts from Table 2.1.1 and Table 2.1.2.

**Table 2.1.4** Categories of some discreet physics numbers

1. Dimensionless integers that seem to be independent of physics constants.
   1.1. Integers correlating with excitation states of the vector potential.
   1.2. D*.
   1.3. Numbers of generations.
   1.4. #ENS.
2. Dimensionless integers that couple with physics constants.
   2.1. 3Q', which we define as charge/($|q_e|$/3).
   2.2. 2S, which equals spin/(ħ/2).
3. Other.
   3.1. $|q_e|$/3.
   3.2. ħ/2.
   3.3. Values of m'.

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This subsection notes successes and possible limitations of some traditional math-based models that people use regarding physics.

People try to develop and use math-based models that people can correlate with observations about nature.

People call one collection of such models the particle-physics Standard Model. (Here, we use the term particle-physics Standard Model to differentiate from uses of the terms cosmology standard model and cosmological standard model. We do not use the terms cosmology standard model and cosmological standard model further in this monograph.) The Standard Model correlates with some observations about elementary particles, composite particles, and interactions between particles.

People say that the Standard Model falls short regarding correlating with various observations and regarding predicting various possible future observations. People might say that, for example, numbers of generations, values of m', and a list of elementary particles can each be considered to be (mainly) inputs to the Standard Model and not (from a standpoint of theory) results of the Standard Model. People might say that, also, the Standard Model (as of 2015) did not adequately predict elementary particles that had yet to be discovered.

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This subsection discusses goals we have regarding the MM1 meta-model and physics models people can produce from the MM1 meta-model.

Table 2.1.5 provides attributes for which we strive, regarding the MM1 meta-model we present and regarding models we develop from the MM1 meta-model.

**Table 2.1.5** Goals regarding a meta-model and regarding physics models

1. Show a math-based meta-model, such that …

- 1.1. People can use the meta-model to develop and explore physics models.
- 1.2. At least one model …
  - 1.2.1. Seems to correlate with data for which traditional physics models do not adequately correlate.
  - 1.2.2. Seems not to significantly require possible assumptions or conclusions that contradict relevant known data.
  - 1.2.3. Seems adequately harmonious with extant successful physics models.
- 1.3. At least one model …
  - 1.3.1. Seems to correlate adequately with traditional physics models that successfully correlate with some data.
- 1.4. People can use the meta-model to explore how to unify new and traditional models.

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This subsection discusses traditional physics uses of quantum harmonic oscillator math-based models.

People might say that the item in Table 2.1.1 about harmonic-oscillator-math raising operators and lowering operators seems to pertain exactly, at least to the extent of some aspects of models for known photonics.

Other uses of quantum harmonic oscillator math-based models in traditional physics represent attempts to layer quantum models on top of models for non-quantum physics. Such attempts feature approximations to physics phenomena. For example, attempts to quantize some aspects of classical physics exactly via harmonic oscillator math would require modeling an infinitely large potential energy.

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This subsection notes reasons for considering trying to find new physics uses for quantum harmonic oscillator math-based models.

Table 2.1.6 pertains. Much of the work in this monograph uses mathematics for quantum harmonic oscillators. (Compare with the first item in Table 2.1.4. Below, we show work that correlates with the second item and the third item in Table 2.1.6.)

**Table 2.1.6** Correlations between quantum harmonic oscillator math-based models and discreet physics numbers

1. Math-based models for harmonic oscillators provide a seemingly exact model for aspects of quantum photonics phenomena.
2. We find, in quantum harmonic oscillator math-based models, a key role for D*.
3. We find that quantum harmonic oscillator math-based models can correlate with generations.

4. We find that quantum harmonic oscillator math-based models may correlate with masses of zero-mass elementary particles and with masses of some non-zero-mass elementary particles.

We note other concepts that may dovetail with attractiveness for trying to use quantum harmonic oscillator math-based models. People might say that, for quantum harmonic oscillators, the kinetic energy and potential energy contribute equally. People might say that such equal contributions correlate with a stationary point or other feature regarding the expression T − V in action (or, Lagrangian) mathematics. Here, T correlates with kinetic energy and V correlates with potential energy. Much traditional modeling related to elementary particles correlates with action-based math.

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This subsection discusses relationships between math, models, and physics that this monograph addresses.

We show and discuss solutions for math describing isotropic pairs of isotropic quantum harmonic oscillators. The set of such solutions has an infinite number of ground-state members. This chapter contains subsections, paragraphs, sentences, and phrases prefaced by the symbol [Physics:]. Work in this chapter uses some of those remarks to limit the ground states this monograph features. After such uses, we phase out using the symbol [Physics:].

[Physics:] Correlations between nature and models this monograph features may span aspects of elementary-particle physics and cosmology. For example, some solutions correlate with all known elementary particles. For some aspects of elementary-particle physics and of cosmology, data about nature are not well-developed.

Work in this chapter allows people to make current and future selections regarding which math solutions to try to correlate with aspects of nature.

[Physics:] For example, some solutions may correlate with yet-to-be-discovered elementary particles. Should future data rule out some such particles, people can, when thinking about particle physics, abandon use of those solutions and of work in this monograph based on those solutions.

Thus, this monograph provides a meta-model (or, platform) for developing models that might match data regarding nature. (See Table 2.1.5.)

~ ~ ~

This subsection provides vocabulary for describing some classes of physics theories and models.

Table 2.1.7 pertains.

**Table 2.1.7** Terminology for classes of some physics theories and models

1. Classical physics.
    1.1. Pre-relativity classical physics.
    1.2. Special-relativistic classical physics.
        1.2.1. This includes (non-quantum mechanical) relativistic electromagnetism.
    1.3. General-relativistic classical physics.
2. Quantum physics.
    2.1. Some models pertaining to objects (such as solids, fluids, molecules, atoms, and atomic nuclei) that people would say contain elementary particles and composite particles.
    2.2. Some models pertaining to elementary particles and composite particles.

People might say that, regarding quantum physics pertaining to elementary particles, Table 2.1.8 pertains. The classes of models the table shows are not necessarily completely thorough or rigorous. We think that the classes are defined adequately for purposes of this monograph.

**Table 2.1.8** Terminology (including QMUSPR, QMPRPR, MM1, and MM1MS1) for classes of some quantum physics models regarding elementary particles

1. We use the term QMUSPR to abbreviate the phrase quantum model that uses (or quantum models that use) assumed elementary-particle properties.
    1.1. Much of traditional quantum mechanics …
        1.1.1. Assumes a list of elementary particles.
        1.1.2. Uses the list of elementary particles as inputs to models.
        1.1.3. Assumes that elementary particles have properties (such as spin, mass, or charge).
        1.1.4. Uses the properties as inputs to models.
        1.1.5. Models interactions between elementary particles and/or objects that include elementary particles.
2. We use the term QMPRPR to abbreviate the phrase quantum model that provides (or quantum models that provide) elementary-particle properties.
    2.1. Some of the work this monograph features …
        2.1.1. Assumes that models can provide lists of known elementary particles and candidates for elementary particles.
        2.1.2. Assumes that, for elementary particles, properties (such as spin, mass, or charge) can (at least, somewhat) correlate with outputs from models.
        2.1.3. Models properties of elementary particles.

        2.1.4. Models interactions between elementary particles.
3. We use the term MM1 to abbreviate the phrase meta-model 1.
    3.1. The term MM1 correlates with the meta-model this monograph discusses.
4. We use the term MM1MS1 to abbreviate the phrase meta-model 1 model set 1.
    4.1. The term MM1MS1 correlates with models that people can develop from the MM1 meta-model.
5. People might say that MM1MS1 models correlate with attempts to develop QMPRPR.

Much of physics collates with describing objects and their relationships with the environments in which the objects exist. For example, some physics might model the motion of a planet within a solar system. Here, there might be many objects. Here, a model might correlate the motion of the planet with an environment characterized by the gravitation fields that the model associates with the other objects in the solar system. Or, for example, some physics might model motions of solids and liquids that make up the planet. A model of earth might consider oceans to be part of the planet. A model of tides might consider gravitational effects correlating with objects (such as the moon and the sun) not associated with the planet and effects (gravitational and otherwise) that the model would correlate with the planet.

Some work, regarding elementary particles, in this monograph features characteristics of the particles that people might characterize as internal properties. Examples of such properties include spin, mass, and charge. Some work, regarding elementary particles, in this monograph features relationships between elementary particles and their environments. Examples correlating with such relationships might include forces that act on particles, perceived energy of particles, and perceived momentum of particles. People might say that making a complete separation between internal and environmental may be inappropriate in some models. Paralleling the example of a planet, its tides, and its solar system, we might agree. However, for purposes of some work in this monograph, we think that Table 2.1.9 provides useful (though not necessarily completely rigorous) distinctions.

**Table 2.1.9** The terms internal (INTERN), fermion-transformational (FERTRA), extended internal (EXTINT), and environmental (ENVIRO), regarding models regarding elementary particles

1. Internal.
    1.1. This term correlates with modeling of aspects (of an elementary particle) that a model treats as being more associated with the particle than with the environment in which the particle exists.
    1.2. Sometimes we use the acronym INTERN.
        1.2.1. INTERN abbreviates the word internal.
2. Extended internal.

2.1. This term also correlates with modeling of aspects (of an elementary particle) that a model treats as being more associated with the particle than with the environment in which the particle exists.
   2.1.1. People might say that extended internal models …
      2.1.1.1. May support theoretically studying correlations between types of elementary-particle properties.
      2.1.1.2. May not be needed for other purposes.
   2.1.2. Sometimes we use the acronym EXTINT.
   2.1.3. EXTINT abbreviates the phrase extended internal.

3. Fermion-transformational.
   3.1. This term correlates with modeling the extent to which an elementary fermion can change into another elementary fermion, based on interactions with non-zero-mass bosons.
   3.2. Sometimes we use the acronym FERTRA.
      3.2.1. FERTRA abbreviates the phrase fermion transformational.
4. Environmental.
   4.1. People might say that this term correlates with modeling of aspects (of an elementary particle) that a model treats as being more associated with the environment in which the particle exists than with the particle.
   4.2. Relevant environments can include the following. (See Table 2.1.10)
      4.2.1. SPATIM.
      4.2.2. FRERAN.
      4.2.3. COMPAR.
      4.2.4. ATOMOL.
   4.3. Sometimes we use the acronym ENVIRO.
      4.3.1. ENVIRO abbreviates the word environmental.
5. This monograph links concepts regarding INTERN models, concepts regarding FERTRA models, and concepts regarding ENVIRO models.

[Physics:] Some aspects of physics distinguish, for a single type of elementary particle, between various environments. For example, for electrons, some aspects of physics pertain for electrons bound in molecules or atoms. Some aspects of physics pertain for electrons that roam over distances inside metals or semiconductors. Some aspects of physics pertain for electrons that travel in a near vacuum. Some aspects of physics regarding electrons span all such environments.

We discuss more than one environment for elementary particles. Table 2.1.10 pertains.

**Table 2.1.10** The terms SPATIM, FRERAN, COMPAR, and ATOMOL regarding ENVIRO models regarding elementary particles

1. SPATIM.

1.1. This term abbreviates the phrase space-time coordinate or the phrase space-time coordinate symmetries.
1.2. People might say that SPATIM ENVIRO models correlate with particles existing in a universe for which space-time coordinates pertain for models.
1.3. We show SPATIM models for all elementary particles.
1.3.1. People might say that some of the elementary particles for which we develop SPATIM models have not been observed to exhibit behavior compatible with some traditional uses of the term free-ranging.
1.3.1.1. Examples of such elementary particles include quarks and gluons.
1.4. We use this term to abbreviate …
1.4.1. SPATIM ENVIRO.

2. FRERAN.
2.1. This term abbreviates the phrase free-ranging.
2.2. People might say that FRERAN models correlate with aspects of traditional quantum physics models regarding free-ranging elementary particles.
2.3. People might say that FRERAN models correlate with aspects (such as some aspects correlating with Feynman diagrams) of traditional quantum physics models that people use regarding aspects of behavior of non-free-ranging elementary particles. (Compare with COMPAR models, to which this table alludes below.)
2.4. We use this term to abbreviate …
2.4.1. FRERAN SPATIM.
2.4.2. FRERAN SPATIM ENVIRO.

3. COMPAR.
3.1. This term abbreviates the phrase composite particles.
3.2. People might say that COMPAR models correlate with aspects (regarding elementary particles) that correlate with composite particles.
3.2.1. Examples of composite particles include pions and the proton.
3.2.2. Examples of elementary particles bound in pions or protons include quarks and gluons.
3.2.2.1. People consider quarks and gluons to be non-free-ranging elementary particles.
3.3. We use this term to abbreviate …
3.3.1. COMPAR SPATIM.
3.3.2. COMPAR SPATIM ENVIRO.

4. This monograph shows, for some elementary particles, both FRERAN and COMPAR models.

- 4.1. [Physics:] For the W boson, FRERAN models differ from COMPAR models.
  - 4.1.1. People might say that FRERAN models pertain to interactions between leptons and W bosons.
  - 4.1.2. People might say that FRERAN models might pertain to some interactions between quarks and W bosons.
  - 4.1.3. People might say that COMPAR models pertain to some interactions between quarks and W bosons.

5. ATOMOL.
   - 5.1. This term abbreviates the phrase atoms and molecules.
   - 5.2. People might say that ATOMOL models correlate with aspects (of elementary particles and atomic nuclei) that correlate with atoms and molecules.
     - 5.2.1. Examples of atoms include the hydrogen atom.
   - 5.3. We use this term to abbreviate …
     - 5.3.1. ATOMOL SPATIM.
     - 5.3.2. ATOMOL SPATIM ENVIRO.

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This subsection notes that this monograph uses two types of representations for quantum harmonic oscillators.

This monograph features models based on math correlating with isotropic quantum harmonic oscillators. Traditionally, people use at least two math-based approaches regarding quantum harmonic oscillators. For some cases, the two approaches yield similar results. (For example, see Table 2.2.14 and Table 2.5.12.) For some cases, the two approaches yield different results. (For example, see Table 2.2.15 and Table 2.5.13.)

One approach features partial differential equations and functions ([Physics:] wave functions) of continuous coordinates ([Physics:] often, spatial coordinates). Here, for multidimensional harmonic oscillators, people may have a choice regarding coordinate systems to use. One choice features using one linear coordinate for each dimension. For an isotropic harmonic oscillator, another choice features using radial coordinates.

This monograph sometimes uses approaches that feature partial differential equations. For multidimensional aspects, we use radial coordinates. For single-dimensional aspects we use a linear coordinate or a projection of a radial coordinate.

We correlate the term DIFEQU with such uses. DIFEQU provides an acronym for (usually) radial-coordinate (generally) multidimensional partial differential equation.

Another approach does not feature spatial coordinates. Such approaches start from, for each dimension, a characterization that features a number that represents the number of times the one-dimensional harmonic oscillator correlating with that dimension is excited. The models can feature raising-operators and lowering-operators that people express in a way that includes a numerical factor that depends on the original number of excitations. People sometimes use the term ladder operator to characterize raising operators and lowering operators. For an isotropic harmonic oscillator, people can develop models that feature a number that represents a total of excitations, with the sum running across the one-dimensional harmonic oscillators that correlate with the various dimensions.

This monograph sometimes uses approaches that feature ladder operators and do not feature partial differential equations. For multidimensional aspects, we generally use representations that feature excitation numbers and ladder operators for the individual component oscillators.

We correlate the term LADDER with such uses. LADDER provides an acronym for ladder operators.

Other approaches pertain regarding harmonic oscillators. For example, for multidimensional isotropic harmonic oscillators expressed in radial coordinates, people use approaches that feature ladder operators. This monograph does not use such approaches.

Table 2.1.11 pertains. Treatments of LADDER representations feature math pertaining to sets of lone (or, one-dimensional) oscillators. Treatments of DIFEQU representations feature radial coordinates and/or linear coordinates and functions ([Physics:] wave functions) expressed in terms of radial and/or linear coordinates.

**Table 2.1.11** Models, for harmonic oscillators, that this monograph uses

1. Models based on LADDER approaches.
2. Models based on DIFEQU approaches.

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This subsection provides definitions of the terms boson and fermion.

Table 2.1.12 shows definitions of boson and fermion. These definitions correlate with traditional physics uses of the two terms.

**Table 2.1.12** Definitions of S, boson, and fermion

1. [Physics:] This monograph uses S to denote spin/ħ.
2. [Physics:] The term boson pertains to any object for which (in math-based models) 2S equals an even nonnegative integer.
3. [Physics:] The term fermion pertains to any object for which (in math-based models) 2S equals an odd positive integer.

~ ~ ~

This subsection lists categories of LADDER solutions this monograph uses and categories of models (based on solutions) this monograph uses.

Table 2.1.13 lists categories of LADDER solutions this monograph uses. [Physics:] Here, regarding the term elementary particles, we deemphasize distinctions people might try to make regarding the term field and the term particle.

**Table 2.1.13** CORMAT and CORPHY categories of LADDER solutions

1. CORMAT solutions …
   1.1. Correlate with the simplest LADDER solutions (to equations involving isotropic pairs of isotropic harmonic oscillators) that we think correlate with known elementary particles or with possible elementary particles.
   1.2. Take their acronym (of CORMAT) from the phrase core relevant mathematical solutions.
2. CORPHY solutions …
   2.1. Correlate with CORMAT solutions in ways such that …
      2.1.1. For boson-related solutions, CORPHY solutions equal CORMAT solutions.
      2.1.2. For fermion-related solutions, CORPHY solutions represent extensions (based on adding two oscillators) to CORMAT solutions.
   2.2. Take their acronym (CORPHY) from the phrase core solutions relevant to elementary-particle physics.
   2.3. Provide a basis for solutions this monograph shows for the following categories of solutions. (See, for example, Table 2.1.14.)
      2.3.1. INTERN LADDER.
      2.3.2. EXTINT LADDER.
      2.3.3. FERTRA LADDER.
      2.3.4. SPATIM LADDER.
      2.3.5. FRERAN LADDER.
      2.3.6. COMPAR LADDER.

~ ~ ~

[Physics:] This subsection previews applications for categories of solutions this monograph uses and for categories of models (based on solutions) this monograph uses.

Table 2.1.14 summarizes some applications for various categories of models or solutions. (See Table 2.2.9, Table 2.5.6, and Section 2.6 for discussions about the term solution. See Table 2.3.4 for a list of families of elementary particles.)

**Table 2.1.14** Some applications of various categories of solutions

1. INTERN LADDER solutions correlate with ...
   1.1. Families of elementary particles.
   1.2. Elementary particles within a family.

   - 1.3. Spin.
   - 1.4. Excitation states, for bosons.
   - 1.5. Charge.
   - 1.6. Some aspects regarding approximate masses, for elementary bosons.
   - 1.7. Zero size for some aspects related to elementary particles.
   - 1.8. Number of color charges, for non-lepton elementary fermions.
   - 1.9. The extent to which the strengths of interactions mediated by G-family particles vary, when the interactions involve fermions, by fermion generation. (See, for example, Table 2.8.2.)
2. DIFEQU solutions correlate with …
   - 2.1. Families of elementary particles.
   - 2.2. Elementary particles within a family.
   - 2.3. Spin.
   - 2.4. Some aspects regarding approximate masses, for elementary bosons.
   - 2.5. Non-zero size for the extent to which fields pertain.
   - 2.6. Zero size for some aspects related to elementary particles.
   - 2.7. The number of generations for each non-zero-mass elementary fermion.
3. FERTRA LADDER solutions correlate with …
   - 3.1. Abilities of fermion elementary particles to transform into other fermion elementary particles via interactions with non-zero-mass elementary bosons.
4. FRERAN LADDER solutions correlate with …
   - 4.1. Symmetries related to special relativity symmetries (and the Poincare group).
   - 4.2. Number of instances of fields for each particle.
     - 4.2.1. Possible explanations for dark-energy stuff and dark matter.
   - 4.3. The notion that, in FRERAN environments, interactions mediated by W-family particles do not change the generations of fermions. (See, for example, remarks preceding Table 2.7.1.)
   - 4.4. For G-family particles, the number of channels pertaining to interactions that the particles intermediate. (See Section 2.13.)
   - 4.5. Aspects related to quarks (and possibly other somewhat similar particles) and gluons (and possibly other somewhat similar particles) bound into composite particles.
5. COMPAR LADDER solutions correlate with …
   - 5.1. Aspects related to quarks (and possibly other somewhat similar particles) and gluons (and possibly other somewhat similar particles) bound into composite particles.

~ ~ ~

[Physics:] This subsection discusses our using different terminology for some aspects this monograph treats than terminology some people use for somewhat similar aspects of traditional physics.

Table 2.1.15 pertains.

**Table 2.1.15** Terminology (MM1MS1-...) that differentiates work in this monograph from some traditional concepts

1. G-family solutions that we correlate with photonics may not exactly correlate with some traditional uses for models for photons. (See Table 2.8.3.)
    - 1.1. The following differences may pertain.
        - 1.1.1. Polarization.
            - 1.1.1.1. We emphasize circular-polarization modes.
                - 1.1.1.1.1. In effect, each polarization (that is, left circular polarization or right circular polarization) correlates with an anti-mode (in the sense of antiparticle) to the other mode.
            - 1.1.1.2. Much of traditional physics seems to emphasize linear polarization modes.
                - 1.1.1.2.1. Linear polarization modes do not function as each other's anti-modes.
        - 1.1.2. Electromagnetism and magnetic dipole moments that are not generated by motions of charges.
            - 1.1.2.1. We treat effects of magnetic dipole moments that correlate with (possibly stationary) elementary particles as correlating with a boson other than the boson we correlate with electromagnetism pertaining to stationary and moving charges.
            - 1.1.2.2. Traditional physics may consider that photons interact with each of ...
                - 1.1.2.2.1. Stationary and moving charges.
                - 1.1.2.2.2. Magnetic dipole moments not generated by the motion of charges.
2. This monograph uses the term MM1MS1-photon to denote some concepts we propose and use.
3. People might say that the following statements characterize some possible differences between MM1MS1-photons and photons (or, QMUSPR-photons).
    - 3.1. MM1MS1-photons may not interact with elementary-particle magnetic dipole moments.
        - 3.1.1. In this monograph, ...

            3.1.1.1. MM1MS1-photons interact with charges and charge-based currents.
            3.1.1.2. A different spin-1 boson interacts with elementary-particle magnetic dipole moments.
    3.2. In traditional physics models, photons interact with charges, charged-based currents, and elementary-particle magnetic dipole moments.
4. This monograph …
    4.1. Discusses a MM1MS1 boson that interacts with elementary-particle magnetic dipole moments.
    4.2. Notes the possibility for correlating this monograph's models for a (conceptual) combination of this MM1MS1 boson and MM1MS1-photons with traditional models for photons.
5. We are less thorough regarding QMUSPR-centric and MM1MS1-centric labelling regarding other elementary particles that correlate with other G-family solutions. (Regarding the G-family, see Table 2.3.4, Section 3.2, and Section 3.3.)
    5.1. For example, traditional physics does not yet encompass gravitons.
        5.1.1. People might say that detection of gravitational waves does not necessarily correlate with detection of individual gravitons.
            5.1.1.1. People might say that this notion has parallels regarding light. People discussed observations correlating with light waves before people discussed detection of photons.
    5.2. Given the lack of a graviton in traditional physics models, …
        5.2.1. We do not try to distinguish between the terms graviton and MM1MS1-graviton.
        5.2.2. In this monograph, we do not further use the term MM1MS1-graviton.
6. We are not certain as to the extent to which MM1MS1-neutrinos differ from neutrinos (or, QMUSPR-neutrinos).
    6.1. MM1MS1-neutrinos have zero mass.
    6.2. People say that the existence of neutrino oscillations (or, neutrino flavor mixing) implies that at least one neutrino has non-zero mass.
        6.2.1. People attribute neutrino oscillations to interactions between neutrinos and gravity.
    6.3. Our models correlate with the notions that …
        6.3.1. MM1MS1-neutrinos can change flavor based on interactions other than interactions with gravity.
        6.3.2. MM1MS1-neutrinos interact with gravity.
        6.3.3. Interactions between MM1MS1-neutrinos and gravity may not lead to neutrino-oscillations.

7. We use MM1MS1 notation for MM1MS1-neutrinos.
8. Generally, various not well-defined terms (such as dark energy) exist.
   8.1. Generally, we do not try to use MM1MS1 notation to distinguish between our uses of such terms and traditional uses of such terms.

# Section 2.2 LADDER models for isotropic quantum harmonic oscillators

Section 2.2 discusses LADDER models for isotropic pairs of isotropic quantum harmonic oscillators. We discuss aspects regarding solutions to equations. We discuss discreet-math models pertaining to 1-dimensional harmonic oscillators. We discuss discreet-math models pertaining to multi-dimensional isotropic quantum harmonic oscillators. We define a constraint, Œ = 0, that each MM1MS1 LADDER model features. We define, in the context of LADDER models, the term solution. We show notation for labeling columns in tables that show aspects of LADDER models. People might say that we show a ground-state solution that traditional physics models may have underutilized.

~ ~ ~

This subsection provides perspective about mathematics that features solutions to equations in general and about mathematics that pertains to isotropic pairs of isotropic quantum harmonic oscillators specifically.

Some branches of mathematics feature solving equations. Some equations have characteristics that people might characterize as isolation and constraints. For example, consider the algebraic equation y = ax + b. Here, people might consider a and b to be constants and x to be an independent variable. Here, people might consider y to be a dependent variable. Or, people might consider that y exists (as an independent variable) in isolation from ax + b and that the equation provides a constraint linking y and x.

People might state that such isolation characterized accounting until around the beginning of the fourteenth century. In effect, people could add numbers and characterize assets. In effect, people could add numbers and characterize liabilities. People might not need to link the two calculations. Starting around the year 1300, people developed and used the practice of double-entry bookkeeping. In double-entry bookkeeping, 0 = assets − liabilities.

Today, much work regarding physics applications of math that pertains to quantum harmonic oscillators exhibits parallels to single-entry practices (or, isolated solutions). People specify some constants and (for one-dimensional quantum harmonic oscillators) a coordinate. (For multi-dimensional quantum harmonic oscillators, there is more than one coordinate.) People solve an equation. For a one-dimensional quantum harmonic oscillator or for an isotropic multi-dimensional quantum harmonic oscillator, a solution features a ground-state energy and an incremental energy that characterizes the differences in energy between any two nearest (but unequal) energy levels.

Work in this section features math for quantum harmonic oscillators. Some work in this section shows isolated solutions. This section makes a transition. Much of the section features an approach that parallels double-entry bookkeeping. People might say that, here, (regarding an equation we show above) each one of y and ax + b correlates with a quantum harmonic oscillator. Here, an analog of a double-entry balance that nets to zero proves useful. People might say that, by analogy, each of y and x is somewhat an independent variable and that $0 = y - (ax + b)$ provides a constraint.

We make use of such a double-entry-like approach to define the math from which we develop physics models.

[Physics:] People might say that the double-entry-like net balance of zero points to how to avoid some possible problems that people state regarding some traditional physics models. One such problem is the seemingly infinite energy that people correlate with a sum of photon ground-state energies. Another such problem correlates with the possibility that the universe includes a seemingly infinite amount of energy.

~ ~ ~

This subsection shows notation for LADDER models for lone quantum harmonic oscillators.

Table 2.2.1 shows well-known information about 1-dimensional harmonic oscillators. Below, this monograph finds uses for negative values of N.

**Table 2.2.1** Notation and math pertaining to a lone quantum harmonic oscillator (LADDER models)

1. People specify the state of a 1-dimensional harmonic oscillator via a linear combination of base states.
2. Each base state features an integer that characterizes that base state.
    2.1. We use the notation $| N >$ for a base state.
        2.1.1. N denotes the integer.
3. Raising operators correlate with increasing the value of N.
    3.1. The symbol $a^+$ denotes a raising operator.
    3.2. The next equation pertains.

$$a^+ \,|\, N > = (1 + N)^{1/2} \,|\, N + 1 > \quad (2.3)$$

4. Lowering operators correlate with decreasing the value of N.
    4.1. The symbol $a^-$ denotes a lowering operator.

4.2. The next equation pertains.

$$a^- \mid N > = N^{1/2} \mid N - 1 > \quad (2.4)$$

5. Traditional treatments of lone harmonic oscillators limit the range of N to nonnegative integers.
   5.1. People might say that the following expressions disconnect possible applicability of negative N from applicability of nonnegative N.

$$a^- \mid 0 > = 0 \mid -1 > \quad (2.5)$$

$$a^+ \mid -1 > = 0 \mid 0 > \quad (2.6)$$

6. People may use the terminology ground state to denote the base state | 0 >.
7. [Physics:] People may associate an energy with each base state.
   7.1. The energy is proportional to the next result.

$$a^+ a^- + 1/2 = N + 1/2 \quad (2.7)$$

8. [Physics:] In physics, the state of a quantum harmonic oscillator can be described as a linear combination of base states …
   8.1. $\Sigma_N b_N \mid N >$, with …
      8.1.1. Each $b_N$ being a complex number.
      8.1.2. $\Sigma_N |b_N|^2 = 1$.

One concern that people might raise regarding negative values of N is a choice of sign for the $(1 + N)^{1/2}$ and $N^{1/2}$ factors associated, respectively, with raising operators and lowering operators. However, a similar choice of sign can pertain for these factors when N is a nonnegative integer. We think that choice of signs is not an issue for work in this monograph.

~ ~ ~

This subsection shows notation for LADDER representations for multi-dimensional isotropic quantum harmonic oscillators.

Table 2.2.2 shows information about multi-dimensional isotropic quantum harmonic oscillators. Below, this monograph finds uses for isotropic pairs of multi-dimensional isotropic quantum harmonic oscillators.

**Table 2.2.2** Notation and math pertaining to LADDER representations for a multi-dimensional isotropic quantum harmonic oscillator

1. For a (#D + 1)-dimensional isotropic quantum harmonic oscillator, …
   1.1. #D denotes a nonnegative integer.
   1.2. The set { Pj | j = 0 , 1 , … , #D − 1 , or #D } provides an index to the lone quantum harmonic oscillators that make up the (#D + 1)-dimensional isotropic quantum harmonic oscillator.

1.3. Numbers N(P0), N(P1), ... correlate with base states of the respective lone harmonic oscillators.
   1.3.1. The lone harmonic oscillators correlate with the (#D + 1)-dimensional isotropic quantum harmonic oscillator.
   1.3.2. Each number N(..) provides part of a description of a base state for the (#D + 1)-dimensional isotropic quantum harmonic oscillator.
2. People specify the state of a (#D + 1)-dimensional isotropic quantum harmonic oscillator via a linear combination of base states for the (#D + 1)-dimensional harmonic oscillator.
3. Traditional treatments of isotropic quantum harmonic oscillators limit the range of each N(..) to nonnegative integers.
4. Below, this monograph uses negative values of N(..).
5. [Physics:] People may associate an energy with each base state of the (#D + 1)-dimensional isotropic quantum harmonic oscillator.
   5.1. The energy is proportional to the sum over the Pj of terms of the following form.

$$(a_{Pj})^{+} (a_{Pj})^{-} + 1/2 = N(Pj) + 1/2 \qquad (2.8)$$

   5.2. The term isotropic correlates with the sum's giving equal weight to each of the N(Pj) + 1/2.

~ ~ ~

This subsection shows notation relevant to LADDER representations for isotropic pairs of isotropic quantum harmonic oscillators.

Table 2.2.3 describes an isotropic pair of isotropic quantum harmonic oscillators. The table shows a relationship between the two paired isotropic quantum harmonic oscillators. Each of the symbols #EMAX and #PMAX comports with Table 2.1.3. #EMAX can denote 'Ø or a nonnegative integer. #PMAX always denotes a nonnegative integer. #EMAX correlates with the term QE-like. #PMAX correlates with the term QP-like. ([Physics:] Think of energy-momentum space. QE-like matches energy-like. The E in QE correlates with use of the symbol E to denote energy. QP-like matches momentum-like. The P in QP correlates with use of the symbol P to denote momentum.)

**Table 2.2.3** Notation and relationships pertaining to an isotropic pair of isotropic quantum harmonic oscillators (LADDER models)

1. Each of #EMAX and #PMAX comports with Table 2.1.3.
   1.1. If #EMAX is an even integer, #PMAX is an even integer.
   1.2. If #EMAX is 'Ø, #PMAX is an even integer.
   1.3. If #EMAX is an odd integer, #PMAX is an odd integer.
2. We define two sets. Each set consists of indices associated with a sequence of consecutive nonnegative integers.

$$\text{SIDE} = \{ \text{E}j \mid j = \#\text{EMAX}, \#\text{EMAX} - 1, \ldots, 1, \text{or } 0 \} \quad (2.9)$$
$$\text{SIDP} = \{ \text{P}j \mid j = 0, 1, \ldots, \#\text{PMAX} - 1, \text{or } \#\text{PMAX} \} \quad (2.10)$$

3. We provide notation for the union of the two sets. (If #EMAX = '∅, SID = SIDP.)

$$\text{SID} = \text{SIDE} \cup \text{SIDP} \quad (2.11)$$

4. N(n) denotes the quantum number for the lone harmonic oscillator n.
   4.1. Each n has one of forms Ej or Pj.
   4.2. Each N(n) + 1/2 term below matches a result (from traditional quantum harmonic oscillator math) for a 1-dimensional harmonic oscillator.
5. We define two sums. (If #EMAX = '∅, SIDE = ∅ and $Æ_{QE} = 0$.)

$$Æ_{QE} = \Sigma_{n \in \text{SIDE}} (N(n) + 1/2) \quad (2.12)$$
$$Æ_{QP} = \Sigma_{n \in \text{SIDP}} (N(n) + 1/2) \quad (2.13)$$

6. The term isotropic pair of isotropic quantum harmonic oscillators correlates with sums giving equal weight to the magnitude of each of the N(Ej) + 1/2 and to the magnitude of each of the N(Pj) + 1/2.
7. We provide two equivalent versions of a constraint that correlates with the pair of isotropic oscillators being isotropic, with each of the two isotropic oscillators being isotropic, and with conditions this monograph imposes. Here, $\pm_n = +1$, for n ∈ SIDE. Here, $\pm_n = -1$, for n ∈ SIDP.

$$0 = Œ = Æ_{QE} - Æ_{QP} \quad (2.14)$$
$$0 = Œ = \Sigma_{n \in \text{SID}} \pm_n (N(n) + 1/2) \quad (2.15)$$

~ ~ ~

This subsection shows a constraint we use to limit models (based on isotropic pairs of isotropic quantum harmonic oscillators) we consider further.

Table 2.2.4 repeats a key feature from Table 2.2.3. This constraint pertains throughout this monograph.

**Table 2.2.4** A constraint on Œ

$$Œ = 0 \quad (2.16)$$

We show LADDER examples of this constraint. People might say that this monograph deemphasizes explicitly pointing out DIFEQU examples of this constraint.

~ ~ ~

This subsection defines the concepts of open oscillator pair and closed oscillator pair.

Table 2.2.5 explains some notation.

**Table 2.2.5** The notation E[j] and P[j]

1. This monograph uses notations E[j] and P[j] in which j can be an arithmetic expression that evaluates to an integer.
    1.1. For example, we might use the notations P[#PMAX −1] and P[#PMAX].

Table 2.2.6 defines the term local-Œ. In the rightmost column, each +1 comes from two instances of +1/2. Each instance of +1/2 correlates with one of the two oscillators in the pair. Here, the term even denotes an even positive integer.

**Table 2.2.6** Definition of local-Œ for a base state correlating with an oscillator pair

| For an oscillator pair … | … for a base state defined by … | … local-Œ equals … |
|---|---|---|
| E[even]-and-E[even − 1] | N(E[even]) and N(E[even − 1]) | N(E[even]) + N(E[even − 1]) + 1. |
| P[even − 1]-and-P[even] | N(P[even − 1]) and N(P[even]) | N(P[even − 1]) + N(P[even]) + 1. |

Table 2.2.7 shows examples in which local-Œ = 0. Here, each of j and k is a complex number.

**Table 2.2.7** Examples in which local-Œ = 0

1. This item exhibits, for an oscillator pair E[even]-and-E[even − 1], amplitudes such that local-Œ = 0.
    1.1. Amplitude …
    j × | N(E[even]) = 0 and N(E[even − 1]) = −1 >
    +
    k × | N(E[even]) = −1 and N(E[even − 1]) = 0 >
    1.2. Such that …
    $|j|^2 + |k|^2 = 1.$
2. This item exhibits, for an oscillator pair P[even − 1]-and-P[even], amplitudes such that local-Œ = 0.
    2.1. Amplitude …
    j × | N(P[even − 1]) = 0 and N(P[even]) = −1 >
    +
    k × | N(P[even − 1]) = −1 and N(P[even]) = 0 >
    2.2. Such that …
    $|j|^2 + |k|^2 = 1.$

Table 2.2.8 exhibits two ways for an oscillator pair to have local-Œ = 0. (Table 2.2.7 discusses j and k.)

**Table 2.2.8** Definitions of open oscillator pair and closed oscillator pair

| For an oscillator pair … | … for state defined (with $|j|^2 + |k|^2 = 1$) by … | … the following term pertains to the oscillator pair |
|---|---|---|
| E[even]-and-E[even − 1] | j = 0 or k = 0 | open |
| E[even]-and-E[even − 1] | j and k are indeterminate | closed |
| P[even − 1]-and-P[even] | j = 0 or k = 0 | open |
| P[even − 1]-and-P[even] | j and k are indeterminate | closed |

Regarding a table such as Table 2.3.11, people can consider to be a closed oscillator pair any E[even]-and-E[even − 1] oscillator pair for which the table shows two blank entries or any P[even − 1]-and-P[even] oscillator pair for which the table shows two blank entries. For example, for 2W in Table 2.3.11, people can consider that the P4L-and-P4R oscillator pair is closed. Also, people can consider to be closed each of the (infinite number of) E[even]-and-E[even − 1] oscillator pairs for which the table does not show columns and each of the (infinite number of) P[even − 1]-and-P[even] oscillator pairs for which the table does not show columns.

~ ~ ~

This subsection defines, for LADDER models, the term solution and discusses possible ways to order lists of solutions for isotropic pairs of isotropic quantum harmonic oscillators.

Table 2.2.9 pertains. This monograph also makes other uses of the word solution. (See, for example, Table 2.5.6.) We think that people can differentiate appropriately among various uses of the term solution.

**Table 2.2.9** Defining the term solution (regarding LADDER model uses for which Œ = 0) and introducing a means for ordering solutions

1. We define a solution by …
   1.1. #EMAX or a similar QE-like limit.
   1.2. #PMAX or a similar QP-like limit.
   1.3. A set of N(n) that satisfies the following constraint.
   $$0 = Œ = Æ_{QE} - Æ_{QP} \tag{2.17}$$
      1.3.1. People might associate the term solution (as in solution to an algebraic equation) with the satisfying of this equality.
2. The number of solutions is infinite.
   2.1. For example, there is an infinity of choices for #EMAX.
3. [Physics:] For this monograph, …
   3.1. The following constraints pertain.
   $$\#EMAX \leq 16 \text{ or } \#EMAX = {}'\emptyset \tag{2.18}$$
   $$\#PMAX \leq 16 \tag{2.19}$$

 3.2. The number of relevant pairs of #EMAX and #PMAX is finite.
4. For a choice of #EMAX and #PMAX, the number of solutions is infinite.
5. [Physics:] Generally (but not always), we exhibit, for isotropic pairs of isotropic quantum harmonic oscillators, solutions that could correlate with ground states that we think are relevant to the physics we address.
 5.1. For a choice of #EMAX (or other QE-like limit) and #PMAX (or other QP-like limit), the number of ground-state solutions this monograph considers is finite.
6. When displaying (in a table) ground-state solutions, this monograph may use (but does not always use) an ordering based on …
 6.1. First, increasing value of #PMAX.
 6.2. Then, other considerations.

~ ~ ~

This subsection introduces notation for displaying LADDER solutions.

Table 2.2.10 shows how some tables use vertical nomenclature to describe names of the lone oscillators that are parts of isotropic pairs of isotropic quantum harmonic oscillators. In Table 2.2.10, the first row shows oscillator names. The next two rows show how we use two rows to show (in tables below) oscillator names. To form a name, append the number from the last row in Table 2.2.10 to the letter from the next-to-last row of Table 2.2.10. We call these names linear-numbering names. Table 2.2.11 extends this work to include an alternative set of names (that is, polarization-centric names) for oscillators.

**Table 2.2.10** Correlations between some linear-numbering names for lone oscillators and the display in tables of column headings correlating with those names

| E6 | E5 | E4 | E3 | E2 | E1 | E0 | P0 | P1 | P2 | P3 | P4 | P5 | P6 | P7 | P8 | Oscillator names |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| E | E | E | E | E | E | E | P | P | P | P | P | P | P | P | P | Linear-numbering |
| 6 | 5 | 4 | 3 | 2 | 1 | 0 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | names |

Table 2.2.11 shows alternative names for oscillators. (Compare with Table 2.2.10. [Physics:] This monograph sometimes uses polarization-centric names.) For polarization-centric names, form the n in the N(n) correlating with a column by appending the last row in the column to the next-to-last row in the column. L denotes left (as in left-circularly polarized). R denotes right (as in right-circularly polarized). For example, the name P8L correlates with the oscillator that this monograph also names P7.

**Table 2.2.11** Polarization-centric names for oscillators

| E6 | E5 | E4 | E3 | E2 | E1 | E0 | P0 | P1 | P2 | P3 | P4 | P5 | P6 | P7 | P8 | Oscillator names |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| E | E | E | E | E | E | E | P | P | P | P | P | P | P | P | P | Polarization-centric |
| 6R | 6L | 4R | 4L | 2R | 2L | 0 | 0 | 2L | 2R | 4L | 4R | 6L | 6R | 8L | 8R | names |

Table 2.2.12 shows how this monograph extends linear numbering beyond E9 and beyond P9. Here, A denotes 10, B denotes 11, …, and G denotes 16. Here, except for columns for E0 and P0, the table shows two oscillators per column. For example, the P9A column correlates with oscillators P9 and PA. This monograph uses notations E[j] and P[j] in which j can be an arithmetic expression that evaluates to one of 0, 1, …, or G. For example, P[10] = PA. (See Table 2.2.5.)

**Table 2.2.12** Extended linear numbering for oscillators and for oscillator pairs

| | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| E | E | E | E | E | E | E | E | E | P | P | P | P | P | P | P | P | P |
| GF | ED | CB | A9 | 87 | 65 | 43 | 21 | 0 | 0 | 12 | 34 | 56 | 78 | 9A | BC | DE | FG |

~ ~ ~

This subsection shows a well-known solution and another ([Physics:] non-traditional) solution.

Table 2.2.13 shows a well-known solution.

**Table 2.2.13** The traditional-physics ground state for 3-dimensional QP-like isotropic quantum harmonic oscillators

1. The following equations characterize the ground state for this solution.

$$\#'E = 0 \tag{2.20}$$

$$\#'P = 2 \tag{2.21}$$

$$N(E0) = 1 \tag{2.22}$$

$$N(P0) = N(P1) = N(P2) = 0 \tag{2.23}$$

2. Œ = 0 results from …
   2.1. A contribution of +3/2 correlating with N(E0) = 1.
   2.2. A contribution of −3/2 correlating with N(P0) = N(P1) = N(P2) = 0.
3. [Physics:] For a traditional ground state of an isotropic quantum harmonic oscillator with 3 spatial (or, QP-like) dimensions, people would correlate the energy with N(E0) + 1/2 = 3/2.

Table 2.2.14 depicts the solution Table 2.2.13 describes.

**Table 2.2.14** Depiction of the ([Physics:] traditional) ground state for 3-dimensional QP-like isotropic quantum harmonic oscillators

| E0 | P0 | P1 | P2 |
|---|---|---|---|
| 1 | 0 | 0 | 0 |

Table 2.2.15 shows a solution for which N(E0) + 1/2 = 0 + 1/2 = 1/2. ([Physics:] People might say that the energy correlates with N(E0) + 1/2 = 1/2. People might dispute this solution. Work below shows another solution for which N(E0) = 0. (See Table 2.5.13.) There, the work uses radial coordinates. People might not dispute that solution. People might apply the term non-traditional to each of these two solutions for which N(E0) = 0.) For the state Table 2.2.15 shows, applying a raising operator to the P0 oscillator produces a state with 0 amplitude. Specifically, $a_{P0}^{+}$ | N(P0) = −1 > = (1 + N(P0))$^{(1/2)}$ | N(P0) = 0 > = 0 | N(P0) = 0 >, because 1 + N(P0) = 0. ([Physics:] Below, we correlate this solution with MM1MS1-photons. The P0 oscillator correlates with longitudinal polarization. This application is consistent with the concept that each of a photon and a MM1MS1-photon has zero longitudinal polarization.)

**Table 2.2.15** A ([Physics:] non-traditional) ground state for 3-dimensional QP-like isotropic quantum harmonic oscillators

| E0 | P0 | P1 | P2 |
|---|---|---|---|
| 0 | −1 | 0 | 0 |

# Section 2.3 INTERN applications of LADDER models

Section 2.3 discusses relevant solutions within models for isotropic pairs of isotropic quantum harmonic oscillators. We feature subsets of the solutions. We focus on INTERN LADDER subsets for which the solutions might correlate with data about elementary particles. We provide names for families of solutions. We characterize solutions pertaining to various families. People might say that we show or point to solutions that correlate with all known and some possible elementary particles. People might say that those solutions correlate with properties that include at least spin and whether rest mass is zero or non-zero.

~ ~ ~

This subsection introduces relevant solutions.

Table 2.3.1 and Table 2.3.3 show some ground-state solutions. For these tables, #'E = #EMAX and #'P = PMAX. For Table 2.3.1, a parameter σ equals +1. For Table 2.3.3, the parameter σ equals −1. [Physics:] Later, we indicate that the subfamilies these tables show correlate with all the elementary particles to which MM1MS1 models point as possibly existing. Later, we correlate σ = +1 with the term free-ranging. Later, we correlate σ = −1 with the term non-free-ranging. Later, we correlate elements of the subfamily names with more-familiar terminology people use regarding elementary particles. Later, we correlate spin/ħ with a parameter S and we indicate that, for subfamilies for which the letter G does not appear in the subfamily name, 2S = #'P.

Table 2.3.1 shows some ground-state solutions for which σ = +1. [Physics:] Here, we show mathematically possible 3C- and 4W-subfamilies. The table shows two 3C rows. We think that, for 3C, the table illustrates a condition we call inappropriate redundancy. (See Table 2.15.8.) We think that, for 3C, a condition we call lack of an appropriate D pertains. (See Table 2.15.8.) Based on either (or both) of these two conditions, we deemphasize 3C solutions. We think that, for 4W, the condition lack of appropriate D pertains. We think that, for 4W, a condition we call inappropriate square of rest mass pertains. (See Table 2.15.8.) Based on at least the first of these two conditions, we deemphasize 4W solutions. Later, we discuss constraints regarding excitations for subfamilies for which N(P0) ≤ −2. Later, we correlate inappropriate redundancy with appropriateness for deemphasizing G-family solutions for which #'P > 8. (See Table 2.15.8. Regarding the term G-family, see Table 2.3.4.)

**Table 2.3.1** CORMAT σ = +1 ground-state solutions and subfamily names, based on #'E and #'P limits on harmonic oscillators

| σ | #'E | E6 | E5 | E4 | E3 | E2 | E1 | E0 | P0 | P1 | P2 | P3 | P4 | P5 | P6 | P7 | P8 | #'P | Solution subfamily |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| +1 | 0 | | | | | | | 0 | 0 | | | | | | | | | 0 | 0H |
| +1 | 'Ø | | | | | | | | 0 | −1 | | | | | | | | 1 | 1C |
| +1 | 'Ø | | | | | | | | −1 | 0 | | | | | | | | 1 | 1N |
| +1 | 2 | | | | | 0 | 0 | 0 | 0 | 0 | 0 | | | | | | | 2 | 2W |
| +1 | 0 | | | | | | | 0 | −1 | 0 | 0 | | | | | | | 2 | 022G |
| ~~+1~~ | ~~'Ø~~ | | | | | | | | ~~0~~ | ~~−1~~ | ~~0~~ | ~~−1~~ | | | | | | ~~3~~ | ~~3C~~ |
| ~~+1~~ | ~~'Ø~~ | | | | | | | | ~~0~~ | ~~0~~ | ~~−1~~ | ~~−1~~ | | | | | | ~~3~~ | ~~3C~~ |
| +1 | 'Ø | | | | | | | | −1 | 0 | 0 | −1 | | | | | | 3 | 3N |
| ~~+1~~ | 4 | | | ~~0~~ | ~~0~~ | ~~0~~ | ~~0~~ | ~~0~~ | ~~0~~ | ~~0~~ | ~~0~~ | ~~0~~ | ~~0~~ | | | | | ~~4~~ | ~~4W~~ |
| +1 | 2 | | | | | 0 | 0 | 0 | −1 | 0 | 0 | 0 | 0 | | | | | 4 | 24..G |
| +1 | 0 | | | | | | | 0 | −2 | 0 | 0 | 0 | 0 | | | | | 4 | 042G |
| +1 | 4 | | | 0 | 0 | 0 | 0 | 0 | −1 | 0 | 0 | 0 | 0 | 0 | 0 | | | 6 | 46..G |
| +1 | 2 | | | | | 0 | 0 | 0 | −2 | 0 | 0 | 0 | 0 | 0 | 0 | | | 6 | 26..G |
| +1 | 0 | | | | | | | 0 | −3 | 0 | 0 | 0 | 0 | 0 | 0 | | | 6 | 064G |
| +1 | 6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | −1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 8 | 68..G |
| +1 | 4 | | | 0 | 0 | 0 | 0 | 0 | −2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 8 | 48..G |
| +1 | 2 | | | | | 0 | 0 | 0 | −3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 8 | 28..G |

| σ | #'E | E 6 | E 5 | E 4 | E 3 | E 2 | E 1 | E 0 | P 0 | P 1 | P 2 | P 3 | P 4 | P 5 | P 6 | P 7 | P 8 | #'P | Solution subfamily |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| +1 | 0 | | | | | | | 0 | −4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 8 | 084G |

Table 2.3.2 defines the term lepton-related solution.

**Table 2.3.2** Definition of lepton-related solution

1. [Physics:] The term lepton pertains to an elementary particle if and only if …
   - 1.1. The elementary particle correlates with a solution Table 2.3.1 shows.
   - 1.2. #'E = 'Ø for that solution.
   - 1.3. Nature exhibits the elementary particle.
     - 1.3.1. Known leptons include …
       - 1.3.1.1. Neutrinos, which correlate with the 1N-subfamily.
       - 1.3.1.2. Electrons, muons, and tauons, which correlate with the 1C-subfamily.
     - 1.3.2. As yet, people have not found or ruled out elementary particles correlating with the 3N-subfamily.

Table 2.3.3 shows some ground-state solutions for which σ = −1. Mathematically, the 0Y solution cannot be excited. (See Table 2.15.8.) We deemphasize the 0Y-subfamily. [Physics:] For each of the cases 1Q, 1R, 3Q, 3I, 3R, and 3D, it matters that N(E[#'E]) = N(P[#'P]), but it does not matter which one of N(E[#'E]) = N(P[#'P]) = 0 or N(E[#'E]) = N(P[#'P]) = −1 we show in this table. (See Table 2.3.14.) Later, we discuss specific properties and constraints regarding values of N(..) for the QE-like oscillators for the 2Y- and 4Y-subfamilies. (See Section 3.6. See, especially, Table 3.6.1 and Table 3.6.8.)

**Table 2.3.3** CORMAT σ = −1 ground-state solutions and subfamily names, based on #'E and #'P limits on harmonic oscillators

| σ | #'E | E 6 | E 5 | E 4 | E 3 | E 2 | E 1 | E 0 | P 0 | P 1 | P 2 | P 3 | P 4 | P 5 | P 6 | P 7 | P 8 | #'P | Solution subfamily |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| −1 | 0 | | | | | | | 0 | 0 | | | | | | | | | 0 | 0O |
| ~~−1~~ | ~~0~~ | | | | | | | ~~−1~~ | ~~−1~~ | | | | | | | | | ~~0~~ | ~~0Y~~ |
| −1 | 1 | | | | | | 0 | 0 | 0 | 0 | | | | | | | | 1 | 1Q |
| −1 | 1 | | | | | | 0 | −1 | −1 | 0 | | | | | | | | 1 | 1R |
| −1 | 2 | | | | | 0 | 0 | 0 | 0 | 0 | 0 | | | | | | | 2 | 2O |
| −1 | 2 | | | | | .. | .. | .. | −1 | −1 | −1 | | | | | | | 2 | 2Y |
| −1 | 3 | | | | −1 | 0 | 0 | 0 | 0 | 0 | 0 | −1 | | | | | | 3 | 3Q |
| −1 | 3 | | | | −1 | −1 | −1 | 0 | 0 | −1 | −1 | −1 | | | | | | 3 | 3I |
| −1 | 3 | | | | −1 | 0 | 0 | −1 | −1 | 0 | 0 | −1 | | | | | | 3 | 3R |
| −1 | 3 | | | | −1 | −1 | −1 | −1 | −1 | −1 | −1 | −1 | | | | | | 3 | 3D |

| σ | #'E | E 6 | E 5 | E 4 | E 3 | E 2 | E 1 | E 0 | P 0 | P 1 | P 2 | P 3 | P 4 | P 5 | P 6 | P 7 | P 8 | #'P | Solution subfamily |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| −1 | 4 | | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | | | | | 4 | 4O |
| −1 | 4 | | | .. | .. | .. | .. | .. | −1 | −1 | −1 | −1 | −1 | | | | | 4 | 4Y |

Each row in Table 2.3.1 and Table 2.3.3 shows a solution subfamily for which the name includes one letter. Each letter denotes the name of one family of solutions.

~ ~ ~

This subsection lists names for families of solutions that this monograph considers further.

Table 2.3.4 discusses names for families of solutions. Technically, it could be permissible to consider the H-family to be a subset of the W-family. ([Physics:] This monograph lists the H- and W-families separately.) People use the term gamma rays to describe some photons. [Physics:] This monograph correlates some solutions from each family, except the I-, R-, D-, and O-families, with known elementary particles. Possibly, the I-family correlates with particles that interact directly with gravity and do not interact directly with electromagnetism. (We use the word directly so as to allow for the possibility of indirect interactions via clouds of virtual particles that would accompany the elementary particles. See Table 2.10.1.) We choose the letter I to correlate with the word invisible. We choose the letter R to correlate with the letter alphabetically next after the letter Q. Possibly, the D-family correlates with particles that do not interact directly with either gravity or electromagnetism. We choose the letter D to correlate with the word dark. We do not much discuss the extent to which O-family particles correlate with people's concepts for leptoquarks.

**Table 2.3.4** Names for families of solutions

| σ | Name | [Physics:] Elementary-particle motivation for the name |
|---|---|---|
| +1 | H-family | Higgs boson. |
| +1 | C-family | Charged leptons. |
| +1 | N-family | Neutrinos. |
| +1 | W-family | Weak-interaction bosons. |
| +1 | G-family | Gamma ray and graviton (a hypothetic type of particle). |
| −1 | Q-family | Quark. |
| −1 | I-family | (No known-particle motivation.) |
| −1 | R-family | (No known-particle motivation.) |
| −1 | D-family | (No known-particle motivation.) |
| −1 | Y-family | The shape of the letter suggests a type of gluon vertex. |
| −1 | O-family | O, as in leptoquark (a hypothetical type of particle). |

~ ~ ~

This subsection introduces notation useful for situations in which families or subfamilies exhibit similarities.

Table 2.3.5 pertains.

**Table 2.3.5** Notation regarding similar σ = +1 families and subfamilies and regarding similar σ = −1 families and subfamilies

1. Regarding some solutions for which σ = +1, …
   1.1. CN denotes either of the C- or N-families.
   1.2. WHO denotes any one of the W-, H-, or O-families.
   1.3. WO denotes either of the W- or O-families.
   1.4. 2WO denotes either of the 2W- or 2O-subfamilies.
   1.5. 4WO denotes either of the 4W- or 4O-subfamilies.
2. Regarding some solutions for which σ = −1, …
   2.1. QIRD denotes any one of the Q-, I-, R-, or D- families.
   2.2. QR denotes either of the Q- or R-families.
   2.3. ID denotes either of the I- or D-families.
   2.4. YO denotes either of the Y- or O-families.
   2.5. 1QR denotes either of the 1Q- or 1R-subfamilies.
   2.6. 3QIRD denotes any one of the 3Q-, 3I-, 3R-, or 3D-subfamilies.
   2.7. 2YO denotes either of the 2Y- or 2O-subfamilies.
   2.8. 4YO denotes either of the 4Y- or 4O-subfamilies.
3. Sometimes, we use the above notations to mean …
   3.1. Some, instead of any one.
   3.2. All, instead of any one.
   3.3. Both, instead of either of.

~ ~ ~

This subsection discusses two uses for family names, for subfamily names, and for names of solutions.

Table 2.3.6 pertains.

**Table 2.3.6** Dual use of family names, subfamily names, and solution names

1. Sometimes, we use family names (such as Q-family) or subfamily names (such as 1Q) to designate sets of solutions.
2. [Physics:] Sometimes, we use (the same) family names or (the same) subfamily names to designate sets of known or possible elementary particles that this monograph correlates with a (respective) set of solutions.
   2.1. Here, the term elementary particle includes the notion of field (associated with the elementary particle).

3. Dual use pertains also for some solution names.
    3.1. [Physics:] However, …
        3.1.1. For Y-family solutions, …
            3.1.1.1. Pairs of 2Y solutions correlate with components of gluons.
            3.1.1.2. Pairs of 4Y solutions correlate with components of possible other particles.
        3.1.2. For G-family solutions and particles, …
            3.1.2.1. For all cases, each solution correlates with two polarization modes.
            3.1.2.2. For some cases, direct correlations between single solutions and single elementary particles do not pertain.
                3.1.2.2.1. For example, direct correlation does not pertain for some cases that feature the SOMMUL-set models. (See Table 2.15.8 and Table 3.2.1.)

~ ~ ~

This subsection discusses names for subfamilies of solutions that this monograph considers further.

For each of the N-, Q-, and R-families, solutions exist for which 2S = 1 and solutions exist for which 2S = 3. This monograph uses the term subfamily to call attention to this concept. For the N-family, the subfamilies are 1N and 3N, respectively. For the Q-family, the subfamilies are 1Q and 3Q, respectively. For the R-family, the subfamilies are 1R and 3R, respectively. In each of these cases, for notation of the form nX, with X denoting a family, the equation n = 2S pertains.

For each of the I- and D-families, solutions exist for which 2S = 3. Here, for notation of the form nX, with X denoting a family, the equation n = 2S = 3 pertains.

For the Y-family, solutions exist for which 2S = 2 and solutions exist for which 2S = 4. For the Y-family, the subfamilies are 2Y and 4Y respectively. ([Physics:] A 0Y solution exists mathematically, but cannot be excited. Therefore, this work deemphasizes the 0Y solution. See Table 2.3.3.)

For the O-family, solutions exist for which 2S = 0, solutions exist for which 2S = 2, and solutions exist for which 2S = 4. For the O-family, the subfamilies are 0O, 2O, and 4O, respectively.

For the G-family, symbols of the form epnG denote subfamilies. Here, e = #'E, p = #'P, and n = 2S. Here, ep..G denotes a subfamily for which more than one solution may pertain and more than one S may pertain. An ep..G-subfamily may include more than one epnG-subfamily.

The C family has one subfamily, 1C. Here, 2S = 1. The W-family has one subfamily, 2W. Here, 2S = 2. The H-family has one solution and, thereby, one subfamily, 0H. Here, 2S = 0.

~ ~ ~

This subsection discusses some aspects of and distinctions between the MM1 meta-model and MM1MS1 models.

Table 2.3.7 pertains. (For more details, see Section 2.15.)

**Table 2.3.7** Some aspects of and distinctions between the MM1 meta-metal and MM1MS1 models

1. The MM1 meta-model ...
    1.1. Embraces as correlating with candidate elementary particles …
        1.1.1. Each of the families (of solutions) to which Table 2.3.4 alludes.
        1.1.2. Each of the subfamilies (of solutions) to which Table 2.3.1 alludes, except …
            1.1.2.1. Those subfamilies the table shows as struck out.
        1.1.3. Each of the subfamilies (of solutions) to which Table 2.3.3 alludes, except …
            1.1.3.1. Those subfamilies the table shows as struck out.
    1.2. Does not embrace as correlating with candidate elementary particles …
        1.2.1. Any other constructs.
2. Each MM1MS1 model comports with the previous item in this table.
3. MM1MS1 models may vary based, for example, on …
    3.1. A subset (of the overall set of solutions that correlate with candidate particles) to consider as correlating with possible particles.
    3.2. The number of fields a model correlates with a solution.
        3.2.1. [Physics:] This difference can lead to possibilities (that can differ by model) regarding …
            3.2.1.1. The nature of dark-energy stuff. (See, for example, Section 4.3.)
            3.2.1.2. The nature of dark matter. (See, for example, Section 4.3.)
    3.3. [Physics:] Modeling and interpretation of models regarding numbers of generations for spin-3/2 elementary fermions. (See Table 2.6.4.)
    3.4. Other considerations. (See Section 2.15.)

~ ~ ~

This subsection discusses #E and #P, which are some limits that we place on oscillators for INTERN LADDER solutions.

Table 2.3.8 discusses reasons why this monograph features solutions for which there are an odd number of QE-like oscillators and an odd number of QP-like oscillators. CORPHY solutions correlate with solutions with an odd number of QE-like oscillators and an odd number of QP-like oscillators.

**Table 2.3.8** Reasoning leading to featuring INTERN LADDER solutions with an odd number of QE-like oscillators and an odd number of QP-like oscillators, plus definitions of #E and #P (CORPHY solutions)

1. [Physics:] For elementary particles, …
   - 1.1. Possibly the following should pertain.
     - 1.1.1. For boson elementary particles, …
       - 1.1.1.1. For each non-G-family boson elementary particle, 2S = #'P.
       - 1.1.1.2. For each G-family boson elementary particle, 2S = 2 or 4.
     - 1.1.2. For each fermion elementary particle, 2S = #'P.
   - 1.2. For each fermion elementary particle, a solution correlating with the particle's antiparticle would use oscillator #'P + 1 and would not use oscillator #'P.
   - 1.3. We limit our choices of numbers of QP-like oscillators to odd numbers.
     - 1.3.1. We deemphasize the notation #'P.
     - 1.3.2. We use the notation #P, with the understanding that #P is a nonnegative even integer.
       - 1.3.2.1. Because oscillator P0 pertains, the number of QP-like oscillators is odd.
   - 1.4. For boson elementary particles, …
     - 1.4.1. For each non-G-family boson elementary particle, …

$$\#P = \#'P \tag{2.24}$$

$$2S = \#P = \#'P \tag{2.25}$$

     - 1.4.2. For each G-family boson elementary particle, …

$$\#P = \#'P \tag{2.26}$$

$$2S = 2 \text{ or } 4 \tag{2.27}$$

   - 1.5. For each fermion elementary particle, …

$$\#P = \#'P + 1 \tag{2.28}$$

$$2S = \#P - 1 = \#'P \tag{2.29}$$

   - 1.6. To maintain Œ = 0, …
     - 1.6.1. We deemphasize the notation #'E.
     - 1.6.2. We use the notation #E, with the understanding that #E is a nonnegative even integer.
     - 1.6.3. For elementary bosons, …

$$\#E = \#'E \tag{2.30}$$

     - 1.6.4. For elementary fermions, …

$$\#E = 0, \text{ if } \#'E = '\emptyset \tag{2.31}$$

$$\#E = \#'E + 1, \text{ otherwise} \tag{2.32}$$

~ ~ ~

This subsection notes that, regarding LADDER solutions, the remainder of this monograph features CORPHY solutions and deemphasizes CORMAT solutions.

Until Table 2.3.8, regarding LADDER solutions, we feature CORMAT solutions. For the remainder of this monograph, regarding LADDER solutions, we feature CORPHY solutions.

For example, regarding Table 2.3.8, CORPHY solutions correlate with use of #EMAX = #E and #PMAX = #P. CORMAT solutions correlate with use of #EMAX = #'E and #PMAX = #'P.

~ ~ ~

This subsection lists ground-state INTERN LADDER solutions that this monograph considers further.

This subsection transforms results in Table 2.3.1 and in Table 2.3.3 from notation based on #'E and #'P to results that correlate with notation based on #E and #P. This correlates with the transition to, for LADDER solutions, featuring CORPHY solutions and deemphasizing CORMAT solutions.

Table 2.3.9 pertains.

**Table 2.3.9** Algorithm for selecting ground-state INTERN LADDER solutions

1. [Physics:] Select solutions that we think are relevant.
   1.1. For example, limit selections to solutions for which #P ≤ 8.
   1.2. For example, do not include solutions that would correlate with a lack of physics-relevance, per Table 2.15.8.
2. Work above and below defines this algorithm.
   2.1. Table 2.3.11, Table 2.3.13, and Table 2.3.14 show results.

We discuss ground-state LADDER solutions for which $\sigma = +1$ and then ground-state LADDER solutions for which $\sigma = -1$. Table 2.3.10 provides information common to those two sets of ground-state LADDER solutions.

**Table 2.3.10** Information pertaining to CORPHY ground-state LADDER solutions

1. [Physics:] N(P0) correlates with aspects related to rest mass.
   1.1. For INTERN LADDER solutions and for ENVIRO LADDER solutions, …
   $$\begin{array}{l} N(P0) = 0 \text{ correlates with } m' \neq 0 \\ N(P0) < 0 \text{ correlates with } m' = 0 \end{array} \tag{2.33}$$
   1.2. For FERTRA LADDER solutions, …
   $$\begin{array}{l} N(P0) = -1 \text{ correlates with } m' \neq 0 \\ N(P0) = -2 \text{ correlates with } m' = 0 \end{array} \tag{2.34}$$

2. For fermion INTERN LADDER solutions, …
    2.1. For P[j], with j = #P − 1 or #P, …
        2.1.1.1. People might say that N(P[j]) = −1 correlates with shutting down oscillator P[j].
3. The following limits pertain to S.

$$0 \le S \le 2 \tag{2.35}$$

    3.1. [Physics:] The upper limit for elementary particles comes from field theory. (Note Table 2.15.8.)

Table 2.3.11 shows CORPHY ground-state INTERN LADDER solutions for which σ = +1. Table 2.3.4 motivates the names of families. The notation ∼ ∼ denotes that (for that row in the table), for the two relevant oscillators, one of the two respective N(..) = 0 and the other N(..) = −1. There are two ways such can happen. Each way correlates with a solution. [Physics:] For that row, people might correlate one solution with the term particle and the other solution with the term antiparticle.

**Table 2.3.11** CORPHY ground-state INTERN LADDER solutions for which σ = +1

| E 6R | E 6L | E 4R | E 4L | E 2R | E 2L | E 0 | P 0 | P 2L | P 2R | P 4L | P 4R | P 6L | P 6R | P 8L | P 8R | (σ = +1) Subfamily |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | 0 | 0 | | | | | | | | | 0H0 |
| | | | | | | 0 | 0 | ∼ | ∼ | | | | | | | 1C |
| | | | | | | −1 | −1 | ∼ | ∼ | | | | | | | 1N |
| | | | | 0 | 0 | 0 | 0 | 0 | 0 | | | | | | | 2W |
| | | | | | | 0 | −1 | 0 | 0 | | | | | | | 022G2& |
| | | | | | | 0 | −1 | 0 | 0 | ∼ | ∼ | | | | | 3N |
| | | | | 0 | 0 | 0 | −1 | 0 | 0 | 0 | 0 | | | | | 24..G |
| | | | | | | 0 | −2 | 0 | 0 | 0 | 0 | | | | | 042G24& |
| | | 0 | 0 | 0 | 0 | 0 | −1 | 0 | 0 | 0 | 0 | 0 | 0 | | | 46..G |
| | | | | 0 | 0 | 0 | −2 | 0 | 0 | 0 | 0 | 0 | 0 | | | 26..G |
| | | | | | | 0 | −3 | 0 | 0 | 0 | 0 | 0 | 0 | | | 064G246& |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | −1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 68..G |
| | | 0 | 0 | 0 | 0 | 0 | −2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 48..G |
| | | | | 0 | 0 | 0 | −3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 28..G |
| | | | | | | 0 | −4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 084G2468& |

Table 2.3.12 provides perspective regarding subfamily names for subfamilies for which only one ground-state solution exists.

**Table 2.3.12** Perspective regarding subfamily names for subfamilies for which only one ground-state INTERN LADDER solution exists

1. For each of the following, the subfamily consists of one solution and Table 2.3.11 shows the name for the solution.
   1.1. 0H0.
      1.1.1. Here, the symbol 0H denotes the subfamily.
   1.2. 022G2&.
   1.3. 042G24&.
   1.4. 064G246&.
   1.5. 084G2468&.

Table 2.3.13 provides perspective about ground-state INTERN LADDER solutions for the G-family. The table provides a bound on which G-family ground-state solutions are physics-relevant. The table provides a means for computing spins for G-family solutions. [Physics:] We use models regarding circular polarization to interpret and use G-family solutions. For example, for a specific MM1MS1-photon (including its observed momentum), the MM1MS1-photon left-circular polarization mode can be multiply excited, the right-circular polarization mode can be multiply excited, but (at any one time) no more than one of the right-circular polarization mode and the left-circular polarization can be excited.

**Table 2.3.13** Perspective regarding spin and other aspects of G-family ground-state INTERN LADDER solutions

1. The following statements describe an algorithm for computing S for G-family solutions.
   1.1. The symbol (0, ..., 0) denotes one of ...
      1.1.1. (0, 0);
      1.1.2. (0, 0, 0, 0);
      1.1.3. (0, 0, 0, 0, 0, 0); or
      1.1.4. (0, 0, 0, 0, 0, 0, 0, 0).
   1.2. Here, the symbol (0, ..., 0) correlates with the set that contains all relevant QP-like oscillators except P0.
   1.3. For a version of (0, ..., 0), use of the list of even integers specified as follows ...
      1.3.1. List = 2; for (0, 0).
      1.3.2. List = 2, 4; for (0, 0, 0, 0).
      1.3.3. List = 2, 4, 6; for (0, 0, 0, 0, 0, 0).
      1.3.4. List = 2, 4, 6, 8; for (0, 0, 0, 0, 0, 0, 0, 0).
   1.4. Select a sub-list from the list.
      1.4.1. For each of the cases 022G2&, 042G24&, 064G246&, and 084G2468&, the sub-list equals the list.

1.4.2. For each of the other cases, except for the following sub-lists, any sub-list pertains.
1.4.2.1. 2, 8.
1.4.2.2. 6.
1.4.2.3. 8.
1.5. For the sub-list, find a combination of arithmetic operations, selected from plus and minus, such that arithmetic combining of the items in the sub-list sums to either 2 or 4.
1.5.1. For example, + 2 = 2.
1.5.2. For example, − 2 + 4 = 2.
1.5.3. For example, − 2 + 6 = 4.
1.5.4. For example, + 2 − 4 + 6 = 4.
1.5.5. For example, − 2 + 4 − 6 + 8 = 4.
1.5.6. Note that the following possibilities would not work because each correlates with S > 2 (See the next item.); hence, the exceptions above.
1.5.6.1. − 2 + 8 = 6.
1.5.6.2. + 6 = 6.
1.5.6.3. + 8 = 8.
1.6. Set 2S equal to the result of the specified arithmetic combining of items in the sub-list.
1.6.1. [Physics:] Here, 2S is formed based on a combination of circular polarizations.
1.6.1.1. A circular polarization carries a spin/ħ equal to the appropriate one of 2/2, 4/2, 6/2, and 8/2.
1.6.1.1.1. Here, the 2, 4, 6, and/or 8 (in the respective numerators) correlate with elements in the sub-list.
1.6.1.2. The combination of circular polarizations is one circular polarization for a G-family particle.
1.6.1.3. The negative of the combination of circular polarizations is the other circular polarization for the G-family particle.
1.6.2. For a symbol of the form epnG..&, …
1.6.2.1. Note that e = #E.
1.6.2.2. Note that p = #P.
1.6.2.3. Note that n = 2S.
1.6.3. For the symbol of the form epnG, …
1.6.3.1. Set e = #E.
1.6.3.2. Set p = #P.
1.6.3.3. Set n = 2S.
2. The number of G-family ground-state solutions for which S = 1 or S = 2 is unbounded (or, infinite).

3. Starting with #P = A = [10], multiple solutions can pertain for a given base state.
    3.1. For example, for 0A2G2468A&, the sub-list 2, 4, 6, 8, and A pertains and each of the following calculations leads to S = 1.
    3.2. 2S = (− 2 − 4 + 6) + (− 8 + 10)
    3.3. 2S = (+ 2 + 4 − 6) + (− 8 + 10)
4. We assume that, for #P ≥ A = [10], G-family solutions are not physics-relevant.
    4.1. We use the inappropriate redundancy criterion. (See Table 2.15.8.)
5. Work in here and elsewhere dovetails with the following statement.
    5.1. [Physics:] For each G-family solution that correlates with an elementary particle, #P ≤ 8.
6. The previous items (that discuss limits on #P for the G-family) dovetail with the following statement.
    6.1. [Physics:] For each G-family solution that correlates with an elementary particle, no other G-family solution has the same combination of #E, #P, 2S, ground-state N(P0), and sub-list.
7. Below, we sometimes denote sub-list by the symbol %.
    7.1. For example, we might use a symbol of the form epnG%& or a symbol of the form nG%&.

Table 2.3.14 characterizes CORPHY ground-state INTERN LADDER solutions for which σ = −1. Table 2.3.4 motivates the names of families. Here, considerations (similar to those regarding Table 2.3.11) for ∼ ∼ apply for adjacent oscillators. Each row for which ∼ ∼ applies has two applications of ∼ ∼ and, therefore correlates with four solutions. People might say that each such row correlates with two particles, and, for each of the two particles, one antiparticle. (See, for example, Table 3.5.3.) For Y-family solutions, Section 3.6 discusses values for N(..) for QE-like oscillators. (See Table 3.6.1 and Table 3.6.8.)

**Table 2.3.14** CORPHY ground-state INTERN LADDER solutions for which σ = −1

| E | E | E | E | E | E | E | P | P | P | P | P | P | P | P | P | (σ = −1) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 6R | 6L | 4R | 4L | 2R | 2L | 0 | 0 | 2L | 2R | 4L | 4R | 6L | 6R | 8L | 8R | Subfamily |
| | | | | | | 0 | 0 | | | | | | | | | 0O |
| | | | | ∼ | ∼ | 0 | 0 | ∼ | ∼ | | | | | | | 1Q |
| | | | | ∼ | ∼ | −1 | −1 | ∼ | ∼ | | | | | | | 1R |
| | | | | .. | .. | .. | −1 | −1 | −1 | | | | | | | 2Y |
| | | | | 0 | 0 | 0 | 0 | 0 | 0 | | | | | | | 2O |
| | | ∼ | ∼ | 0 | 0 | 0 | 0 | 0 | 0 | ∼ | ∼ | | | | | 3Q |
| | | ∼ | ∼ | −1 | −1 | 0 | 0 | −1 | −1 | ∼ | ∼ | | | | | 3I |
| | | ∼ | ∼ | 0 | 0 | −1 | −1 | 0 | 0 | ∼ | ∼ | | | | | 3R |
| | | ∼ | ∼ | −1 | −1 | −1 | −1 | −1 | −1 | ∼ | ∼ | | | | | 3D |
| | | .. | .. | .. | .. | .. | −1 | −1 | −1 | −1 | −1 | | | | | 4Y |

| E 6R | E 6L | E 4R | E 4L | E 2R | E 2L | E 0 | P 0 | P 2L | P 2R | P 4L | P 4R | P 6L | P 6R | P 8L | P 8R | ($\sigma = -1$) Subfamily |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | | | | | 4O |

~ ~ ~

This subsection discusses generations and color charges.

People might say that the concept of generation correlates with an INTERN property of elementary fermions. People might say that the concept of color charge correlates with an INTERN property of, at least, quarks.

Table 2.6.4 discusses numbers of generations for elementary fermions. Section 2.14 provides possible insight regarding generations, numbers of color charges, and possible EXTINT LADDER models that show aspects regarding generations and color charges.

# Section 2.4 One MM1 meta-model and various MM1MS1 models

Section 2.4 discusses relationships between the MM1 meta-model and MM1MS1 models.

~ ~ ~

This subsection discusses relationships between the MM1 meta-model and MM1MS1 models.

Table 2.4.1 pertains.

**Table 2.4.1** Relationships between the MM1 meta-model and MM1MS1 models

1. The MM1 meta-model …
    1.1. Features CORPHY ground-state INTERN LADDER solutions.
    1.2. Outputs …
        1.2.1. Solutions that might correlate with elementary particles.
        1.2.2. Families and subfamilies (of solutions) that might correlate with elementary particles.
        1.2.3. The following numbers, which may correlate with properties of elementary particles.
            1.2.3.1. $\sigma$.
                1.2.3.1.1. +1 or −1.
            1.2.3.2. S.

1.2.3.2.1. 0, 1/2, 1, 3/2, or 2.

1.2.3.3. m'.

1.2.3.3.1. = 0 or ≠ 0.

2. Each MM1MS1 model …
    2.1. Comports with the MM1 meta-model.
    2.2. Likely, accepts, as inputs, outputs from the MM1 meta-model.
        2.2.1. To the extent the MM1MS1 model uses ground-state INTERN LADDER solutions, …
            2.2.1.1. The MM1MS1 model uses CORPHY ground-state INTERN LADDER solutions.
        2.2.2. For each of some CORPHY ground-state INTERN LADDER solutions, the MM1MS1 model …
            2.2.2.1. May ignore the solution.
            2.2.2.2. May assume the solution does not correlate with an elementary particle.
            2.2.2.3. May assume the solution might (or does) correlate with an elementary particle.
    2.3. Possibly, accepts, as inputs, results that other MM1MS1 models produce.
    2.4. Likely, applies models other than INTERN LADDER models.
    2.5. Possibly, produces, as outputs, …
        2.5.1. Candidate values for particle properties beyond properties the MM1 meta-model produces.
        2.5.2. Candidates for interactions in which particles partake.
        2.5.3. Interpretations of and/or candidate results regarding aspects of …
            2.5.3.1. Elementary-particle physics.
            2.5.3.2. Cosmology.
            2.5.3.3. Astrophysics.

# Section 2.5 DIFEQU models for isotropic quantum harmonic oscillators

Section 2.5 discusses DIFEQU models for isotropic quantum harmonic oscillators. We discuss continuous-math models pertaining to lone harmonic oscillators. We discuss continuous-math models pertaining to multi-dimensional isotropic quantum harmonic oscillators. We discuss projecting representations into smaller numbers of dimensions than originally pertain to the representations. We discuss, in the context of DIFEQU models, the notion of solution. We note that the topic of normalization has physics-relevance. We show two types of solutions that normalize. People might say that we show solutions that traditional physics models may have underutilized. We discuss ranges of parameters that pertain for each of traditional solutions and non-traditional solutions. People might say that solutions we characterize correlate with known elementary particles and with possible elementary particles. People might say that we show integers that we later correlate with masses of elementary bosons.

~ ~ ~

This subsection provides notation for DIFEQU models for lone quantum harmonic oscillators.

Table 2.5.1 shows well-known information about 1-dimensional harmonic oscillators.

**Table 2.5.1** Notation and math pertaining to a lone quantum harmonic oscillator (DIFEQU solutions)

1. The symbol x0 denotes a coordinate.
   1.1. The domain for x0 is $-\infty < x0 < \infty$.
2. The symbol $\Psi(x0)$ denotes a function of x0.
   2.1. The range of $\Psi$ is the set of complex numbers.
3. The following equation correlates with the lone quantum harmonic oscillator.
   $$\xi\, \Psi(x0) = (\xi'/2)\, (\, -\eta^2\, \partial^2/\partial(x0)^2 + \eta^{-2}\, (x0)^2\, )\, \Psi(x0) \qquad (2.36)$$
4. $\xi$ and $\xi'/2$ denote numbers.
5. [Physics:] The following may pertain.
   5.1. $\Psi(x0)$ denotes a wave function.
   5.2. The variable x0 has dimensions of length.
   5.3. $\eta$ denotes a parameter with dimensions of length.
   5.4. $V = (\xi'/2)\, \eta^{-2}\, (x0)^2$.
      5.4.1. People call this the potential.
   5.5. Generally, $\eta^2(\xi'/2)$ and $\eta^{-2}(\xi'/2)$ are constants that are not directly related to each other.
   5.6. Sometimes, people correlate $\xi$ and $\xi'/2$ with numbers having dimensions of energy.
      5.6.1. People correlate $\xi'/2$ with the term ground-state energy.
6. People characterize solutions by ...
   6.1. $\Psi$ has the form of a Hermite polynomial (in the variable x0) multiplied by $\exp(\, -(x0)^2/(2\eta^2)\, )$.

~ ~ ~

This subsection discusses an aspect of a difference between some uses of harmonic oscillator models in traditional physics and some uses of harmonic oscillator models in this monograph.

Table 2.5.2 pertains. People might say that Table 6.1.5 and Table 6.1.6 provide perspective about and harmonization regarding the possibly significant difference to which Table 2.5.2 points.

**Table 2.5.2** Differences in units for ξ (and ξ') between some aspects of traditional physics and some aspects of work in this monograph

1. For some uses of harmonic oscillator models in traditional physics, …
    1.1. ξ and ξ'/2 have dimensions of energy. (Note, for example, Table 2.5.1.)
2. For some uses of harmonic oscillator models in this monograph, …
    2.1. ξ and ξ'/2 have dimensions of energy squared. (See, for example, (2.58) in Table 2.9.4.)

~ ~ ~

This subsection shows notation for DIFEQU solutions for multi-dimensional isotropic quantum harmonic oscillators.

Table 2.5.3 shows information about multi-dimensional isotropic quantum harmonic oscillators.

**Table 2.5.3** Notation and math pertaining to a DIFEQU solution for a multi-dimensional isotropic quantum harmonic oscillator

1. For a (#D + 1)-dimensional isotropic quantum harmonic oscillator, …
    1.1. #D denotes a nonnegative integer.
    1.2. The set { j | j = 0 , 1 , … , #D − 1 , or #D } provides …
        1.2.1. An index to the lone quantum harmonic oscillators that make up the (#D + 1)-dimensional isotropic quantum harmonic oscillator.
    1.3. The set { xj | j = 0 , 1 , … , #D − 1 , or #D } provides …
        1.3.1. A set of #D + 1 coordinates, with each coordinate pertaining to the respective lone quantum harmonic oscillator.
    1.4. Numbers N(0), N(1), …, and N(#D) describe base states of the lone harmonic oscillators.
        1.4.1. The lone harmonic oscillators correlate with the (#D + 1)-dimensional isotropic quantum harmonic oscillator.

1.4.2. Each number N(..) provides part of a description of a base state for the (#D + 1)-dimensional isotropic quantum harmonic oscillator.

2. People specify the state of a (#D + 1)-dimensional isotropic quantum harmonic oscillator via a linear combination of base states for the (#D + 1)-dimensional harmonic oscillator.
3. [Physics:] People may associate an energy with each base state of the (#D + 1)-dimensional isotropic quantum harmonic oscillator.
   3.1. The energy is proportional to the sum over the relevant j of terms of following form.

$$(a_j)^+ (a_j)^- + 1/2 = N(j) + 1/2 \qquad (2.37)$$

   3.2. The term isotropic correlates with the sum's giving equal weight to each of the N(j) + 1/2.

For such a (#D + 1)-dimensional isotropic quantum harmonic oscillator, people can express representations for which sets of component oscillator are grouped into subsets (of the entire set of relevant harmonic oscillators) and for which radial coordinates can pertain. Traditionally, radial coordinates can pertain within each subset that includes at least two harmonic oscillators. We find uses for radial coordinates for subsets that include just one harmonic oscillator. (See, for example, Table 2.5.5.)

Table 2.5.4 pertains.

**Table 2.5.4** Process for creating and preparing to use a partly linear-coordinate, partly radial-coordinate representation for a DIFEQU isotropic quantum harmonic oscillator

1. Subdivide the set { j | j = 0 , 1 , … , #D − 1 , or #D } (See Table 2.5.3.) into subsets such that …
   1.1. Each j appears in exactly one subset.
   1.2. No subset is the empty set.
2. For each subset that has exactly 1 member, …
   2.1. Anticipate using math that Table 2.5.1 and Table 2.5.7 describe or using math that Table 2.5.6 describes.
3. For each subset that has more than 1 member, …
   3.1. Anticipate using math that Table 2.5.6 describes.
      3.1.1. For example, the expression $r^2$ equals the sum of the squares of the relevant xj. (See, for example, (2.38) in Table 2.5.6.)
   3.2. Anticipate (for purposes of this monograph) not necessarily trying to correlate LADDER solutions with DIFEQU solutions.

~ ~ ~

This subsection anticipates uses of partly linear-coordinate, partly radial-coordinate representations for DIFEQU isotropic quantum harmonic oscillators.

Table 2.5.5 pertains. (Regarding D*, see, for example, Table 2.1.1.)

**Table 2.5.5** Possible applications for models correlating with D* equaling each of 3, 2, and 1

1. We anticipate using subsets (of the set { j | j = 0 , 1 , … , #D − 1 , or #D }) that have 3 elements.
    1.1. [Physics:] For example, some of these applications correlate with notions of at least one of the following.
        1.1.1. D* = 3.
        1.1.2. Projections of Ψ from D > 3 dimensions into D* = 3 dimensions.
        1.1.3. Three QP-like dimensions correlating with people's notions of space-time coordinates (or, possibly, notions of space-time).
2. We anticipate using subsets that have 2 elements.
    2.1. [Physics:] For example, some of these applications correlate with notions of at least one of the following.
        2.1.1. Projections of Ψ from D* = 3 dimensions into D* = 2 dimensions.
        2.1.2. Projections of Ψ from D ≥ 3 dimensions into D* = 2 dimensions.
        2.1.3. Integers that may correlate approximately with squares of masses for WHO-family particles.
3. We anticipate using subsets that have 1 element.
    3.1. [Physics:] For example, some of these applications correlate with notions of at least one of the following.
        3.1.1. Projections of Ψ from D* = 3 dimensions into D* = 1 dimension.
        3.1.2. Projections of Ψ from D ≥ 3 dimensions into D* = 1 dimension.
        3.1.3. Integers that may correlate approximately with squares of masses for WHO-family particles. (People might say that this correlation is somewhat more indirect than the corresponding correlation for subsets that have 2 elements.)

~ ~ ~

This subsection discusses radial-coordinate representations for isotropic quantum harmonic oscillators.

This subsection discusses radial-coordinate math-based models for QP-like isotropic quantum harmonic oscillators. This subsection does not consider normalization. (Later, we discuss normalization.) The use of QP-like notation does not imply a lack of applicability of similar math-based models to QE-like aspects of models. Some remarks in this subsection pertain to QE-like notation. For possible QE-like applications, see Table 2.6.6.

The math pertains to solutions this monograph denotes by $\Psi(r)$. Here, r is the radial coordinate. $\Psi$ can be a function also of coordinates (that is, angular coordinates) other than r. (Regarding multiple definitions of the term solution, see remarks preceding Table 2.2.9.)

Table 2.5.6 pertains. The formulation is equivalent to traditional formulations in which $\eta^2(\xi'/2)$ and $\eta^{-2}(\xi'/2)$ are constants that are not directly related to each other.

**Table 2.5.6** Math and notation for radial-coordinate solutions for QP-like isotropic quantum harmonic oscillators

1. The following equation pertains.
   $$\xi\,\Psi(r) = (\xi'/2)\,(\,-\eta^2\,\nabla^2 + \eta^{-2}\,r^2\,)\,\Psi(r) \qquad (2.38)$$
2. People call the following part of the above equation the Laplacian operator for D dimensions.
   $$\nabla^2 = r^{-(D-1)}(\partial/\partial r)(r^{D-1})(\partial/\partial r) - \Omega r^{-2} \qquad (2.39)$$
3. $\xi$ and $\xi'/2$ denote numbers.
   3.1. [Physics:] In some applications, $\xi$ and $\xi'$ have dimensions of energy.
4. $\Psi(r)$ denotes a function ([Physics:] or, wave function).
5. r denotes a variable ([Physics:] with dimensions of length).
6. $\eta$ denotes a parameter ([Physics:] with dimensions of length).
7. $\Omega$ denotes a number.
8. D denotes a positive integer.
9. $V = (\xi'/2)\,\eta^{-2}\,r^2$.
   9.1. People call this the potential.

Table 2.5.7 pertains.

**Table 2.5.7** Solutions, based on Hermite polynomials, for quantum harmonic oscillators for which D = 1

1. For D = 1 (in Table 2.5.6), some solutions feature the following.
   1.1. $\Omega = 0$.
   1.2. The range $-\infty < r < \infty$ pertains.
   1.3. $\Psi$ has the form of a Hermite polynomial (in the variable r) multiplied by $\exp(\,-r^2/(2\eta^2)\,)$.

Work below tends not to emphasize solutions people associate with Table 2.5.7. Below, the range $0 \leq r < \infty$ pertains for traditional solutions. ([Physics:] People might dispute work below pertaining to D = 1. However, such non-traditional work regarding D = 1 possibly matches some physics notions. For example, see Table 2.5.5. Table 2.6.6

lists possible applications for QE-like solutions for which D = 1. For QE-like D = 1 applications, we might use the variable t' (for time) in place of r. And, we would use a symbol (with units of time) other than η in place of η.)

Table 2.5.8 describes solutions other than solutions people traditionally associate with Table 2.5.7.

**Table 2.5.8** Some radial-coordinate solutions for isotropic quantum harmonic oscillator math

1. The following relationship characterizes these solutions.

$$\Psi(r) \propto r^{\nu} \exp(-r^2 / (2\eta^2))$$

2. For these solutions, …
   2.1. The following algebraic equations pertain.

$$\xi = (D + 2\nu)(\xi'/2) \tag{2.40}$$

$$\Omega = \nu(\nu + D - 2) \tag{2.41}$$

   2.2. The parameter η does not appear in the two equations.
3. The following pertain to some ([Physics:] traditional) solutions.
   3.1. ν is nonnegative.
   3.2. ν is an integer.
   3.3. Ω is nonnegative.
4. Each of the following items points to other ([Physics:] non-traditional) solutions.
   4.1. ν can be negative.
   4.2. ν can be other than an integer.
5. For D > 2, the condition ν < 0 is necessary (but not sufficient) for the next item to pertain.
   5.1. Ω can be negative.
6. For D = 2, …
   6.1. $\Omega = \nu^2 \geq 0$.

~ ~ ~

This subsection discusses normalization.

[Physics:] Traditional physics does not include wave functions that do not normalize.

People say that a function such as Ψ normalizes if, and only if, integration (throughout the domain of the function) of the expression consisting of the product of the function and its complex conjugate produces a finite result.

For DIFEQU solutions, this monograph does not consider to be physics-relevant any wave function that does not normalize.

~ ~ ~

This subsection shows that solutions for which D + 2ν > 0 normalize.

Table 2.5.9 pertains.

**Table 2.5.9** Normalization for solutions for which $D + 2\nu > 0$

1. The following relationships characterize behavior of the r-related normalization integrand near $r = 0$.

$$(\Psi(r))^* \Psi(r)\, r^{D-1} \propto r^{D-1+2\nu} \exp(-2r^2 2^{-1} \eta^{-2}) \to r^{D-1+2\nu}, \text{ as } r \to 0 \quad (2.42)$$

   1.1. Here, * denotes complex conjugate.
   1.2. Here, the term $r^{D-1}$ comes from the expression $\int \ldots r^{D-1} dr$.
2. The integral is finite …
   2.1. If $-1 < D - 1 + 2\nu$.
   2.2. Or, equivalently, if $D + 2\nu > 0$.
3. $\Psi$ normalizes if (but not only if) $D + 2\nu > 0$.

~ ~ ~

This subsection shows that solutions for which $D + 2\nu = 0$ normalize. Table 2.5.10 pertains.

**Table 2.5.10** Normalization for solutions for which $D + 2\nu = 0$

1. Wolfram Alpha (2014a) provides the following definition of the Dirac delta function.

$$\delta(r) = \lim_{\varepsilon \to 0+} (1/(2(\pi\varepsilon)^{1/2})) \exp(\, -r^2/(4\varepsilon)\,) \quad (2.43)$$

2. This work makes the following association.

$$4\varepsilon = \eta^2 \quad (2.44)$$

3. The next item supplements a result in Table 2.5.9.
4. $\Psi$ normalizes if (but not only if) $D + 2\nu = 0$.

~ ~ ~

This subsection provides terminology for characterizing some DIFEQU solutions.

Table 2.5.11 pertains. Here, we anticipate discussing a specific solution (for which the exponent $\nu$ pertains) in the contexts of more than one number of dimensions. Here, the terms inside and edge correlate with normalization pertaining for the solution in a context of D dimensions. Here, the term outside correlates with normalization not being possible in a context of D dimensions.

**Table 2.5.11** Definitions, regarding a solution and a number of dimensions, of inside, edge, and outside (DIFEQU solutions)

1. For a solution characterized by the exponent $\nu$, …
   1.1. For a number of dimensions D, …
      1.1.1. Inside denotes $D + 2\nu > 0$.
      1.1.2. Edge denotes $D + 2\nu = 0$.
      1.1.3. Outside denotes $D + 2\nu < 0$.

People might think of the terms inside, edge, and outside as correlating, for a value of $\nu$, with a domain of D for which normalization pertains. People might think of the terms inside, edge, and outside as correlating, for a value of D, with a domain of $\nu$ for which normalization pertains.

~ ~ ~

This subsection shows some solutions.

This subsection focuses on math for which $D^* = 3$. [Physics:] $D^* = 3$ matches key elementary-particle data about spin. (See, (2.1) in Table 2.1.1.) Here, as elsewhere, S correlates with spin/$\hbar$.

Table 2.5.12 shows some ([Physics:] traditional) solutions.

**Table 2.5.12** Some ([Physics:] traditional) solutions (DIFEQU solutions)

1. The next items describe some ([Physics:] traditional) solutions.
   1.1. $D^* = 3$.
   1.2. $D = 3$.
   1.3. $S = \nu$, for some nonnegative integer $\nu$.
   1.4. $\Omega = \nu(\nu + D - 2) = S(S + D^* - 2) = S(S + 1)$.
   1.5. 2S+1 angular solutions pertain.
2. For example, the next items describe a solution that correlates with the solution Table 2.2.13 shows. ([Physics:] Here, people would state that $3\times(\xi'/2)$ equals the energy.)
   2.1. $D^* = 3$.
   2.2. $D = 3$.
   2.3. $S = \nu = 0$.
   2.4. $\Omega = \nu(\nu + D - 2) = 0$.
   2.5. 1 angular solution pertains.
3. The next items describe the ground state and the excited states for the previous example.
   3.1. $\xi = (D + 2\nu)\,(\xi'/2) = (D^*/2 + \nu)\,\xi' = (3/2 + S)\,\xi'$.
   3.2. S is a nonnegative integer.
   3.3. $\Omega = S(S + 1)$.
   3.4. $\Psi(r) \propto r^S \exp(\,-r^2 / (2\eta^2)\,)$.
   3.5. For the ground state, $S = 0$.

Table 2.5.13 shows math that allows for other ([Physics:] non-traditional) solutions for which $D^* = 3$. In particular, solutions allow $D \neq D^*$ (with D a positive integer) and solutions allow 2S to be an odd nonnegative integer.

**Table 2.5.13** Some ([Physics:] non-traditional) solutions, including an example of a non-traditional ground state for 3-dimensional isotropic quantum harmonic oscillators (DIFEQU solutions)

1. The next items allow for some non-traditional solutions.
   1.1. $D^* = 3$.
   1.2. $D^* + 2\nu > 0$, with $2\nu$ being an integer.
   1.3. $\Omega = \nu(\nu + D - 2)$, for some positive integer D.
   1.4. $|\Omega| = S(S + D^* - 2) = S(S + 1)$, for some S with 2S being a nonnegative integer.
   1.5. $2S + 1$ angular solutions pertain.
2. For example, the next items show a solution for which $S \neq \nu$. For r near 0, the normalization integral behaves like a non-zero factor multiplied by $\int r^0 dr$. This monograph considers this non-traditional solution to correlate with a ground state. ([Physics:] People might say that $1\times(\xi'/2)$ equals the ground-state energy. People might say that the energy matches the energy of the state Table 2.2.15 shows. We find distinctly different applications for the two solutions.)
   2.1. $D^* = 3$.
   2.2. $\nu = -1$.
   2.3. $\Psi(r) \propto r^{-1} \exp( -r^2 / (2\eta^2) )$.
   2.4. $D = 3$.
   2.5. $\Omega = \nu(\nu + D - 2) = -1(0) = 0$.
   2.6. $S = 0$.
   2.7. $\Omega = S(S + D^* - 2) = S(S + 1) = 0(1) = 0$.
   2.8. $\xi = (1/2)\,\xi'$.
   2.9. 1 angular solution pertains.

~ ~ ~

This subsection shows ranges of parameters for solutions based on radial coordinates.

Table 2.5.14 lists parameters that characterize solutions. Here, this monograph does not discuss traditional solutions for which $D = 1$. For non-traditional solutions that this monograph considers, this monograph limits $2\nu$ to be an integer, limits D to be a positive integer, and (for $D > 1$) limits $D + 2\nu$ to be nonnegative. For each non-traditional solution, for one of $\pm = +$ and $\pm = -$, $\Omega$ must satisfy both of the equations the last row in the table shows. [Physics:] Applications for which $\xi' < 0$ may pertain regarding some aspects of models correlating with masses of non-zero-mass elementary bosons for which $S = 1$ or $S = 2$. (See Section 5.7 and Section 5.8.)

**Table 2.5.14** Parameters for solutions (based on radial coordinates) that can be normalized (DIFEQU solutions)

| Parameter | Value range ([Physics:] correlating with traditional solutions) | Value range ([Physics:] including non-traditional solutions) |
|---|---|---|
| $\xi'$ | $> 0$ | $\neq 0$ |
| $\eta$ | $\neq 0$ | $\neq 0$ |
| D* | Positive integer | Positive integer |
| D | = D* | Positive integer |
| 2S | Even nonnegative integer | Nonnegative integer |
| $2\nu$ | = 2S | Integer, with $2\nu \geq -D$ |
| $\Omega$ | $= S(S + D^* - 2)$ | $= \nu(\nu + D - 2)$<br>$= \pm S(S + D^* - 2)$ |

Table 2.5.15 discusses the symbol $\sigma$. The symbol correlates with the sign of $\Omega$. (Compare with Table 2.3.1 and Table 2.3.3.) [Physics:] Examples of free-ranging elementary particles include electrons and photons. Examples of non-free-ranging elementary particles include quarks and gluons.

**Table 2.5.15** The number $\sigma$

1. $\Omega = \sigma S(S + 1)$, with …
   1.1. [Physics:] For free-ranging particles, …
      1.1.1. $\sigma = +1$, for solutions for which $S > 0$.
      1.1.2. $\sigma = +1$, for solutions for which $S = 0$.
   1.2. [Physics:] For particles that do not range freely, …
      1.2.1. $\sigma = -1$, for solutions for which $S > 0$.
      1.2.2. $\sigma = -1$, for solutions for which $S = 0$.

In some aspects of this monograph, uses of $\sigma$ correlate with $D^* = 3$ and, hence, with $\Omega = \sigma S(S + 1)$. For other purposes, people might define $\sigma$ to correlate with $\Omega = \sigma S(S + D^* - 2)$.

~ ~ ~

This subsection lists non-traditional solutions, including solutions for which $S \leq 2$ and $D^* = 3$, 2, or 1.

Table 2.5.16 shows some cases of relationships between parameters. The table shows examples for which $D^* = 3$. Here, the work ignores the parameter $\xi'$. For $\nu = -1/2$ and for $\nu = -1$, any non-zero $\eta$ can pertain. For $\nu = -3/2$, physics-relevant solutions pertain only with respect to one of the limits $\eta \to 0^+$ and $\eta \to 0^-$. Here, physics-relevance correlates with a solution's being able to be normalized. For each of three values of $\nu$, the table shows how to compute D, given $\Omega$. For each of those three values of $\nu$, the table describes each candidate solution for which $S \leq 2$. ([Physics:] We use the term candidate solution to describe a solution that may correlate with an elementary particle.) For each of those values of ν, no mathematical upper limit exists for −Ω. For each of those values of ν, no mathematical upper limit exists for S.

**Table 2.5.16** Relationships between some parameters, for some solutions for which D* = 3 (DIFEQU solutions)

| D* | ν | D* + 2ν | D | S | Ω | σ | D | D + 2ν | 2S + 1 |
|---|---|---|---|---|---|---|---|---|---|
| 3 | −1/2 | 2 | (5 − 4Ω)/2 | 1/2 | 3/4 | +1 | 1 | 0 | 2 |
| " | " | " | " | 1/2 | −3/4 | −1 | 4 | 3 | 2 |
| " | " | " | " | 3/2 | −15/4 | −1 | 10 | 9 | 4 |
| " | " | " | " | … | | | | | |
| " | −1 | 1 | 3 − Ω | 1 | 2 | +1 | 1 | −1 | 3 |
| " | " | " | " | 0 | 0 | +1 | 3 | 1 | 1 |
| " | " | " | " | 0 | 0 | −1 | 3 | 1 | 1 |
| " | " | " | " | 1 | −2 | −1 | 5 | 3 | 3 |
| " | " | " | " | 2 | −6 | −1 | 9 | 7 | 5 |
| " | " | " | " | … | | | | | |
| " | −3/2 | 0 | (21 − 4Ω)/6 | 1/2 | 3/4 | +1 | 3 | 0 | 2 |
| " | " | " | " | 1/2 | −3/4 | −1 | 4 | 1 | 2 |
| " | " | " | " | 3/2 | −15/4 | −1 | 6 | 3 | 4 |
| " | " | " | " | … | | | | | |

Table 2.5.17 shows some cases of relationships between parameters. The table shows examples having D* = 2. We do not anticipate using these solutions directly to identify candidate elementary particles. We anticipate using the solutions in ways that pertain to candidate elementary particles we identify via other means. For some candidate boson particles, we anticipate projecting solutions for which D* = 3 and ν = −1 into functions that correlate with D* = 2 and ν = −1. We anticipate that such projections correlate with masses for zero-mass elementary bosons and with approximate masses for non-zero-mass elementary bosons. (See, for example, Section 3.7.) For these applications, needs for normalization correlate with the respective solutions for which D* = 3 and do not necessarily correlate with the solutions for which D* = 2. For some of these applications, the limit S ≤ 2 does not pertain. For some of these applications, we think there is no need, for the S = 0 solution, to assign a value for σ. Here, for ν = −1, Ω = $\pm S^2$.

**Table 2.5.17** Relationships between some parameters, for some solutions for which D* = 2 (DIFEQU solutions)

| D* | ν | D* + 2ν | D | S | Ω | σ | D | D + 2ν | 2S + 1 |
|---|---|---|---|---|---|---|---|---|---|
| 2 | −1/2 | 1 | (5 − 4Ω)/2 | 1/2 | 1/4 | +1 | 2 | 1 | 2 |
| " | " | " | " | 1/2 | −1/4 | −1 | 3 | 2 | 2 |
| " | " | " | " | 3/2 | −9/4 | −1 | 7 | 6 | 4 |
| " | " | " | " | 5/2 | −25/4 | −1 | 15 | 14 | 6 |

| D* | ν | D* + 2ν | D | S | Ω | σ | D | D + 2ν | 2S + 1 |
|---|---|---|---|---|---|---|---|---|---|
| " | " | " | " | 7/2 | −49/4 | +1 | 27 | 26 | 8 |
| " | " | " | " | 9/2 | −81/4 | −1 | 43 | 42 | 10 |
| " | " | " | " | … | | | | | |
| " | −1 | 0 | 3 − Ω | 1 | 1 | +1 | 2 | 0 | 3 |
| " | " | " | " | 0 | 0 | | 3 | 1 | 1 |
| " | " | " | " | 1 | −1 | −1 | 4 | 2 | 3 |
| " | " | " | " | 2 | −4 | −1 | 7 | 5 | 5 |
| " | " | " | " | 3 | −9 | −1 | 12 | 10 | 7 |
| " | " | " | " | 4 | −16 | −1 | 19 | 17 | 9 |
| " | " | " | " | 5 | −25 | −1 | 28 | 26 | 11 |
| " | " | " | " | 6 | −36 | −1 | 39 | 37 | 13 |
| " | " | " | " | 7 | −49 | −1 | 52 | 50 | 15 |
| " | " | " | " | 8 | −64 | −1 | 67 | 65 | 17 |
| " | " | " | " | … | | | | | |

Table 2.5.18 shows a formula pertaining to values of S that could arise for a choice of D* = 1. For some purposes of this monograph, we think we do not need to consider issues related to normalization. (See, for example, Table 2.6.2 and Table 2.10.8.)

**Table 2.5.18** Values of S that could arise for the choice D* = 1

1. Ω = ±S(S + D* − 2) = ±S(S − 1), with …
   1.1. D* = 1.
   1.2. S(S − 1) = 0, 0, 2, 6, …, respectively, for S = 0, 1, 2, 3, … .
   1.3. S(S − 1) would be −1/4 for S = 1/2.
   1.4. S(S − 1) = 3/4, 15/4 …, respectively, for S = 3/2, 5/2, … .
2. To the extent one considers just algebraic relationships (and not necessarily physics and not necessarily DIFEQU considerations), …
   2.1. S(S − 1) = …, 6, 2, respectively, for S = …, −2, −1.
3. Thus, for the sequence S = .., −2, −1, 0, 1, 2, 3, .., (See Table 2.10.8.) …
   3.1. The sequence of Ω is .., 6, 2, 0, 0, 2, 6, .. .
   3.2. Each of these Ω is nonnegative.
   3.3. Each of these Ω correlates with solutions for which D* = 3 and S (regarding D* = 3) is a nonnegative integer.

~ ~ ~

**Reference 1** Wolfram Alpha (2014a)
Wolfram Alpha, computational knowledge engine, Wolfram Alpha LLC, (2014). http://mathworld.wolfram.com/DeltaFunction.html.

# Section 2.6 Applications of DIFEQU models

Section 2.6 discusses, in the context of DIFEQU models, solutions. We focus on DIFEQU subsets for which the solutions might correlate with data about elementary particles. We correlate some subsets with the physics terms fermion and boson. We correlate some subsets with the physics terms fields and particles. We discuss aspects of projecting mathematical representations into smaller than the original numbers of dimensions. We show factors contributing to numbers of physics-relevant solutions. We list, for some DIFEQU subsets, numbers of relevant solutions. People might say that we discuss possible time-like uses of radial coordinates.

~ ~ ~

[Physics:] This subsection discusses concepts related to particles and fields.

People might say that physics uses of the term field include notions of constructs that extend broadly (with respect to space-time coordinates), that correlate with forces that impact elementary particles, and/or that correlate with the creation and destruction of elementary particles. Physics includes other uses of the term field.

People might say that physics uses of the term particle include notions of constructs that have properties (such as charge or mass) and/or that have localized existence (at particular times, with respect to space-time coordinates).

People might say that, for elementary particles, no internal component particles should exist. People might say that, for an elementary particle, internal properties should not vary. Perhaps, such non-variance of internal properties correlates with a matter of terminology. For example, people might debate the extent to which color charge is an internal property of quarks.

We prefer, absent math-based models, to deemphasize some detailed discussions of concepts of particles and fields.

People might say that work in this section provides working definitions of concepts related to the term particle and of concepts related to the term field. (See, for example, Table 2.6.1.)

~ ~ ~

This subsection discusses QE-like and QP-like aspects of DIFEQU solutions that correlate with MM1MS1 models.

People might say that Section 2.5 features QP-like notation, coordinates, and models.

[Physics:] In traditional physics, wave functions are functions of both QE-like coordinates and QP-like coordinates.

For MM1MS1 solutions that do not explicitly state QE-list aspects (such as coordinates), the monograph assumes QE-like aspects pertain. (See, for example, Table 2.6.2 and Table 2.6.6.)

Sometimes, we explicitly include QE-like coordinates. (See, for example, Section 2.10.)

~ ~ ~

[Physics:] This subsection previews some physics uses of DIFEQU solutions for which $D^* = 3$ or $D^* = 2$.

People use models to discuss elementary particles. People include, in some such discussions, math-based models for fields with which people say particles dovetail.

For this monograph's work, Table 2.6.1 pertains. For this monograph's work, some QP-like radial-coordinate representations feature $D^* = 3$ or $D^* = 2$. (For perspective regarding uses of DIFEQU representations, see Table 2.7.5.)

**Table 2.6.1** Correlations among $\nu$, $\eta$, particles, and fields, for solutions based on DIFEQU representations (for $D^* = 3$ or $D^* = 2$)

1. For $D^* = 3$, …
   1.1. $\nu = -1/2$ correlates with fermion fields.
   1.2. $\nu = -1$ correlates with boson fields.
   1.3. $\nu = -3/2$ correlates with fermion particles.
2. For $D^* = 2$, …
   2.1. $\nu = -1$ correlates with boson particles.
3. For fields, one of the following ranges pertains.

$$0 < \eta < \infty \tag{2.45}$$

$$0 < -\eta < \infty \tag{2.46}$$

4. For particles, the following limit pertains.

$$\eta^2 \to 0 \tag{2.47}$$

5. People might say that each of the following correlates with traditional-physics concepts of spatial extent.
   5.1. A range $0 < |\eta| < \infty$ for fields.
   5.2. A size $\eta = 0$ for particles.
6. Regarding sizes for elementary particles, …
   6.1. People might say that the uncertainty principle correlates with non-zero sizes for elementary particles.
   6.2. The uncertainty principle correlates with non-zero-sized localization correlating with models that feature supposition of base-state wave functions.
   6.3. People might say that much traditional QED (or, quantum electrodynamics) correlates with point-like (not non-zero-volume-like) interaction vertices.
   6.4. Work in this monograph …

6.4.1. Features MM1MS1 base states.
6.4.2. Tends not to discuss superposition.
6.4.3. Discusses point-like interaction vertices. (See Section 2.10.)

Table 2.6.2 pertains. (See Section 2.10.)

**Table 2.6.2** Correlations between particles and dimensions, for boson solutions based on radial coordinates (for D* = 3)

1. [Physics:] For a DIFEQU solution for which D* = 3 and ν = −1 (that is, for a boson field), the following discussion pertains.
   1.1. Projecting a radial-coordinate solution into a ν = −1, D* = 2 solution correlates with aspects pertaining to the boson as a particle.
      1.1.1. People might say that …
         1.1.1.1. The projection correlates with an interaction vertex.
         1.1.1.2. The projection maps into the 2 QP-like (or, spatial) dimensions perpendicular to the particle's direction of motion. Here, that direction pertains to motion either before the vertex (which would correlate with a vertex that correlates with an interaction that destroys the boson) or after the vertex (which would correlate with a vertex that correlates with an interaction that creates the boson).
         1.1.1.3. For the projection, D* + 2ν = 0, with D* = 2.
            1.1.1.3.1. This aspect correlates with some integer arithmetic pertaining to masses for non-zero-mass elementary bosons. (See Table 2.5.17, Section 2.10, Section 3.7, and Section 3.8.)
   1.2. People might expect that projecting the solution into a remaining 1 (= 3 − 2) dimension might pertain.
      1.2.1. Possibly, projection into a remaining 1 dimension dovetails with a projection of QE-like aspects of the wave function into 1 QE-like dimension.
      1.2.2. People might say that …
         1.2.2.1. The QP-like projection maps into the dimension parallel to the particle's direction of motion.
         1.2.2.2. In effect, projecting the radial-coordinate solution into a point-like construct correlating with a D* = 1 construct correlates with aspects pertaining to the boson as a particle.

1.2.2.2.1. For this projection to correlate with $D^* + 2\nu = 0$, $D^*$ would need to equal 2.
1.2.2.2.2. Such a projection could correlate with oscillators E0 and P0 in LADDER models.
1.2.2.2.3. A QE-like analog of $\nu$ might equal −1.
1.2.2.3. For the combined (QE-like and QP-like) projection, a $D^* + 2\nu = 0$ construct correlates with some integer arithmetic pertaining to masses for non-zero-mass elementary bosons. (See Table 2.5.17, Section 2.10, Section 3.7, and Section 3.8.)
1.2.2.3.1. Here, the choice of S (as is $\{D + 2\nu\}(2,S,\Omega)$) need not match S for the particle. (See Table 2.10.8.)

~ ~ ~

This subsection shows factors contributing to numbers of candidate solutions for $D^* = 3$ and $\nu = -1/2$ and for $D^* = 3$ and $\nu = -3/2$.

Each of $\nu = -1/2$ and $\nu = -3/2$ correlates with 2S equaling an odd integer (or, equivalently, S equaling a half-integer). Here, for $\Psi(r)$, $r^\nu$ has two possible values. The expressions $|r^\nu|$ and $-|r^\nu|$ denote the possible values. (This contrasts with $\nu = -1$ cases for which $r^\nu$ has just one value.) Recall that $\eta$ can have a positive value or a negative value. We think that the relative signs of $r^\nu$ and $\eta$ have meaning. ([Physics:] In terms of physics models, for $\nu = -3/2$, this work considers the case of Dirac elementary fermions. Here, one relative sign correlates with a particle and the other relative sign correlates with the particle's antiparticle. In terms of physics models, for $\nu = -1/2$, this work considers the case of fermion fields. Here, we think that one linear combination of terms correlates with a particle-and-antiparticle pair creation operator and that an orthogonal linear combination of terms correlates with a particle-and-antiparticle pair destruction operator.)

Table 2.6.3 provides a definition. Below, we use applications for which j and k are nonnegative integers.

**Table 2.6.3** Definition of min(j, k)

1. We define min(j, k) by the following.

$$\min(j, k) = j, \text{ if } j \le k \tag{2.48}$$

$$\min(j, k) = k, \text{ if } j \ge k \tag{2.49}$$

Table 2.6.4 pertains. Each item can provide a factor contributing to a calculation of a maximum number of solutions. This work excludes the case D = 1 because the math for D = 1 resembles math correlating with Table 2.5.7. To the extent the subgroup relationship $SU(4) \supset SU(2) \times SU(2) \times U(1)$ is relevant, #GEN(3/2) could be other than 15, 9, or 6.

**Table 2.6.4** Factors contributing toward maximum numbers of solutions (for $D^* = 3$ and $\nu = -1/2$ and for $D^* = 3$ and $\nu = -3/2$)

1. For cases for which $D^* = 3$ and $D > 1$, …
   1.1. For cases for which $\nu = -1/2$ or $\nu = -3/2$, …
      1.1.1. $\min(D, 2(2S + 1))$ provides a factor for calculating the maximum number of candidate solutions.
   1.2. [Physics:] People might say that this factor correlates (in some sense) with correlating numbers of particle-INTERN states and numbers of particle spin states.
2. For cases in which $D^* = 3$ and $\nu = -3/2$, …
   2.1. For solutions corresponding to one sign of $\eta$, …
      2.1.1. A function of $2S + 1$ provides another factor for calculating the number of solutions.
      2.1.2. Correlating with models pertaining for $|\eta| > 0$, the number of spin states is $2S + 1$.
      2.1.3. Perhaps, the following modeling pertains.
         2.1.3.1. In the limit $\eta^2 \rightarrow 0$, considerations related to $2S + 1$ spin states correlate with considerations related to the number of generators of $SU(2S + 1)$.
      2.1.4. For spin-1/2 elementary particles, …
         2.1.4.1. The factor (for use in calculating the number of solutions) correlates with the number of generations (or, for MM1MS1-neutrinos, with the number of flavors).
         2.1.4.2. [Physics:] The number of generations is 3.
         2.1.4.3. $2S + 1 = 2$.
         2.1.4.4. The number of generators of $SU(2S + 1)$ (or, of $SU(2)$) is 3.
      2.1.5. For spin-3/2 elementary particles, we use the parameter #GEN(3/2) to denote the factor (for use in calculating the number of solutions), which equals the number of generations. Possibly, one of the following two cases pertains.
         2.1.5.1. For the case we designate as GEN(3/2)15, …
            2.1.5.1.1. We use concepts this table states above.
            2.1.5.1.2. $2S + 1 = 4$.
            2.1.5.1.3. The function is the number of generators of $SU(4)$.
            2.1.5.1.4. The number of generators of $SU(4)$ is 15.
            2.1.5.1.5. #GEN(3/2) = 15.
         2.1.5.2. For the case we designate as GEN(3/2)≠15, …
            2.1.5.2.1. $2S + 1 = 4$.
            2.1.5.2.2. Possibly, the function correlates with the relevance of 2 instances of $SU(2)$.

2.1.5.2.2.1. People might say that choices for numbers of generators correlate with …
  2.1.5.2.2.1.1. #GEN(3/2) = 9, with 9 being the square of the number of generators of SU(2).
  2.1.5.2.2.1.2. #GEN(3/2) = 6, with 6 being the sum, over 2 instances of SU(2), of the number of generators for each instance.
  2.1.5.2.2.1.3. #GEN(3/2) possibly being other than 15, 9, or 6.

~ ~ ~

This subsection discusses numbers of candidate solutions for D* = 3.

This work extends Table 2.5.16 to show numbers of solutions.

Table 2.6.5 pertains. Here, #GEN denotes the number of generations. For the row for which ν = −1/2 and D = 1, math correlating with Table 2.5.7 may pertain. For this row, Table 2.6.5 shows possible numbers of candidate solutions, based on a minimum number of 1 (1 = D) and a maximum number of 4 (4 = 2(2S + 1)). For the row for which ν = −1 and D = 1, math correlating with Table 2.5.7 may pertain. For this row, Table 2.6.5 shows possible numbers of candidate solutions, based on a minimum number of 1 (1 = D) and a maximum number of 3 (3 = 2S+1). Otherwise, for boson particles and fields, the number of candidate solutions equals 2S + 1. ([Physics:] The number of spin states for a non-zero-mass elementary particle is 2S + 1.) For each of the three ν = −3/2 rows Table 2.6.5 shows, min(D, 2(2S + 1)) = D. [Physics:] People might say that, regarding ν = −3/2, to the extent nature exhibits spin-3/2 fermions, nature might exhibit fewer than 6×#GEN spin-3/2 fermions. (See Section 3.5.)

**Table 2.6.5** Numbers of candidate solutions, for some solutions based on radial coordinates (for D* = 3)

| D* | ν | D | Ω | σ | S | 2S + 1 | Generations (#GEN) | Number of candidate solutions |
|---|---|---|---|---|---|---|---|---|
| 3 | −1/2 | 1 | 3/4 | +1 | 1/2 | 2 | | 1, 2, 3, or 4 |
| " | " | 4 | −3/4 | +1 | 1/2 | 2 | | 4 |
| " | " | 10 | −15/4 | −1 | 3/2 | 4 | | 8 |
| " | " | | | | … | | | |
| " | −1 | 1 | 2 | +1 | 1 | 3 | | 1, 2, or 3 |
| " | " | 3 | 0 | +1 | 0 | 1 | | 1 |
| " | " | 3 | 0 | −1 | 0 | 1 | | 1 |
| " | " | 5 | −2 | −1 | 1 | 3 | | 3 |
| " | " | 9 | −6 | −1 | 2 | 5 | | 5 |

| D* | ν | D | Ω | σ | S | 2S + 1 | Generations (#GEN) | Number of candidate solutions |
|---|---|---|---|---|---|---|---|---|
| " | " | | | | … | | | |
| " | −3/2 | 3 | 3/4 | +1 | 1/2 | 2 | 3 | 9 = 3×#GEN |
| " | " | 4 | −3/4 | −1 | 1/2 | 2 | 3 | 12 = 4×#GEN |
| " | " | 6 | −15/4 | −1 | 3/2 | 4 | #GEN(3/2) | 6×#GEN |
| " | " | | | | … | | | |

~ ~ ~

This subsection discusses possibly useful concepts about generations.

Assume, for this subsection, that the number of generations equals the number of generators of SU(2S+1). The concept of generations pertains only for ν = −3/2 solutions. These solutions correlate with the limit $\eta^2 \rightarrow 0$. Generally, away from $\eta^2 \rightarrow 0$ or for solutions that correlate with fields, the number 2S + 1 provides a relevant factor regarding number of solutions. People might want to consider the extent to which implications exist for math-based models that correlate a number (2S + 1) of solutions that extend significantly away from r ≈ 0 with a number (the number of generators of SU(2S + 1)) of solutions that, in some sense, pertain for r = 0 only.

[Physics:] People might say that, perhaps, generations correlate with a particle-INTERN counterpart to aspects of particle-ENVIRO wave functions. (To compare with EXTINT LADDER models, see Section 2.14.)

~ ~ ~

This subsection suggests reasons for considering QE-like uses of radial-coordinate math.

Table 2.6.6 pertains.

**Table 2.6.6** Perspective regarding possible QE-like uses of DIFEQU models

1. Possible uses with models for the big bang or with models involving consequences of the big bang.
   1.1. [Physics:] The existence of the big bang correlates with a lower limit for the value of a coordinate people correlate with time.
   1.2. Use of radial coordinates for QE-like math correlates with models that can be compatible with the big bang.
2. Possible uses with models involving interaction vertices.
   2.1. [Physics:] People might say that some of our work regarding interaction vertices correlates with projecting QP-like solutions for which D* = 3 into functions for which D* = 1 in a way that also involves projecting QE-like solutions into functions for which D* = 1. (See, for example, Table 2.6.2.)

2.2. [Physics:] Some of our work regarding sizes of interaction vertices suggests that (for some vertices) a QE-like (or, temporal) size is zero. (See Section 2.10.) Perhaps such work correlates with models for which considering a lower limit (for particle creation) or upper limit (for particle destruction) regarding a temporal coordinate (or, a QE-like radial coordinate) is appropriate.

3. Possible uses regarding physics correlating with $\sigma = -1$.

3.1. [Physics:] Possibly, some considerations (correlating with physics for which $\sigma = -1$) correlate with more than one QE-like coordinate and, possibly therefore, with uses (within models) of QE-like radial coordinates.

3.1.1. Some of our work regarding $\sigma = -1$ phenomena correlates with symmetries we denote by (1;3)<. (See, for example, Table 2.13.33.)

3.1.2. These symmetries have similarities to the (1;3)> symmetries we correlate with various $\sigma = +1$ phenomena. (See, for example, Table 2.13.6.)

3.1.3. For $\sigma = +1$ phenomena for which (1;3)> symmetries pertain, use of QP-like radial coordinates can pertain.

3.2. Possibly, some uses of QE-like radial coordinates can pertain with regard to (1;3)< phenomena.

3.2.1. (See discussion pertaining to developing Table 2.13.33.)

3.2.2. (See discussion pertaining to developing Table 2.13.36.)

3.3. Possibly, some uses of QE-like radial coordinates can pertain regarding color charge.

3.3.1. People might say that (2.116) and (2.117) in Table 2.14.1 correlate with possible modeling involving the following.

3.3.1.1. #E + 1 QE-like radial coordinates.

3.3.1.2. Symmetries related to SU(#E + 1).

3.3.1.3. A relevant number of gluons or gluon-like analogs correlating with …

3.3.1.3.1. 1, for #E = 0.

3.3.1.3.2. 8, for #E = 2.

3.3.1.3.3. 25, for #E = 4.

4. Possible uses with models correlating with masses for W-, H-, and O-family elementary particles.

4.1. People might say that models correlating with masses for WHO-family elementary particles could be based on more than one QE-like coordinate. (See Table 2.10.7, Section 3.7, and Section 3.8.)

4.1.1. Here, possibly, each of some uses of negative contributions toward the square of a mass correlates with a pair of QE-like coordinates.

5. Possible uses correlating with ranges of the weak interaction.

5.1. People might say that masses for W-family bosons correlate with the various $R_0$ for W-family bosons and, therefore, correlate with the range of the weak interaction. (See, for example, Table 2.10.8.)

# Section 2.7 FERTRA and other models linking LADDER and DIFEQU models

Section 2.7 discusses correlations between LADDER models and DIFEQU models. We correlate LADDER solutions and DIFEQU solutions for the WHO-families of solutions. We discuss overlaps and gaps in the applicability of LADDER and DIFEQU models. We discuss the concept of composite particles. We show FERTRA LADDER solutions for the CN-families and for the QIRD-families.

~ ~ ~

This subsection discusses bridging between DIFEQU models and LADDER models.

Table 2.1.14 summarizes some applications for DIFEQU models and LADDER models. That table lists each of some applications more than once. This section discusses some overlaps between DIFEQU models and LADDER models.

~ ~ ~

This subsection starts to bridge between DIFEQU models and LADDER models for the WHO-families.

For $D^* = 3$ and $\nu = -1$, 2S equals an even integer (or, equivalently, S equals an integer).

Table 2.7.1 re-depicts, as LADDER ground-state solutions, DIFEQU ground states for $\nu = -1$ solutions that Table 2.5.16 shows. For each row in Table 2.7.1, each of the columns SUBF, S, Ω, and σ correlates with DIFEQU results. (See Table 2.5.16.) For each row in Table 2.7.1, each of the columns SUBF, S, and σ correlates with LADDER results. (See Table 2.3.1.) For each row in Table 2.7.1, the number of QP-like oscillators matches 2S + 1. Here, the symbol 'I denotes a 0. Uses of 'I correlate with instance-related harmonic oscillators. (See, for example, Table 2.13.4.) Here, the symbol 'G denotes a 0. Uses of 'G correlate with symmetries related to interactions with elementary fermions. A pair of 'G symbols correlates with an additional 3-generator (SU(2)-related) symmetry. [Physics:] People might say that the 3-generator symmetry related to the 2W-subfamily correlates with conservation of generation for interactions between leptons and 2W bosons. For example, in an interaction in which the emission of a $W^-$ boson converts a muon into a neutrino, the neutrino is a muon-neutrino and not, for example, an electron-neutrino. People consider muons to be generation-2 leptons. Muon-neutrinos are generation-2 leptons. We associate an occurrence of four 'G symbols with the symmetry that correlates with #GEN(3/2). (See Table 2.6.4 and Section 2.8.)

**Table 2.7.1** Ground states for D* = 3, ν = −1, and S ≤ 2

| E | E | E | E | E | E | E | P | P | P | P | P | P | P | P | P | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 6R | 6L | 4R | 4L | 2R | 2L | 0 | 0 | 2L | 2R | 4L | 4R | 6L | 6R | 8L | 8R | SUBF | S | Ω | σ |
| | | | | 'G | 'G | 'I | 0 | 0 | 0 | | | | | | | 2W | 1 | 2 | +1 |
| | | | | | | 'I | 0 | | | | | | | | | 0H | 0 | 0 | +1 |
| | | | | | | 0 | 'I | | | | | | | | | 0O | 0 | 0 | −1 |
| | | | | 0 | 0 | 0 | 'I | 'G | 'G | | | | | | | 2O | 1 | −2 | −1 |
| | | 0 | 0 | 0 | 0 | 0 | 'I | 'G | 'G | 'G | 'G | | | | | 4O | 2 | −6 | −1 |

~ ~ ~

This subsection discusses some differences between fermions and bosons.

[Physics:] In physics, a boson state can be excited an indefinite number of times.

Ground states, such as those Table 2.7.1 shows, seem appropriate.

[Physics:] In physics, a fermion state can be either not excited or excited exactly once.

~ ~ ~

This subsection discusses concepts relevant to FERTRA LADDER solutions.

It might seem inappropriate to try to depict some aspects of each fermion solution via the type of LADDER representations Table 2.3.11 shows or Table 2.3.14 shows. (See 1C, 1N, and 3N in Table 2.3.11. See 1Q, 1R, 3Q, 3I, 3R, and 3D in Table 2.3.14.)

People might say that subtracting 1 from each N(..) might lead to LADDER solutions that represent some aspects of fermions. Each value of N(..) would be −1 or −2. A lone oscillator for which N(..) = −1 cannot be excited to a state with N(..) = 0. A lone oscillator for which N(..) = −2 can be excited to a state with N(..) = −1. A lone oscillator for which N(..) = −1 could be de-excited to a state with N(..) = −2. Such considerations seem at least somewhat appropriate for fermions.

To develop FERTRA LADDER solutions, we make such subtractions. For leptons, we maintain Œ = 0 by opening QE-like oscillators and assigning values of N(..) = −1. (See Table 2.7.3.)

Table 2.7.2 pertains.

**Table 2.7.2** Concepts pertaining to FERTRA LADDER solutions

1. People might say that each FERTRA LADDER solution correlates with no or little more information than does the corresponding INTERN LADDER solution.

2. People might say that FERTRA LADDER solutions have some similarity to Y-family solutions.
    - 2.1. People might say that pairs of Y-family solutions correlate with nature. (See Section 3.6.)
        - 2.1.1. For example, a gluon correlates with a sum of terms, with each term correlating with a pair of Y-family solutions.
            - 2.1.1.1. One Y-family solution in the pair correlates with erasing a color charge from a quark.
            - 2.1.1.2. The other Y-family solution in the pair correlates with painting a color charge on to a quark.
    - 2.2. People might say that FERTRA LADDER solutions correlate with erasing some properties from fermions. (See Table 3.5.2, Table 3.5.3, and Table 3.5.5.)
    - 2.3. People might say that representations similar to FERTRA LADDER solutions correlate with painting some properties on to fermions. (See Table 3.5.4 and Table 3.5.5.)

~ ~ ~

This subsection discusses FERTRA LADDER solutions for the CN-families and the QIRD-families.

Table 2.7.3 shows some FERTRA LADDER solutions for the CN-families and the QIRD-families. We use the word some to allude to the need to develop similar results that include reversing, for each solution, N(P[#P − 1]) and N(P[#P]) and that include reversing, for each QIRD-family solution, N(E[#E]) and N(E[#E − 1]). Such reversals would double the number of C- and N-family rows and quadruple the number of QIRD-family rows.

**Table 2.7.3** Some FERTRA LADDER solutions for the CN-families and the QIRD-families

| σ | E 8R | E 8L | E 6R | E 6L | E 4R | E 4L | E 2R | E 2L | E 0 | P 0 | P 2L | P 2R | P 4L | P 4R | P 6L | P 6R | P 8L | P 8R | Subfamily |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| +1 | | | | | −1 | −1 | −1 | −1 | −1 | −1 | −1 | −2 | | | | | | | 1C |
| +1 | | | | | −1 | −1 | −1 | −1 | −2 | −2 | −1 | −2 | | | | | | | 1N |
| +1 | −1 | −1 | −1 | −1 | −1 | −1 | −1 | −1 | −1 | −2 | −1 | −1 | −1 | −2 | | | | | 3N |
| −1 | | | | | | | −2 | −1 | −1 | −1 | −1 | −2 | | | | | | | 1Q |
| −1 | | | | | | | −2 | −1 | −2 | −2 | −1 | −2 | | | | | | | 1R |
| −1 | | | | | −2 | −1 | −1 | −1 | −1 | −1 | −1 | −1 | −1 | −2 | | | | | 3Q |
| −1 | | | | | −2 | −1 | −2 | −2 | −1 | −1 | −2 | −2 | −1 | −2 | | | | | 3I |
| −1 | | | | | −2 | −1 | −1 | −1 | −2 | −2 | −1 | −1 | −1 | −2 | | | | | 3R |
| −1 | | | | | −2 | −1 | −2 | −2 | −2 | −2 | −2 | −2 | −1 | −2 | | | | | 3D |

~ ~ ~

This subsection shows a parameter that correlates with whether, for a LADDER solution, this monograph discusses a matching DIFEQU solution.

Table 2.7.4 discusses correlations between rest mass (m') and N(P0).

**Table 2.7.4** Correlations between m' and N(P0)

1. m' = 0, if …
    1.1. For INTERN LADDER solutions, …
        1.1.1. N(P0) ≤ −1.
    1.2. For FERTRA LADDER solutions, …
        1.2.1. N(P0) = −2.
2. m' ≠ 0, if …
    2.1. For INTERN LADDER solutions, …
        2.1.1. N(P0) ≥ 0.
    2.2. For FERTRA LADDER solutions, …
        2.2.1. N(P0) = −1.
3. [Physics:] m' denotes rest mass.
4. For elementary fermions, …
    4.1. This monograph provides FERTRA LADDER solutions that correlate with the extent to which elementary fermions interact with elementary bosons of the 2W-, 2O-, and 4O-subfamilies. (See Section 3.5.)

Table 2.7.5 pertains.

**Table 2.7.5** Correlations between solutions this monograph discusses and m', plus extrapolations from DIFEQU results for some solutions to similar results for other LADDER solutions

1. Regarding solutions for which m' ≠ 0, ...
    1.1. This monograph discusses LADDER solutions.
    1.2. This monograph discusses DIFEQU solutions.
2. Regarding solutions for which m' = 0, …
    2.1. This monograph discusses LADDER solutions.
    2.2. We may infer characteristics (that people might correlate with DIFEQU solutions) by extrapolating from DIFEQU results that correlate with solutions for which m' ≠ 0.

~ ~ ~

This subsection notes that this monograph includes, in some tables, listings for composite particles.

[Physics:] This monograph contains tables that list, in columns with labels such as subfamily, at least two more types of concepts (other than subfamily). The concepts include mesons and baryons. (The concepts also encompass tetraquarks, pentaquarks, and other possible composite particles having more than one quark.) This monograph does not develop solutions correlating with these listings. The listings correlate with composite particles and not with elementary particles. For mesons, 2S is an even nonnegative integer. For baryons, 2S is an odd nonnegative integer.)

# Section 2.8 Generation, color charge, and INTERN models for bosons

Section 2.8 discusses the extents to which elementary bosons impact, for elementary fermions, the properties of generation and color charge. We consider INTERN models correlating with G-family solutions. We discuss conservation of fermion generation for some interactions involving elementary fermions and G-family bosons. We consider INTERN models correlating with WHO-family solutions. We discuss conservation of fermion generation for some interactions involving elementary fermions and WHO-family bosons. We consider INTERN models correlating with Y-family solutions. We note a lack of conservation of fermion color charge for Y-family-related interactions involving fermions for which σ = −1.

~ ~ ~

This subsection shows ground-state INTERN LADDER solutions for the G-family.

Table 2.8.1 repeats (from Table 2.3.11) ground-state INTERN LADDER solutions for the G-family. Table 2.8.1 shows oscillators EG = E[16] through PG = P[16]. For each oscillator-related column other than columns E0 and P0, the column pertains to an oscillator pair. The symbol 00 correlates with (depending on the solution and the pair) values for the respective two N(..) of 0 and 0, 0 and @, @ and 0, or @ and @. Sometimes, the symbol @ denotes a 0 that does not change (from the ground-state value) for excited states correlating with the solution. Sometimes, the symbol @ denotes a 0 that does not change (from the ground-state value) for excited states correlating with a polarization mode that correlates with a solution. (See, for example, Table 3.2.2 and the explanation that precedes Table 3.2.2.) The case @-and-@ pertains for exactly (#P)/2 − |N(P0)| oscillator pairs.

**Table 2.8.1** G-family ground-state INTERN LADDER solutions

| E | E | E | E | E | E | E | E | E | P | P | P | P | P | P | P | P | P | Solution or |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| GF | ED | CB | A9 | 87 | 65 | 43 | 21 | 0 | 0 | 12 | 34 | 56 | 78 | 9A | BC | DE | FG | subfamily |
| | | | | | | | | 0 | −1 | 00 | | | | | | | | 022G2& |
| | | | | | | | 00 | 0 | −1 | 00 | 00 | | | | | | | 24..G |

| E GF | E ED | E CB | E A9 | E 87 | E 65 | E 43 | E 21 | E 0 | P 0 | P 12 | P 34 | P 56 | P 78 | P 9A | P BC | P DE | P FG | Solution or subfamily |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | 0 | −2 | 00 | 00 | | | | | | | 042G24& |
| | | | | | | 00 | 00 | 0 | −1 | 00 | 00 | 00 | | | | | | 46..G |
| | | | | | | | 00 | 0 | −2 | 00 | 00 | 00 | | | | | | 26..G |
| | | | | | | | | 0 | −3 | 00 | 00 | 00 | | | | | | 064G246& |
| | | | | | 00 | 00 | 00 | 0 | −1 | 00 | 00 | 00 | 00 | | | | | 68..G |
| | | | | | | 00 | 00 | 0 | −2 | 00 | 00 | 00 | 00 | | | | | 48..G |
| | | | | | | | 00 | 0 | −3 | 00 | 00 | 00 | 00 | | | | | 28..G |
| | | | | | | | | 0 | −4 | 00 | 00 | 00 | 00 | | | | | 084G2468& |

Table 2.8.2 restates Table 2.8.1. We convert each QE-like 00 to the symbol "G. We choose the symbol "G to correlate with the use of 'G for correlating with fermion generations. (This parallels use, regarding the 2W-subfamily, of 'G in Table 2.7.1.) Here, an instance of "G correlates with a pair of 'G. (Here, an occurrence of "G does not correlate with a notion of generations for bosons. The concept of generations does not pertain for bosons.) For a row in Table 2.8.2, the number of occurrences of "G correlates with the extent to which a boson (for which the row pertains) differentiates, when interacting with an elementary fermion, by generation of the fermion. For example, the electron, muon, and tauon correlate respectively with 3 generations of (in some sense) one fermion. 022G2& (or, a MM1MS1-photon) interacts equally strongly with the charge of an electron, muon, or tauon. For 022G2&, the table shows zero occurrences of "G. In contrast, 244G4& (or, a graviton) interacts more strongly with a tauon than with a muon and more strongly with a muon than with an electron. For 244G4&, the table shows one occurrence of "G. We denote the number of occurre-nces of "G by the symbol #"G.

**Table 2.8.2** G-family ground-state INTERN LADDER solutions, showing QE-like possible interaction-related symmetries

| E GF | E ED | E CB | E A9 | E 87 | E 65 | E 43 | E 21 | E 0 | P 0 | P 12 | P 34 | P 56 | P 78 | P 9A | P BC | P DE | P FG | Solution or subfamily |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | 0 | −1 | 00 | | | | | | | | 022G2& |
| | | | | | | | "G | 0 | −1 | 00 | 00 | | | | | | | 24..G |
| | | | | | | | | 0 | −2 | 00 | 00 | | | | | | | 042G24& |
| | | | | | | "G | "G | 0 | −1 | 00 | 00 | 00 | | | | | | 46..G |
| | | | | | | | "G | 0 | −2 | 00 | 00 | 00 | | | | | | 26..G |
| | | | | | | | | 0 | −3 | 00 | 00 | 00 | | | | | | 064G246& |
| | | | | | "G | "G | "G | 0 | −1 | 00 | 00 | 00 | 00 | | | | | 68..G |
| | | | | | | "G | "G | 0 | −2 | 00 | 00 | 00 | 00 | | | | | 48..G |
| | | | | | | | "G | 0 | −3 | 00 | 00 | 00 | 00 | | | | | 28..G |
| | | | | | | | | 0 | −4 | 00 | 00 | 00 | 00 | | | | | 084G2468& |

~ ~ ~

This subsection interprets aspects correlating with G-family INTERN LADDER solutions.

People might say that Table 2.8.3 correlates data with solutions. The leftmost five columns reflect work in this monograph. The S column shows spins. The SDI column shows spatial dependences of interactions. (See Table 2.13.23, Table 3.3.10, and Table 3.3.11.) Here, we count each "G symmetry independently from any other "G symmetry that pertains. We try to correlate known phenomena with G-family particles. Each of the electron, muon, and tauon has the same charge. (People might say that, for purposes of this table, we do not need to take into consideration anomalous magnetic dipole moments.) The rightmost column notes that, for electromagnetism, the strength of the interaction does not vary by generation of the lepton involved. The rightmost column notes that, for gravitation, the strength of the interaction varies by generation of the lepton involved. Each of the electron, muon, and tauon has a different mass. We think that, somewhat paralleling work regarding the weak interaction, the case in which exactly one "G pertains correlates with variation by generation for interaction vertices involving spin-1/2 fermions. (See discussion regarding Table 2.7.1.) We think that work, so far, covers electromagnetic interactions with stationary and moving charges but not necessarily electromagnetic interactions with elementary-particle magnetic dipole moments. Presumably, a boson interacting with an elementary-particle magnetic dipole moment carries information about the spin of the elementary particle. People might say that 2G2& cannot perform that function. We seek a particle for which % has at least 2 elements, spin-1 pertains, an interaction with a spatial dependence of $r^{-4}$ pertains, and #"G is an even number. That spatial dependence matches characteristics of magnetic dipoles. So far, 2G24& and 2G68& might qualify. For 2G24&, $2 \in$ %. We think that aspects related to 2G24& can scale to describe magnetic dipole moments correlating with spins of systems of multiple spin-entangled charged particles. For 2G68&, $2 \notin$ %. We think that 2G68& couples with spin but not with charge. We correlate 2G24& with vertices in which electromagnetism interacts with the magnetic dipole moments of elementary particles.

**Table 2.8.3** Interaction-related phenomena, G-family bosons that intermediate the interactions, and numbers of "G symmetries

| G-family particle (%68both, per Table 3.3.3) | S | SDI | Subfamily or solution (per Table 2.8.2) | Number of "G (per Table 2.8.2) | Known phenomena | Known variation by fermion generation (for charged leptons) |
|---|---|---|---|---|---|---|
| 2G2& | 1 | $r^{-2}$ | 022G2& | 0 | Electromagnetic interaction | No |
| 4G4& | 2 | $r^{-2}$ | 24..G | 1 | Gravitational interaction | Yes |

| G-family particle (%68both, per Table 3.3.3) | S | SDI | Subfamily or solution (per Table 2.8.2) | Number of "G (per Table 2.8.2) | Known phenomena | Known variation by fermion generation (for charged leptons) |
|---|---|---|---|---|---|---|
| 2G24& | 1 | $r^{-4}$ | 042G24& | 0 | Charged-lepton magnetic dipole moment | No |
| 4G26& | 2 | $r^{-4}$ | 26..G | 1 | | |
| 2G46& | 1 | $r^{-4}$ | 26..G | 1 | | |
| 2G246& | 1 | $r^{-6}$ | 064G246& | 0 | | |
| 4G48& | 2 | $r^{-4}$ | 48..G | 2 | | |
| 2G68& | 1 | $r^{-4}$ | 48..G | 2 | | |
| 2G248& | 1 | $r^{-6}$ | 28..G | 1 | | |
| 4G268& | 2 | $r^{-6}$ | 28..G | 1 | | |
| 2G468& | 1 | $r^{-6}$ | 28..G | 1 | | |
| 4G2468& | 2 | $r^{-8}$ | 084G2468& | 0 | | |

Table 2.8.4 generalizes patterns from Table 2.8.3. We assume that the appearance in a Table 2.8.2 row of two "G symbols correlates with #GEN(3/2) (and, possibly with SU(4) symmetries) and elementary fermions for which 2S = 3. The symbol #"G denotes the relevant number of "G symbols.

**Table 2.8.4** Assumptions about generation-based variation of elementary-fermion properties, regarding G-family interactions

| #"G | Property (or, vertex-strength) variation by fermion generation (for spin-1/2 elementary fermions) | Property (or, vertex-strength) variation by fermion generation (for spin-3/2 elementary fermions) |
|---|---|---|
| 0 | No | No |
| 1 | Yes | No |
| 2 | No | Yes |
| 3 | Yes | Yes |

~ ~ ~

This subsection discusses two distinct sets of models, each of which might pertain to G-family elementary particles.

This monograph notes possibilities for EACUNI models and for SOMMUL models for aspects of G-family elementary particles. (See, for example, Table 2.13.11 and Table 3.3.3.)

People might say that, for EACUNI models, results (in Table 2.8.3 and Table 2.8.4) related to #"G correlate with some known physics and are not incompatible with known physics. People might say that results (in Table 2.8.3 and Table 2.8.4) related to #"G do not correlate with the combination of known physics and SOMMUL models.

This monograph discusses some possibilities regarding SOMMUL models. This monograph does not try to model an analog (for SOMMUL models) to #"G. This monograph deemphasizes SOMMUL models.

~ ~ ~

This subsection points to aspects correlating with WHO-family INTERN LADDER solutions.

Table 2.7.1 shows information about ground-state WHO-family INTERN LADDER solutions. (People might say that, regarding that table, technically, the column labelled Ω might not pertain to LADDER solutions.) Table 2.8.5 pertains.

**Table 2.8.5** Values of #"G for WHO-family elementary bosons

| Subfamily | #"G |
|---|---|
| 2W | 1 |
| 0H | 0 |
| 0O | 0 |
| 2O | 1 |
| 4O | 2 |

~ ~ ~

This subsection interprets aspects correlating with WHO-family INTERN LADDER solutions.

Table 2.8.6 pertains.

**Table 2.8.6** Aspects regarding generation-based variation of elementary-fermion properties, regarding interactions with WHO-family bosons

1. Regarding interactions in FRERAN environments, …
    1.1. Each interaction vertex that includes a spin-1/2 elementary fermion and a 2W boson features …
        1.1.1. Measuring the generation of the fermion.
        1.1.2. Conserving the generation of the fermion.
    1.2. Each interaction vertex comports with results from Table 2.8.4 and Table 2.8.5.
2. Regarding interactions in COMPAR environments, …
    2.1. For interactions involving a pair of spin-1/2 elementary fermions and a pair of 2W bosons, …
        2.1.1. At least one fermion can change generation.
    2.2. We are uncertain as to the extent that …

2.2.1. Each interaction vertex that includes a spin-1/2 elementary fermion and a 2O boson features …

2.2.1.1. Measuring the generation of the fermion.

2.2.1.2. Conserving the generation of the fermion.

2.2.2. Some interaction vertices might comport with results from Table 2.8.4 and Table 2.8.5.

~ ~ ~

This subsection interprets aspects correlating with Y-family INTERN LADDER solutions.

Table 2.8.7 summarizes relevant information from Table 2.3.14.

**Table 2.8.7** Y-family ground-state INTERN LADDER solutions

| E | E | E | E | E | E | E | P | P | P | P | P | P | P | P | P | (σ = −1) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 6R | 6L | 4R | 4L | 2R | 2L | 0 | 0 | 2L | 2R | 4L | 4R | 6L | 6R | 8L | 8R | Subfamily |
| | | | | .. | .. | .. | −1 | −1 | −1 | | | | | | | 2Y |
| | | .. | .. | .. | .. | .. | −1 | −1 | −1 | −1 | −1 | | | | | 4Y |

Table 2.8.8 reinterprets Table 2.8.7. In Table 2.8.8, each occurrence of the symbol 'C correlates with a QP-like occurrence of −1 in Table 2.8.7. Regarding Table 2.8.8, we posit that #P + 1 correlates with the number of color charges for relevant elementary fermions. (See, for example, Table 2.14.1 and Section 3.6.)

**Table 2.8.8** Reinterpretation of Y-family ground-state INTERN LADDER solutions

| E | E | E | E | E | E | E | P | P | P | P | P | P | P | P | P | (σ = −1) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 6R | 6L | 4R | 4L | 2R | 2L | 0 | 0 | 2L | 2R | 4L | 4R | 6L | 6R | 8L | 8R | Subfamily |
| | | | | .. | .. | .. | 'C | 'C | 'C | | | | | | | 2Y |
| | | .. | .. | .. | .. | .. | 'C | 'C | 'C | 'C | 'C | | | | | 4Y |

~ ~ ~

This subsection interprets aspects correlating with Y-family INTERN LADDER solutions.

Table 2.8.9 pertains.

**Table 2.8.9** Lack of conservation of color charge

1. Y-family solutions differ from G-family and WHO-family solutions in the following regard.

1.1. Each G-family solution does or might correlate with an elementary particle.

    1.2. Each WHO-family solution does or might correlate with an elementary particle.
    1.3. Each elementary particle that correlates with Y-family solutions correlates with a sum of terms, with each term being comprised of a product of two constructs, each correlating with a Y-family solution. One such solution correlates with erasing a color charge (or color-charge analog) from a spin-1/2 (or spin-3/2, respectively) fermion. The other such solution correlates with painting a color charge (or, color-charge analog).
2. Y-family solutions differ from G-family and WHO-family solutions in the following regard.
    2.1. For each G-family solution, the notion of 'G does not pertain to the E0 oscillator.
        2.1.1. Here, $\sigma = +1$.
    2.2. For each WHO-family solution, the notion of 'G does not pertain to the E0 oscillator (if $\sigma = +1$) and does not pertain to the P0 oscillator (if $\sigma = -1$).
    2.3. For each Y-family solution, the notion of 'C pertains regarding the P0 oscillator.
        2.3.1. Here, $\sigma = -1$.
3. People might say that such thinking does not correlate with a notion, for the Y-family, of conservation of color charge.
    3.1. People might say that the case of leptons provides an exception. (See Table 2.14.1.)

~ ~ ~

This subsection notes a possible lack of correlation between generation and color charge.

People might say that nature exhibits, for elementary fermions for which $\sigma = -1$, no correlation between generation and color charge.

# Section 2.9 Schwarzschild radius, Planck length, and $R_0$

Section 2.9 discusses a series of lengths. The series features the Schwarzschild radius, the Planck length, and a length we call $R_0$. People might say that we show physics-relevance for $R_0$.

~ ~ ~

This subsection provides perspective about pattern recognition.

People turn perceived patterns into bases for models and theories. To some extent, the more elements that people think correlate with a possible pattern, the more confidence people might have that people can develop useful models.

Sometimes, even if the number of elements in a pattern is small, people can develop useful models.

In this section, we discuss patterns regarding small numbers of elements. We think the patterns point to useful results. Table 2.9.1 pertains.

**Table 2.9.1** Types of patterns this section discusses

1. A series of formulas, with each formula based on a product of powers of factors common to all the formulas.
   1.1. Most of the factors are physics numbers..
2. A series of lengths with which the previous series correlates.
   2.1. Here, we look for physics phenomena that people might correlate with the lengths.

~ ~ ~

This subsection provides perspective about the Schwarzschild radius, the Planck length, and the rest mass of electrons.

In traditional classical physics pertaining to black holes, people ascribe significance to a distance people call the Schwarzschild radius. People say that (absent quantum effects) energy and matter that exist closer to the center of a (spherically symmetric, non-rotating) black hole than the Schwarzschild radius cannot escape from the black hole. (For this discussion, we ignore possible dissipation of the black hole. We discuss black-hole dissipation in Section 4.6.) A formula for that radius involves the so-called mass of the black hole. The formula does not involve ħ.

In traditional quantum physics, people try to attribute significance to a distance people call the Planck length. The length is approximately $1.6\times10^{-35}$ meters. The constant ħ appears in a formula for that length. That formula does not contain a factor correlating with the mass of anything. People correlate ħ/2 with a minimum unit of spin.

In this monograph, the rest mass of an electron is the smallest rest mass for a known non-zero-mass elementary particle and may be the smallest rest mass for any non-zero-mass elementary particle. (We assume that MM1MS1-neutrinos have rest masses of zero.)

~ ~ ~

This subsection defines a series of lengths that includes the Schwarzschild radius and the Planck length.

Table 2.9.2 defines a series of lengths that includes the Schwarzschild radius and the Planck length.

**Table 2.9.2** A series of lengths that includes the Planck length and a length derived by applying the formula for the Schwarzschild radius to the mass of an electron

1. The next equation notes a traditional physics length, the Planck length. This work provides a symbol, $R_2$, for that length. Here, $G_N$ denotes the gravitational constant, ħ denotes Planck's constant (reduced), and c denotes the speed of light. This work adds to the traditional statement two factors, each of value 1. The first such factor is $m_e^0$. The symbol $m_e$ denotes the rest mass of an electron. The second such factor is $2^0$.
$$R_2 = G_N^{1/2}\, m_e^0\, \hbar^{1/2}\, c^{-3/2}\, 2^0 \tag{2.50}$$
2. The next equation applies a traditional formula to a property (rest mass) associated with electrons. The formula represents the Schwarzschild radius. Traditionally, people apply the Schwarzschild-radius formula to black holes. Traditionally, people do not apply the formula to objects people claim have not enough mass to form black holes. This work adds to the traditional statement one factor with value 1. That factor is $\hbar^0$.
$$R_4 = G_N^1\, m_e^1\, \hbar^0\, c^{-2}\, 2^1 \tag{2.51}$$
3. The next equation shows a ratio of the above two lengths.
$$Z = R_2/R_4 = G_N^{-1/2}\, m_e^{-1}\, \hbar^{1/2}\, c^{1/2}\, 2^{-1} \approx 1.1945\times10^{22} \tag{2.52}$$
4. The next equation defines a series of lengths.
$$R_j = R_2\, Z^{(2-j)/2}, \text{ with j being an even integer} \tag{2.53}$$
5. The next equation provides a formula for $R_0$.
$$R_0 = G_N^0\, m_e^{-1}\, \hbar^1\, c^{-1}\, 2^{-1} \tag{2.54}$$
6. The next equation restates the previous item.
$$R_0\, m_e c^2 = \hbar^1\, c^1\, 2^{-1} \tag{2.55}$$

Table 2.9.3 shows factors and values (for electrons and positrons) for a series of lengths. These approximate lengths are the products of the factors indicated by the five columns having labels k for $l^k$ (for some l). Times are computed via time = length/c. The time-centric column shows the log-base-10 of times (in seconds). The time since the big bang is $\sim10^{17.6}$ seconds. The j column values indicate possibly interesting correlations between items and G-family solutions ejjGj&. (Elsewhere, this monograph does not use the notion of a 000G0& solution. Because each would correlate with S > 2, elsewhere this monograph deemphasizes ejjGj& solutions for which j > 4.) The j' column values indicate possibly interesting correlations between items and properties of objects. (See Table 3.13.2.) In effect, j' = j + 2.

**Table 2.9.3** A series of lengths relevant to electrons and positrons

| j | $R_j$ | Length (m) | $Log_{10}$ (time (sec)) | Concept | k for $G_N^k$ | k for $m_e^k$ | k for $\hbar^k$ | k for $c^k$ | k for $2^k$ | j' |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $3.3\times10^{53}$ | +45 | | −1.5 | −4 | 2.5 | 0.5 | −4 | |
| | | $2.7\times10^{31}$ | +23 | | −1 | −3 | 2 | 0 | −3 | |
| | | $2.3\times10^{9}$ | +0.88 | | −0.5 | −2 | 1.5 | −0.5 | −2 | |
| 0 | $R_0$ | $1.9\times10^{-13}$ | −21 | $R_0$ | 0 | −1 | 1 | −1 | −1 | 2 |
| 2 | $R_2$ | $1.6\times10^{-35}$ | −43 | Planck length | 0.5 | 0 | 0.5 | −1.5 | 0 | 4 |
| 4 | $R_4$ | $1.4\times10^{-57}$ | −65 | Schwarzschild radius | 1 | 1 | 0 | −2 | 1 | 6 |
| 6 | $R_6$ | $1.1\times10^{-79}$ | −87 | | 1.5 | 2 | −0.5 | −2.5 | 2 | 8 |
| 8 | $R_8$ | $9.5\times10^{-102}$ | −109.5 | | 2 | 3 | −1 | −3 | 3 | |
| | | $7.9\times10^{-124}$ | −132 | | 2.5 | 4 | −1.5 | −3.5 | 4 | |

~ ~ ~

This subsection discusses possible significances of lengths $R_0$.

Possibly, the formula for $R_0$ pertains to other than electrons and positrons. For pions, $R_0$ may have significance. For Z and W bosons, $R_0$ may have significance. For the Higgs boson, $R_0$ may have significance. Table 2.9.4 pertains.

**Table 2.9.4** Concepts indicating possible significance for lengths $R_0$

1. For a non-zero mass particle pp, this formula defines $R_0(pp)$.

$$R_0(pp)\ (m'(pp))c^2 = \hbar c/2 \tag{2.56}$$

   1.1. m' denotes rest mass.
2. The charged-pion $R_0$ approximates the charged-pion charge radius.
   2.1. A charged-pion $R_0$ would be $\sim0.70\times10^{-15}$ meters.
      2.1.1. Here, we substitute, in the formula for $R_0$(electron), m'(charged pion) for $m_e$.
      2.1.2. The charged-pion $R_0$ is a factor ~139.6 / 0.511 or ~273.2 smaller than $R_0$ for electrons.
   2.2. An experimental charge radius for charged pions is $0.78\,^{+0.09}_{-0.10}\times10^{-15}$ meters. (See G. T. Adylov (1977).)

3. $R_0$ for weak-interaction bosons may correlate with a range of the weak interaction.
    - 3.1. A Z-boson $R_0$ is $\sim 2\times10^{-18}$ meters.
        - 3.1.1. A W-boson $R_0$ is $\sim 1.1\ R_0$(Z boson).
    - 3.2. People measure spatial dependence for interactions mediated by the weak interaction.
        - 3.2.1. For a separation of $\sim 10^{-18}$ meters between two interacting particles, the weak interaction and the electromagnetic interaction have similar magnitudes. (See Particle Data Group (2014).)
        - 3.2.2. At a separation of $\sim 3\times10^{-17}$ meters, the weak interaction is less by approximately a factor of $10^4$. (See Particle Data Group (2014).)
4. These formulas pertain for the Higgs boson. (See remarks pertaining to Table 3.7.3 and see Table 3.7.4.)

$$(m'(\text{Higgs})c^2)^2 = (1/4)\ (\hbar c)^2\ /\ (R_0(m'(\text{Higgs})))^2 \tag{2.57}$$

$$\xi'/2 = (1/4)\ (\hbar c)^2\ /\ (R_0(m'(\text{Higgs})))^2 \tag{2.58}$$

~ ~ ~

This subsection shows values of $R_0$ for various particles.

Table 2.9.5 extends concepts in Table 2.9.3 and Table 2.9.4.

**Table 2.9.5** Some values of $R_0$, for various particles

| Particle | $R_0$(particle) |
|---|---|
| MM1MS1-photon | undefined or infinite |
| MM1MS1-neutrino | undefined or infinite |
| electron | $\sim 2\times10^{-13}$ m |
| pion | $\sim 7\times10^{-16}$ m |
| proton | $\sim 1\times10^{-16}$ m |
| Z boson | $\sim 2\times10^{-18}$ m |
| Higgs boson | $\sim 8\times10^{-19}$ m |

~ ~ ~

This subsection posits possibilities for research regarding $R_0$ for various particles.

We do not much discuss further herein, some possible lines of inquiry that Table 2.9.6 notes. Regarding uncertainty, in traditional physics, uncertainty relationships often pertain to a superposition of amplitudes. Much of the work in this monograph focuses on base states.

**Table 2.9.6** Possible lines of inquiry related to the concept of $R_0$ for elementary particles

1. To what extent does $R_0$ for a weak-interaction boson correlate with a size that characterizes a cloud of virtual particles that accompanies the boson? (See Table 2.10.1.)
2. To what extent does $R_0$ for an electron have significance?
   2.1. For example, to what extent does $R_0$ for electrons correlate with a size that characterizes the cloud of virtual particles that accompany an electron?
3. At least from a standpoint of theory, for zero-mass bosons, to what extent does an unbounded $R_0$ have significance?
   3.1. For example, to what extent might an unbounded $R_0$ correlate with some concept of (perhaps field-related) interconnectedness, for example, of particles correlating with gravity? Or, of photons? Or, of all G-family particles?
4. Regarding the following relationship, for non-zero-mass elementary bosons (See Section 2.10. See, for example, Table 2.10.9.), …

$$\langle t^2 \rangle (m'c^2)^2 \sim \hbar^2/4 \tag{2.59}$$

   4.1. To what extent does the relationship …
      4.1.1. Generalize?
      4.1.2. Correlate with a notion of uncertainty?
      4.1.3. Correlate with a notion of certainty?
   4.2. People might say that, here, $(\langle t^2 \rangle)^{1/2}$ correlates with a lifetime for the boson.
5. To what extent should people consider that, for non-zero-mass elementary particles and for composite particles, an uncertainty-like relationship for which lengths exceed the Planck length ($\sim 1.6 \times 10^{-35}$ m) pertains?
6. To the extent there is a meaningful such uncertainty-like relationship (regarding, at least, non-zero-mass elementary bosons), to what extent should people consider that this uncertainty-like relationship correlates with sums of some kind of base states that correlate with a concept (perhaps related to internal particle properties or to interaction-centric characteristics) that people might explore further?
7. To what extent should people consider that possible relevance of COMPAR models (See Section 2.13.) for phenomena that correlate with scale size $R_0$ augurs poorly (or well) for attempts to build theories based on possible phenomena for which the scale size $R_2$ (the Planck length) would be key?

~ ~ ~

**Reference 2** G. T. Adylov (1977)
G. T. Adylov, et. al., A measurement of the electromagnetic size of the pion from direct elastic pion scattering data at 50 GeV/c, *Nuclear Physics B*, Volume 128, Issue 3, 3 October 1977, pages 461-505. (http://dx.doi.org/10.1016/0550-3213(77)90056-6)

**Reference 3** Particle Data Group (2014)
Particle Data Group, Electroweak (web page), The Particle Adventure, Lawrence Berkeley National Laboratory, (2014), http://www.particleadventure.org/electroweak.html.

# Section 2.10 Models related to vertices and to particle sizes, masses, and ranges

Section 2.10 discusses models related to sizes, masses, and ranges of elementary particles. We discuss clouds of virtual particles. We discuss MM1MS1-currents related to energy and momentum. We discuss interactions between gravity and MM1MS1-currents related to energy and momentum. We discuss aspects of models for interaction vertices (or, for vertices in Feynman diagrams). People might say that our work correlates with notions that, in some models, elementary particles have sizes of zero. We point to models and integers related to actual or approximate masses of elementary bosons. People might say that we show models that correlate with the spatial ranges of elementary bosons.

~ ~ ~

This subsection discusses aspects of people's thoughts about sizes of elementary particles.

People discuss possibilities that elementary particles have zero size. People discuss possibilities that elementary particle have non-zero sizes that correlate with the Planck length. People discuss the extent to which wave-like models may correlate with sizes relevant to particle-like models.

Perhaps, one interpretation of particle size correlates with models that address the topic of the extent (with respect to QE-like {or, temporal} coordinates and QP-like {or, spatial} coordinates) of interaction vertices.

In this section, we focus on some aspects of size related to interactions in which particles partake.

~ ~ ~

This subsection discusses aspects of interaction vertices.

In traditional quantum physics, people model interactions between (for example) elementary fermions and elementary bosons. A Feynman diagram may depict, for example, an interaction vertex in which an electron comes in and a neutrino and a $W^-$ boson leave. For such vertices, the net number of entering fermions equals the net number of exiting fermions. Here, net denotes the number of matter fermion particles minus the number of antimatter fermion antiparticles.

~ ~ ~

This subsection provides perspective about this section.

This section discusses models for some aspects of how elementary particles interact. We suggest roles for clouds of virtual particles. Traditional physics provides concepts regarding such clouds.

This section provides results we later use for modeling approximate ratios of masses of non-zero-mass elementary bosons. (See Section 3.7 and Section 3.8.) As far as we know, traditional physics models do not estimate the ratios we approximate.

Some math this section shows pertains to models people use in traditional physics. Some of our applications of that math are non-traditional.

~ ~ ~

This subsection discusses the concept of clouds of virtual particles.

Table 2.10.1 discusses the concept of clouds of virtual particles and discusses some uses of the terms E and P.

**Table 2.10.1** The concept of clouds of virtual particles, plus some uses of the terms E and P

1. [Physics:]
   - 1.1. For an elementary particle (or any object) for which $\sigma = +1$ and $m' \neq 0$ pertain, traditional quantum physics provides that …
     - 1.1.1. The concept of a cloud of virtual particles that accompany any $m' \neq 0$ elementary particle or any object pertains.
     - 1.1.2. The following equation pertains.

$$E^2 - c^2P^2 = (m')^2c^4 \tag{2.60}$$

     - 1.1.3. The value of E measured by one observer need not equal the value of E measured by another observer.
   - 1.2. We assume that E and P include effects of the traditional quantum-physics concept of such a cloud of virtual particles.

1.3. People might say that the following pertain.
   1.3.1. For $\sigma = +1$ and $m' = 0$, …
      1.3.1.1. The term rest (as in not moving) in the phase rest mass does not apply.
      1.3.1.2. This monograph makes the following uses of the terms E and P. (See Table 2.1.1.)
         1.3.1.2.1. E = energy.
         1.3.1.2.2. P = the magnitude of the momentum.
      1.3.1.3. Here, the following equations pertain.

$$E = cP \tag{2.61}$$

$$E^2 - c^2P^2 = (m')^2c^4 \tag{2.62}$$

   1.3.2. The value of E measured by one observer need not equal the value of E measured by another observer.
   1.3.3. This monograph correlates E and P with the concept of a cloud of virtual particles.
1.4. For $\sigma = -1$, …
   1.4.1. Questions such as those Table 6.3.2 mentions may pertain.
   1.4.2. People might say that distinguishing FRERAN SPATIM models from COMPAR SPATIM models may help regarding addressing such questions.

2. [Physics:]
   2.1. People might say that, regarding traditional quantum physics, models for aspects of clouds of virtual particles generally correlate with …
      2.1.1. Invariances and symmetries compatible with special relativity.

~ ~ ~

This subsection discusses notions of currents.

In electromagnetism, people, when discussing motions of charged objects, may use the term current. People might use the term electric current.

More generally, physics applies the word current to a combination of the motion of an object and a property that object exhibits. For currents that people model via 4-vectors, a name for the current may correlate with the name for the property people associate with the QE-like component of the vector for an object that (relative to an observer) is not moving. The case of electric charge provides an example. People might use the term charge current.

Below, we consider other currents. For example, we consider a current correlating with the energy-momentum 4-vector pertaining (relative to an observer) to motion of energy related to an object.

~ ~ ~

This subsection discusses gravitational interactions with elementary particles and objects.

Table 2.10.2 extends discussion from Table 2.10.1.

**Table 2.10.2** Energy-momentum MM1MS1-currents, rest mass m', and gravity

1. [Physics:]
   1.1. We posit that gravity interacts with energy-momentum MM1MS1-currents.
      1.1.1. Our interpretation correlates with gravity's interacting with light via the energy and momentum associated with clouds of virtual particles associated with photons.
      1.1.2. People might say that our interpretation does not correlate with the notion that gravity interacts with rest mass.
   1.2. The following statements pertain. (See, for example, Table 2.7.4.)
      1.2.1. Regarding some σ = +1 elementary particles for which m' = 0, …
         1.2.1.1. Interactions between a MM1MS1-photon and gravity correlate with E and P (and the relationship E = cP) for, at least, the MM1MS1-photon.
            1.2.1.1.1. We use the term at least, because an interaction between a MM1MS1-photon and a graviton presumably correlates also with E and P for the graviton.
         1.2.1.2. Interactions between a MM1MS1-neutrino and gravity correlate with E and P (and the relationship E = cP) for, at least, the MM1MS1-neutrino.
      1.2.2. For all σ = +1 elementary particles for which m' = 0, …
         1.2.2.1. Interactions between a particle and gravity correlate with E and P (and the relationship E = cP) for, at least, the particle.
      1.2.3. For σ = +1 elementary particles for which m' ≠ 0 specifically and for σ = +1 objects for which m' ≠ 0 generally, …
         1.2.3.1. Strengths of interactions between an elementary particle or other object and gravity correlate with E and P for the particle or object.
      1.2.4. See, for example, Section 4.6.

~ ~ ~

This subsection discusses the possibility that some MM1MS1-currents correlate with tensors of rank greater than or equal to 2.

In traditional physics, gravity interacts with rest mass or RESENE. People might say that RESENE is a scalar quantity. People might say that this scalar quantity correlates with a rank-0 tensor.

In general relativity, effects of gravity correlate with a stress-energy tensor of rank-2. General relativity correlates that stress-energy tensor with curvature of space-time.

People might say that photons interact with a 4-vector charge current.

Above, we discuss the concept that gravity interacts with an energy-and-momentum 4-vector current. This MM1MS1-current correlates with a rank-1 tensor.

Later, we discuss matters related to curvature (and, thereby, possibly to rank-2 tensors). (See for example, Section 6.4. See, also, Table 6.1.5 and Table 6.1.6.)

~ ~ ~

This subsection discusses traditional quantum-physics notions of interaction vertices.

When working with quantum models for interactions between elementary particles, people may use techniques people correlate with the term Feynman diagrams. People model a specific vertex of an interaction as involving entering particles and exiting particles. The entering particles and their quantum states pertain before the specific vertex. The exiting particles and their quantum states pertain after the specific vertex. Away from and at vertices, the existence and motions of particles correlate with lines or other somewhat linear symbols.

An overall interaction can involve more than one vertex. For example, people envision an interaction in which an electron enters and a neutrino (more precisely, an electron-neutrino) exits as having 2 vertices. For example, at one vertex, the electron disappears, a neutrino appears, and a $W^-$ both gets created and carries off a unit of negative charge. At another vertex, the $W^-$ disappears and something else (for example, conversion of an up quark to a down quark) occurs.

For some such interactions, people diagram each vertex by using a point.

~ ~ ~

This subsection discusses notions regarding waves, particles, and sizes of elementary particles.

People might correlate the notion of an interaction vertex with notions of particles. Away from vertices, people might use terms such as particle, field, and/or wave.

We prefer to deemphasize some aspects of such discussion.

People might say that this monograph shows models for which notions of particle and zero-size correlate with interaction vertices.

~ ~ ~

This subsection discusses models that correlate with sizes that might correlate with interaction vertices.

Table 2.10.3 pertains.

**Table 2.10.3** Some correlations between $\eta^2 \to 0$ and sizes associated with interaction vertices

1. For non-zero-mass elementary fermions, …
   1.1. Work above correlates particles with DIFEQU solutions for which $\eta^2 \to 0$ pertains.
   1.2. People might say that models that require $\eta^2 \to 0$ correlate with the notion that, at least in the sense of the model, non-zero-mass elementary fermions have zero spatial size.
   1.3. People might say that Table 2.10.4 provides an alternative approach that leads to a model-based conclusion of zero-spatial size.
   1.4. People might say that a notion of zero size dovetails with concepts people traditionally correlate with models for interaction vertices.

Table 2.10.4 derives the result $\langle r^2 \rangle = 0$ for some solutions for which $\nu = -3/2$ and $D^* = 3$. Also, the table derives the result $\langle r^2 \rangle = 0$ for some solutions for which $\nu = -1$ and $D^* = 2$. (See Table 2.6.1.) Here, models feature $\eta^2 \to 0$.

**Table 2.10.4** $\langle r^2 \rangle = 0$ for some $\nu = -3/2$, $D^* = 3$ solutions and $\langle r^2 \rangle = 0$ for some $\nu = -1$, $D^* = 2$ solutions

1. This result follows from Table 2.5.6.
$$\xi = (\xi'/2)\,(\,\eta^2\langle p_r^2\rangle + \eta^{-2}\langle r^2\rangle\,) \quad (2.63)$$
   1.1. Here, $\langle j \rangle$ denotes the expected value of j.
   1.2. Here, $p_r^2$ denotes $r^{-(D-1)}(\partial/\partial r)(r^{D-1})(\partial/\partial r) - \Omega r^{-2}$.
2. We assume $\xi' \neq 0$.
3. This result follows from Table 2.5.8.
$$D + 2\nu = \eta^2\langle p_r^2\rangle + \eta^{-2}\langle r^2\rangle \quad (2.64)$$
4. For 1C-subfamily particles, …
   4.1. Models feature $D^* = 3$ and $\nu = -3/2$.
   4.2. $D = 3$. (See the $\nu = -3/2$ row in Table 2.6.5.)
   4.3. $D + 2\nu = 0$.
   4.4. We can consider each of $\eta^2$, $\langle p_r^2\rangle$, $\eta^{-2}$, and $\langle r^2\rangle$ to be non-negative.
   4.5. The next equation pertains.
$$\langle r^2\rangle = 0 \quad (2.65)$$
5. For G-family particles, …
   5.1. Models correlate with $D^* = 2$ and $\nu = -1$.

5.2. All uses (with respect to $D^* = 2$) of $D + 2\nu$ feature $D + 2\nu = 0$. (See Section 3.3.)

5.3. Somewhat similarly to results for 1C-subfamily particles, …

$$\langle r_2{}^2 \rangle = 0 \tag{2.66}$$

5.3.1. Here, we use the symbol $r_2$ to call attention to the notion that this radial coordinate correlates with $D^* = 2$ (or, 2 dimensions).

People might say that, here, $(\langle r^2 \rangle)^{1/2}$ correlates with a size for an interaction vertex. People might say that, here, $(\langle r^2 \rangle)^{1/2}$ correlates with a size for an elementary particle.

Table 2.10.5 restates and extrapolates results from Table 2.10.4.

**Table 2.10.5** Zero-length sizes for some aspects of some interaction vertices

1. $\langle r^2 \rangle = 0$ pertains for interaction vertices for leptons for which $m' \neq 0$.
2. $\langle r_2{}^2 \rangle = 0$ pertains for interaction vertices for the 2 dimensions that an observer would say are perpendicular to the direction of motion of a G-family boson.
   2.1. Here, we use the symbol $r_2$ to call attention to combination of notions that …
      2.1.1. This radial coordinate correlates with $D^* = 2$ (or, 2 dimensions).
      2.1.2. For the vertex, $D^* = 3$.

~ ~ ~

This subsection discusses some types of interaction vertices, including vertices representing interactions between electrons and MM1MS1-photons.

We consider the example of an interaction vertex involving an electron and a G-family elementary boson. Table 2.10.6 pertains. Here, we extrapolate from Table 2.10.5. Here, QE-like variables come into play.

**Table 2.10.6** Aspects of a model for a vertex correlating with an interaction between an electron and a G-family elementary boson

1. People might say that , for the electron, the next equation follows from $\langle r^2 \rangle = 0$.
$$\langle t^2 \rangle = 0 \tag{2.67}$$
2. People might say that, for the G-family particle, the next equations follows from $\langle r^2 \rangle = 0$ and $\langle t^2 \rangle = 0$ for the electron.
$$\langle r^2 \rangle = 0 \tag{2.68}$$
$$\langle t^2 \rangle = 0 \tag{2.69}$$
3. The next equation, for the G-family particle, follows from work above in this table.

$$<x^2> = 0 \quad (2.70)$$

3.1. Here, x aligns with the direction of motion (either before the vertex destroys the particle or after the vertex creates the particle) of the G-family particle. Compare with Table 2.10.5.)

4. The vertex correlates with a single point with respect to space-time coordinates.
5. People might say that, for an interaction vertex, models correlate with a collapse (or, disappearance) of the clouds of virtual particles that models otherwise would correlate with the vertex-entering particles that correlate with the vertex.

~ ~ ~

This subsection develops some aspects of quantum kinematics that dovetail with quantized masses for elementary bosons.

[Physics:] The operator $\partial^2/\partial^2x$ correlates (within a factor) with $P^2$, in which P denotes an operator people associate with linear momentum. The operator $\partial^2/\partial^2t$ correlates (within a factor) with $E^2$, in which E denotes an operator people associate with energy.

[Physics:] Each of those two factors includes a factor of $\hbar^2$.

For INTERN solutions that correlate with non-G-family elementary bosons, #E = #P.

[Physics:] People might say that Table 2.10.7 pertains. (See Table 2.6.2, Section 3.7, and Section 3.8.) Here, we, in effect, add (to work in Table 2.6.2 related to $D^* = 2$) concepts that people might correlate with a notion of a QE-like $D^* = 2$ and/or with a notion that $\xi'$ can effectively be negative (at least, for some terms in a calculation for which other terms in the calculation correlate with contributions for which $\xi' > 0$). Here, c denotes the speed of light. For the G-family, Table 2.10.7 assumes $(D + 2\nu)_j = 0$ when j correlates with an oscillator pair EkR-and-EkL for which k is an even integer and #E < k ≤ #P.

**Table 2.10.7** Models for masses for elementary bosons for which either $\sigma = +1$ or the bosons are related to the Y-family of solutions

1. This equation pertains for the cases stated.

$$E^2 - c^2P^2 = (m')^2c^4 \quad (2.71)$$

1.1. For $\sigma = +1$ elementary particles.

1.2. For composite particles.

1.3. For elementary particles correlating with Y-family solutions.

2. For elementary bosons for which $\sigma = +1$, we assume that the following equation pertains.

$$E^2 - c^2P^2 \approx (\xi'/2)\, \Sigma_j \pm_j(D + 2\nu)_j \quad (2.72)$$

2.1. Here, …
 2.1.1. $j \in$ { oscillator pairs E[#E]R-and-E[#E]L, …, E2R-and-E2L, E0-and-P0, P2L-and-P2R, …, P[#P]L-and-P[#P]R }.
 2.1.2. $\pm_j = \pm 1$.
 2.1.3. Each value of $(D + 2\nu)_j$ pertains to a $D^* = 2$ and $\nu = -1$ solution, as per Table 2.5.5, Table 2.5.17, and Table 2.6.2.
  2.1.3.1. We use the notation $\{D + 2\nu\}(2,S,\Omega)$ to denote the value of $D + 2\nu$ that correlates with $D^* = 2$, with a value of S, and with a value of Ω.
  2.1.3.2. People might express concern regarding applying this concept for the E0-and-P0 oscillator pair.
   2.1.3.2.1. Table 2.10.8 discusses this use.
2.2. For zero-mass elementary bosons (that is, particles correlating with the G- or Y-families), …
 2.2.1. Each $(D + 2\nu)_j$ equals 0.
 2.2.2. That is, we use only $\{D + 2\nu\}(2,1,1)$.
2.3. For the known non-zero mass elementary bosons (that is, particles correlating with the H- or W-families), …
 2.3.1. $\xi'/2$ correlates with experimental results via a formula for $R_0$. (See Table 2.9.4.)

People might say that Table 2.10.8 discusses concepts that correlate with some aspects of Table 2.10.7.

**Table 2.10.8** $(D + 2\nu)_j$ correlating with the E0-and-P0 oscillator pair

1. People might say that this use of $\{D + 2\nu\}(2,S,\Omega)$ correlates with a selection of numbers that fits data about the masses of the Higgs, Z, and W bosons. (See Section 3.7.)
2. For oscillator pairs other than E0-and-P0, work correlating with Table 2.10.7 correlates with notions of (for coordinates for energy-momentum space) …
 2.1. For QE-like aspects, …
  2.1.1. $(e[even])^2 + (e[even - 1])^2$.
 2.2. For QP-like aspects, …
  2.2.1. $c^2(p[even - 1])^2 + c^2(p[even])^2$.
3. People might say that, for oscillator pairs other than E0-and-P0, work correlating with Table 2.10.7 correlates with notions of (in extensions to space-time coordinates) …
 3.1. $c^2(t[even])^2 + c^2(t[even - 1])^2$.

3.2. $(x[\text{even} - 1])^2 + (x[\text{even}])^2$.

4. People might say that, for each of the various oscillator pairs (other than the E0-and-P0 oscillator pair) this table mentions above, …
   4.1. Math features a sum of two terms, with each term featuring the square of a coordinate.
   4.2. Math pertaining to $D^* = 2$ pertains.
5. For the oscillator pair E0-and-P0, traditional physics might correlate with notions of, …
   5.1. In QE-like and QP-like coordinates for energy-momentum space, …
      5.1.1. $(e0)^2 - c^2(p0)^2$.
   5.2. In space-time coordinates (relative to an interaction vertex), …
      5.2.1. $c^2(t0)^2 - (x0)^2$.
6. For an interaction vertex involving a non-zero-mass boson, …
   6.1. For r characterizing the spatial extent of the vertex, $\langle r^2 \rangle$ correlates with $\eta$ for the fermion.
   6.2. The vertex correlates with $\eta^2 \rightarrow 0$.
   6.3. $\langle r^2 \rangle = 0$, for the vertex.
      6.3.1. This contrasts with $\langle r^2 \rangle \sim (R_0)^2$ (See Table 2.10.9.), for which …
         6.3.1.1. $\langle r^2 \rangle$ pertains to the range of the non-zero-mass boson.
         6.3.1.2. $R_0$ pertains to the non-zero-mass boson.
   6.4. Therefore, …

$$\langle (x0)^2 \rangle = 0 \tag{2.73}$$

   6.5. Similarly, for the variable t characterizing a temporal extent of the vertex, …

$$c^2 \langle t^2 \rangle = 0 \tag{2.74}$$

   6.6. Therefore, …

$$c^2 \langle (t0)^2 \rangle = 0 \tag{2.75}$$

   6.7. Therefore, …

$$c^2 \langle (t0)^2 \rangle - \langle (x0)^2 \rangle = 0 \tag{2.76}$$

   6.8. We posit that …
      6.8.1. $c^2 \langle (t0)^2 \rangle - \langle (x0)^2 \rangle = 0$ correlates with $E^2 - c^2P^2 = (m')^2c^4$.
      6.8.2. This difference (involving $c^2 \langle (t0)^2 \rangle$ and $\langle (x0)^2 \rangle$) contrasts with sums, pertaining to other relevant oscillator pairs, of terms, with each term involving two coordinates.
      6.8.3. $\{D + 2\nu\}(2,S',\Omega')$ numbers, for which S' need not match S, can pertain.

7. We note the following. (See Table 2.5.17, Section 3.7, and Section 3.8.)
   7.1. For the 2W family, the choice S' = 3 seems appropriate because ...
   $$\{D + 2\nu\}(2,3,\Omega') = 10 \tag{2.77}$$
   7.2. For the 0H family, the choice S' = 4 seems appropriate because ...
   $$\{D + 2\nu\}(2,4,\Omega') = 17 \tag{2.78}$$
8. We note that the rows of Table 2.7.1 show a sequence for which, respectively, …
   $$S = 1, 0, 0, 1, 2\ \ldots \tag{2.79}$$
   $$-\Omega = -2, 0, 0, 2, 6, \ldots \tag{2.80}$$
9. We note that Table 2.5.18 correlates with a concept for how to address a series of S for which …
   $$|S| = 1, 0, 0, 1, 2, \ldots \tag{2.81}$$
   $$S = -1, 0, 0, 1, 2, \ldots \tag{2.82}$$
10. We posit that the following pertain.
   10.1. S' for the 2W-subfamily correlates with $\nu = -1$ and S = 3 in Table 2.5.17.
   10.2. S' for the 0H-subfamily correlates with $\nu = -1$ and S = 4 in Table 2.5.17.
   10.3. S' for the 0O-subfamily correlates with $\nu = -1$ and S = 5 in Table 2.5.17.
   10.4. S' for the 2O-subfamily correlates with $\nu = -1$ and S = 6 in Table 2.5.17.
   10.5. S' for the 4O-subfamily correlates with $\nu = -1$ and S = 7 in Table 2.5.17.

~ ~ ~

This subsection discusses models correlating with rest masses for elementary fermions.

For zero-mass elementary fermions (that is, the 1N-, 3N-, 1R-, 3R-, and 3D-subfamilies), as for zero-mass elementary bosons, LADDER solutions feature N(P0) ≤ −1. For zero-mass elementary fermions, in INTERN LADDER solutions, N(P0) = −1. Regarding rest mass, models similar to the models in Table 2.10.7 for zero-mass bosons may pertain. Here, each relevant $(D + 2\nu)_j$ equals 0.

For non-zero mass elementary fermions (that is, the 1C-, 1Q-, 3Q-, and 3I-subfamilies), relevant models differ from models for the WHO-family masses.

For each W-, H-, and O-family ground-state INTERN LADDER solution, each open oscillator pair has local-Œ = 1. For each 1C, 1Q, 3Q, and 3I INTERN LADDER solution, the P[#P − 1]-and-P[#P] oscillator pair is open and has local-Œ = 0.

For each of 1C, 1Q, 3Q, and 3I, the difference (compared to the WHO-families) regarding local-Œ for the P[#P − 1]-and-P[#P] oscillator pair is significant regarding the sizes of rest masses. For 1C and 1Q particles, this monograph shows (in Section 3.9) an approximate formula for rest masses. For generation-1 1C and 1Q particles, rest mass varies (at least approximately) as $m(0,0)\times e^{-k|Q'|}$, in which m(0,0) is a positive number with units of mass, k is a positive number, and Q' is the charge in units of $|q_e|/3$. Also, for other generations of 1C and 1Q particles, exponential functions involving functions of Q' pertain.

Perhaps, the rest mass for an elementary fermion particle for which $m' \neq 0$ correlates conceptually with a notion of a degree of some sort of effort to maintain the P[#P − 1]-and-P[#P] oscillator pair as being open, compared to the many ways for this pair to be closed. Perhaps, such degree of effort correlates with an exponential function. Perhaps, such effort correlates with generation-1 $m' \neq 0$ elementary fermions having less rest mass than $m' \neq 0$ elementary bosons have. In Section 3.9, we possibly correlate generation-1 $m' \neq 0$ elementary fermion particle masses with functions of the form $\exp(-k(D + 2\nu)_j)$, in which the $(D + 2\nu)_j$ correlate with $(D + 2\nu)_j$ for solutions for which $D^* = 2$ and $\nu = -1/2$, as per Table 2.5.17.

~ ~ ~

This subsection discusses spatial ranges for elementary bosons.

Work above correlates with the notion that elementary bosons have zero size with respect to an interaction vertex. Regarding elementary bosons for which the rest mass is non-zero, MM1MS1 models correlate with the notion that various $(D + 2\nu)_j$ are non-zero.

Table 2.10.9 pertains. The table repeats some results. (See Table 2.10.8.) Table 2.10.9 discusses spatial ranges for elementary bosons.

**Table 2.10.9** Results regarding sizes pertaining to interaction vertices for interactions between an elementary-particle fermion and an elementary-particle boson and regarding distances between two related interaction vertices

1. For a non-zero-mass elementary fermion, vertex sizes feature $\langle r^2 \rangle = 0$.
   - 1.1. This matches the notion that the limit $\eta^2 \to 0$ pertains for fermion elementary particles.
2. For a non-zero-mass elementary fermion, vertex sizes feature $\langle t^2 \rangle = 0$.
   - 2.1. This matches a notion that $c^2\langle t^2 \rangle = \langle r^2 \rangle = 0$ pertains for non-zero-mass fermion elementary particles.
3. For a non-zero-mass elementary boson for which $\sigma = +1$, vertex sizes feature $\langle x^2 \rangle = 0$ and $\langle t^2 \rangle = 0$.
   - 3.1. This matches that notion that a projection, for the boson, to $D^* = 1$ dimensions correlates with $\langle r^2 \rangle = 0$ for the fermion.
4. For a non-zero-mass elementary boson for which $\sigma = +1$, vertex sizes feature $\langle (r_2)^2 \rangle = 0$. (See Table 2.10.5.)

   4.1. This matches that notion that a projection, for the boson, to $D^* = 2$ dimensions correlates with $<r^2> = 0$ for the fermion.
5. For a $\sigma = +1$ non-zero-mass elementary boson, at a vertex, …
   5.1. Some $(D + 2\nu)_j$ are non-zero. (See Table 2.5.17.)
   5.2. Therefore, for the range associated with the boson, …
      5.2.1. $<r^2> \neq 0$.
      5.2.2. We posit that that the following pertains for the field.

$$c^2<t^2> \sim <r^2> \sim R_0^2 \quad (2.83)$$

6. For a $\sigma = +1$ non-zero-mass elementary boson, near a vertex, …
   6.1. We posit that …
      6.1.1. The boson participates in another interaction vertex.
      6.1.2. People might say that MM1MS1 models correlate with the other vertex being close to the (original) vertex, with closeness correlating with (in a frame of reference that correlates with the pair of vertices) correlating with the following expressions.

$$c^2<t^2> \sim R_0^2 \quad (2.84)$$

$$<r^2> \sim R_0^2 \quad (2.85)$$

$$(<r^2>)^{1/2} \sim R_0 \quad (2.86)$$

   6.2. The leaves the possibility that, for the boson, $<r^2> \sim R_0^2$ correlates with a need for the other interaction (in which the boson participates) to be nearby.
   6.3. People might say that $c^2<t^2> \sim R_0^2$ for the boson correlates with the following approximation for a lifetime of the boson.

$$(<t^2>)^{1/2} \sim R_0/c \quad (2.87)$$

7. For a $\sigma = +1$ zero-mass elementary boson, …
   7.1. Vertex sizes feature $<r^2> = 0$.
      7.1.1. This matches the notion that each $\{D + 2\nu\}_j$ that correlates with boson is 0.
      7.1.2. This also matches $<r^2> = 0$ for the fermion.
   7.2. Vertex sizes feature $<t^2> = 0$.
      7.2.1. This matches $<r^2> = 0$ for the boson and $<t^2> = 0$ for the fermion.
   7.3. People might say that $R_0 = \infty$.
      7.3.1. This correlates with the boson being able to travel an indefinitely large distance before interacting with another fermion.

# Section 2.11 SPATIM symmetries

Section 2.11 discusses symmetries that correlate with the Poincare group and with special relativity. We provide perspective regarding possible uses of symmetries. We introduce SPATIM models correlating with symmetries that can correlate with solutions. We discuss rules for how, for a field that correlates with a combination of a fermion (elementary particle) field and a boson (elementary particle) field, to compute symmetries for the combined field based on the symmetries of the two component fields.

~ ~ ~

This subsection provides perspective about a possible use for symmetries.

People might say that LADDER solutions we show above tend to pertain to INTERN aspects of elementary particles and to INTERN aspects of the fields associated with the elementary particles. These solutions seem not to correlate with concepts such as a direction in which a particle might be traveling or such as a momentum that an observer might attribute to the particle.

We seek a technique for developing solutions that correlate with ENVIRO aspects related to LADDER solutions we develop above.

People might say that this technique correlates with a mathematical notion of mapping LADDER solutions from a quantum energy-momentum space into LADDER solutions in a space that correlates with applications of space-time coordinates.

The technique we develop correlates with the Poincare group, which is a mathematical construct that people correlate with symmetries that correlate with traditional physics models based on special relativity.

~ ~ ~

This subsection provides perspective about symmetries and about this section.

People are familiar with concepts of symmetries. People characterize some symmetries as exact. On a flat tabletop, rotate a circle in a way that the center of the circle stays at its original point. The circle seems the same. People characterize some symmetries as approximate. A mirror image of a person's face may seem somewhat the same as the original image.

Symmetries correlate with constraints. The more exact a symmetry, the smaller (in some sense) the set of symmetric entities.

Various aspects of physics theory feature symmetries. Some sets of symmetries correlate with models that use space-time coordinates that people correlate with concepts of time and space.

Models correlating with special relativity generally exhibit more constraints and more symmetries than do somewhat similar models correlating with Newtonian physics. People characterize symmetries correlating with special relativity by invoking a mathematical construct people call the Poincare group.

This section introduces concepts and notation this monograph correlates with symmetries related to special relativity.

~ ~ ~

This subsection defines SPATIM (or, space-time coordinate) symmetries.

[Physics:] People might say that SPATIM symmetries correlate with mappings from a quantum energy-momentum space into space-time coordinates.

Table 2.11.1 defines and discusses concepts related to some SPATIM symmetries. Work in subsequent sections focuses on (a`;b`)> symmetries and on (a`;b`)< symmetries.

**Table 2.11.1** Definitions of and notation for SPATIM (or, space-time coordinate) symmetries for which a` ≤ 1 and b` ≤ 3

1. [Physics:] SPATIM abbreviates space-time coordinate (symmetries).
2. FRERAN SPATIM (or, free-ranging space-time coordinate) symmetries correlate with the Poincare group.
3. People correlate with the Poincare group the following symmetries.
    - 3.1. A 1-generator symmetry.
    - 3.2. Three 3-generator symmetries.
4. [Physics:] People might correlate the Poincare group symmetries with conservation laws and symmetries, as follows.
    - 4.1. The 1-generator symmetry correlates with conservation of energy.
    - 4.2. One 3-generator symmetry correlates with conservation of momentum.
    - 4.3. One 3-generator symmetry correlates with conservation of angular momentum.
        - 4.3.1. For this symmetry, we think that this monograph need not much address the distinction between each of the following two interpretations.
            - 4.3.1.1. Vector symmetry, which correlates with rotation symmetry and reflection symmetry.
            - 4.3.1.2. Pseudovector symmetry, which correlates with rotation symmetry but not reflection symmetry.
    - 4.4. One 3-generator symmetry correlates with the applicability of special relativity (as opposed to, say, Newtonian classical physics). (See Section 2.12.)
        - 4.4.1. For this symmetry, we use the term conservation of RESENE.
            - 4.4.1.1. We use the acronym RESENE to abbreviate the phrase rest energy.

5. For this monograph, we correlate with Poincare group symmetries (and solutions we discuss) the following.
    5.1. The symbol a` denotes the applicable integer number of 1-generator symmetries.
        5.1.1. 0 ≤ a` ≤ 1.
        5.1.2. This monograph does not attempt to correlate this 1-generator symmetry with a mathematical group.
    5.2. The symbol b` denotes the applicable integer number of 3-generator symmetries.
        5.2.1. 0 ≤ b` ≤ 3, for some SPATIM applications.
        5.2.2. Each 3-generator symmetry correlates with an occurrence of the group SU(2).
            5.2.2.1. The group SU(2) correlates with 3 generators.
        5.2.3. This monograph may correlate such a 3-generator symmetry with either of the following.
            5.2.3.1. A P[even − 1]-and-P[even] oscillator pair, for σ = +1.
            5.2.3.2. An E[even]-and-E[even − 1] oscillator pair, for σ = −1.
    5.3. For a solution for which σ = +1, the next item denotes relevant SPATIM symmetries.
        5.3.1. (a`;b`)>.
    5.4. For a solution for which σ = −1, …
        5.4.1. Aspects of the next item can pertain for FRERAN SPATIM models.
            5.4.1.1. (a`;b`)>.
        5.4.2. The next item denotes relevant COMPAR SPATIM symmetries.
            5.4.2.1. (a`;b`)<.

Table 2.11.2 shows rules we assume pertain to determining ENVIRO symmetries for pairs of fields. (See, for example, the transitions from to Table 2.13.31 to Table 2.13.33 to Table 2.13.34.) In Table 2.11.2, j and k are integers.

**Table 2.11.2** Rules for combining SPATIM symmetries

1. For identical or adequately similar fields for which (a`;b`)> pertain, the following combining of symmetries pertains.
$$(a`;b`)> + (a`;b`)> = (a`;b`)> \quad (2.88)$$
2. Otherwise, …
    2.1. Regarding a`, …
    $$(j;…)> + (k;…)> = (…;(j + k))>, \text{ for } 0 ≤ j, 0 ≤ k, \text{ and } j + k ≤ 1 \quad (2.89)$$
    $$(j;…)< + (k;…)< = (…;(j + k))>, \text{ for } 0 ≤ j, 0 ≤ k, \text{ and } j + k ≤ 1 \quad (2.90)$$
    2.2. Regarding b`, …

(…;j)> + (…;k)> = (…;(j + k))>, for 0 ≤ j, 0 ≤ k, and j + k ≤ 3 (2.91)

(…;j)< + (…;k)< = (…;(j + k))<, for 0 ≤ j, 0 ≤ k, and j + k ≤ 3 (2.92)

2.3. Regarding COMPAR solutions and combining pre-composites (for which σ = −1 {See Table 2.13.33.}) to form composite particles (for which σ = +1), …

(1;3)< + (1;3)< = (1;3)> (2.93)

3. Regarding σ = +1, …

(1;3)> + (1;3)> = (1;3)> (2.94)

3.1. (1;3)> correlates with the Poincare group.

# Section 2.12 Invariances, symmetries, and conservation laws

Section 2.12 discusses the extent to which invariances, symmetries, and conservation laws pertain. We discuss relationships among concepts of invariances, symmetries, and conservation laws. We provide perspective regarding invariances, symmetries, and conservation laws that correlate with SPATIM-correlated phenomena. We show examples of MM1MS1-related symmetries that traditional physics models may not feature. One such symmetry correlates with conservation of fermion generation for some interactions. We suggest categories for conservation laws. We suggest a quantum-related basis for symmetries correlating with conservation of angular momentum. People might say that we provide new perspective regarding approximate conservation laws people associate with the terms C-symmetry, P-symmetry, and T-symmetry. We compare MM1MS1 models and traditional interpretations regarding conservation of charge.

~ ~ ~

This subsection discusses some aspects of conservation laws.

People entwine concepts of conservation laws with models for physics. People might say that, currently, it could be difficult to recast traditional physics in ways that do not include, for example, conservation of energy, conservation of momentum, and conservation of angular momentum. People entwine these three conservation laws with classical-physics theories and with quantum-physics theories.

~ ~ ~

This subsection discusses some aspects regarding invariances, symmetries, and conservation laws.

People entwine conservation laws with mathematics that correlates with models for physics. For example, people correlate conservation of energy with mathematics related to adding, regarding all measurements, an arbitrary constant to a coordinate for time. Or, people correlate conservation of momentum with mathematics related to adding an arbitrary three-dimensional displacement to coordinates for position in space. To the extent such displacements or other changes yield no change in the relevant physics, people say that a system is invariant under the changes.

People correlate notions of symmetry with such invariances.

People might also want to consider invariances regarding choice of observer.

~ ~ ~

This subsection points to results that correlate with and summarize some work in this section.

People might say that, based on context that Table 6.1.5 and Table 6.1.6 set, Table 6.1.7 summarizes some aspects of invariances that correlate with $\sigma = +1$, symmetries that correlate with $\sigma = +1$, conservation laws, and internal properties of elementary particles for which $\sigma = +1$.

~ ~ ~

This subsection provides perspective on math-based models related to uniform displacements.

Perhaps, at least two trains of thought correlate with models related to uniform displacements.

One train of thought correlates with math for which people use the term calculus. This train of thought leads to the series the people might characterize as position, velocity, acceleration, and so forth. People might discuss this series in terms of a number of derivatives (with respect to a coordinate correlating with time) of the expected value of position. A zero number of derivatives correlates with position. One derivative correlates with velocity. And so forth. People find many uses for applications based on this train of thought.

One train of thought correlates with classical-physics measurements for collective phenomena. We consider a series based on expected values of powers of position. The series consists of expected value of 1 (or, position to the zeroth power), expected value of position, expected value of the square of position (perhaps, as measured away from an expected value of position), and so forth. The first item does not change. People can say that this item evolves linearly with time, with a zero rate of change. For an object that does not experience significant forces, the second term changes linearly with time. Under some circumstances, for a collection of objects that diffuse from a small region, the third term evolves linearly with time. Here, each of the first few members of the series has applications, though not necessarily to the same specific physical systems.

People might say that the first train of thought correlates with classical mechanics (and extensions to classical mechanics). People might say that the second train of thought correlates with statistical mechanics.

~ ~ ~

This subsection provides perspective on FRERAN SPATIM invariances, symmetries, and conservation laws.

People tend to correlate applications of invariance with the first train of thought the previous subsection describes. Here, we look at invariance from perspectives that feature measurements.

Consider an observer who makes measurements of an object. The observer might be an astronomer. The object might be a star or galaxy. We focus on observations and deductions the astronomer might make. We do thought experiments based changes in temporal and spatial distances between the astronomer and the object. We idealize that the astronomer has full use of subtle data and not just partial use of optically available data. We idealize, for purposes of discussion, that the universe obeys fully deterministic laws of (say, classical) physics.

For a baseline, assume the astronomer does thorough work based on observations made around some time and from some point. Here, thorough work means, for example, deducing the evolution of the object during a significant period of the object's existence.

Under what circumstances might the astronomer do essentially the same thorough work? At a somewhat different time? Yes. From a somewhat different location? Yes. From a somewhat different velocity of motion perpendicular to a radius from the object to the astronomer? Yes. From a somewhat different velocity along the radius from the object to the astronomer? Yes. From a somewhat different rate of change of the tangential velocity? Yes. From somewhat different rate of change of radial velocity? Yes.

People might say that, correlating with the first train of thought, the first three of the invariances respectively correlate with conservation of energy, conservation of momentum, and conservation of angular momentum. Different observers can observe different energies, momenta, and/or angular momenta. For any one observer, each of conservation of energy, conservation of momentum, and conservation of angular momentum pertains.

For models that do not take into account special-relativistic effects (or, generally, for Newtonian models), for the something that correlates with fourth invariance, we might suggest the term conservation of relative velocity. Any relative velocity can pertain. In particular, speeds faster than light speed might pertain. Different observers can observe different relative velocities. For any one observer, conservation of relative velocity pertains.

For Newtonian physics, each of conservation of momentum, conservation of angular momentum, and conservation of relative velocity would correlate with a 3-vector.

For models based on special relativity, any relative velocity for which the speed does not equal or exceed the speed of light can pertain.

Perhaps, limiting speeds correlates with truncating the series we are discussing.

Perhaps, we can focus on an invariance that involves a quantity all observers can agree on.

We suggest that conservation of RESENE pertains. To the extent special relativity pertains, all observers can deduce the same the same rest energy.

For elementary particles, for rest energy, people use $c^2$ times rest mass. People might say that conservation of rest energy correlates with special relativity.

Regarding this phenomenon, for the notions of invariance and symmetry, people sometimes use the term boost.

For special relativity, models correlating with conservation of energy and conservation of momentum correlate with a 4-vector. Models correlating with conservation of angular momentum correlate with a 3-vector. Models correlating with conservation of relative velocity (or with conservation of RESENE) can correlate with a 3-vector.

~ ~ ~

This subsection provides perspective regarding invariances or symmetries correlating with SPATIM-related invariances and with FRERAN phenomena.

Table 2.12.1 pertains.

**Table 2.12.1** SPATIM-related invariances (or symmetries) for FRERAN phenomena

1. FRERAN SPATIM-related symmetries.
    - 1.1. The symmetries (1;3)> pertain.
        - 1.1.1. In this table, we use notation that decomposes the symbol (1;3)> in to four components. Respectively, the components are the following.

$$(1\times1;\ \ldots,\ \ldots,\ \ldots)> \tag{2.95}$$

$$(\ldots;\ 1\times3,\ \ldots,\ \ldots)> \tag{2.96}$$

$$(\ldots;\ \ldots,\ 1\times3,\ \ldots)> \tag{2.97}$$

$$(\ldots;\ \ldots,\ \ldots,\ 1\times3)> \tag{2.98}$$

    - 1.2. People might say that these symmetries correlate with mathematical-modeling possibilities for coordination between energy-momentum representations of fields (and/or particles) and space-time representations of wave functions.
    - 1.3. Invariance with respect to temporal displacement correlates with (1×1; …, …, …)>.
        - 1.3.1. Here, displacement involves a 1-vector construct.

- 1.4. Invariance with respect to spatial displacement correlates with (...; 1×3, ..., ...)>.
  - 1.4.1. Here, displacement involves a 3-vector construct.
- 1.5. Invariance with respect to rotation correlates with (...; ..., 1×3, ...)>.
  - 1.5.1. Here, displacement involves a 3-vector construct.
- 1.6. Invariance with respect to boost correlates with (...; ..., ..., 1×3)>.
  - 1.6.1. Here, displacement involves a 3-vector construct.
  - 1.6.2. Here, relative velocity features a 3-vector value of velocity.
    - 1.6.2.1. People might think of the 3-vector as correlating with velocity/c. (See Table 2.12.8.)
  - 1.6.3. People might say that this invariance correlates with at least one (and perhaps all) of the following.
    - 1.6.3.1. The classical-physics special-relativistic notion of Lorentz invariance (or, of boost-related symmetry).
    - 1.6.3.2. That the speed of light, c, is a constant.
    - 1.6.3.3. Limitations that speeds cannot exceed c.
    - 1.6.3.4. Concepts of zero-curvature, with respect to notions of curvature inherent in uses of space-time coordinates.

Table 2.12.2 provides symbols we define to correlate with some point and global symmetries.

**Table 2.12.2** The symbols #b' and #b" and correlations with point and global symmetries

1. The symbol #b' correlates with point symmetries.
   - 1.1. For point symmetries (a`;b`)>, the next equation pertains.

     $$\#b' = b` \qquad (2.99)$$

     - 1.1.1. We assume that #b' ≥ 2.
       - 1.1.1.1. People might say that a value of less than 2 correlates with a violation of at least one of conservation of momentum and conservation of angular momentum.
2. The symbol #b" correlates with global symmetries.
   - 2.1. For global symmetries (a`;b`)>, the next equation pertains.

     $$\#b'' = b` \qquad (2.100)$$

     - 2.1.1. We assume that #b' ≥ 2.

2.1.1.1. People might say that a value of less than 2 correlates with a violation of at least one of conservation of momentum and conservation of angular momentum.

Table 2.12.3 pertains.

**Table 2.12.3** Assumptions regarding #b' and #b"

1. In Section 2.10, we show that elementary-particle interaction vertices correlate with points, with respect to space-time coordinates.
2. In Table 2.12.5, we correlate #b' = 3 with invariance (with respect to observers) of RESENE (or, rest energy). (See, for example, Table 2.11.1.)
3. For much of this monograph, …
    3.1. We assume the following equation, for all interactions involving (at least) elementary particles for which σ = +1,

$$\#b' = 3 \tag{2.101}$$

4. For some of this monograph (See Table 2.13.43.), …
    4.1. We discuss possibilities correlating with the following equation.

$$\#b' = 2 \tag{2.102}$$

5. We take up the topic of #b" in Section 6.1 and Section 6.4.

~ ~ ~

This subsection provides perspective regarding the extent to which some conservation laws pertain.

This monograph correlates (1;3)> local symmetries with a set of conservation laws. The set includes at least conservation of energy, conservation of momentum, and conservation of angular momentum.

For ENS48 models, ENS6 models, and ENS1 models, the set of conservation laws pertains for all σ = +1 phenomena.

Conservation of energy, momentum, and angular momentum correlate with symmetries people might symbolize by (1;2)>. People might raise the question of the extent to which the applicability of (1;3)> symmetries correlates with an additional QP-like conservation law (in addition to conservation of momentum and conservation of angular momentum). People might say that, to the extent σ = +1 and #b' = 3 pertain, that conservation law pertains to interactions between elementary particles.

MM1MS1 models point to other possibly useful possible new symmetries and conservation laws. Table 2.12.4 shows examples of possibly new symmetries. Elsewhere, we show a more comprehensive concept of conservation of fermion generation. (See Section 2.8.)

**Table 2.12.4** Examples of symmetries that traditional ENVIRO models may not feature

1. For σ = −1 models, we symbolize various symmetries by (a`;b`)<.
    1.1. For example, sometimes (1;3)< pertains.
    1.2. These symmetries correlate with COMPAR solutions. (See Table 2.13.31.)
    1.3. People might say that various (a`;b`)< symmetries correlate with (possibly non-traditional) conservation laws or with other mathematical physics pertaining to QCD (or, quantum chromodynamics).
2. We might symbolize symmetries related to 2W-subfamily and to conservation of fermion generation by (1×3, …; …, …, …)>.
    2.1. Here, the notion of conversation of fermion generation pertains to an interaction between a boson and a spin-1/2 fermion such that the fermion does not change generation.
    2.2. Here, compared to Table 2.12.1, we add a QE-like component. (See Table 2.13.4.)
    2.3. People might say that (1×3, …; …, …, …)> correlates with (possibly non-traditional) conservation laws or with other mathematical physics pertaining to (at least) interaction vertices in which a 2W particle either …
        2.3.1. Interacts with a lepton.
        2.3.2. Creates or destroys a matter-lepton-and-antimatter-lepton pair of particles.
3. We might symbolize some possible symmetries related to particles and fields in atoms and molecules by (…; …, …, …, …)>.
    3.1. To denote these symmetries, we use the symbol (a`;4)>.
    3.2. These symmetries correlate with ATOMOL solutions.
    3.3. Here, compared to Table 2.12.1, we add a QP-like component. (See Section 6.2.)
    3.4. People might say that those (a`; …, …, …, …)> symmetries correlate with the group SO(4).
    3.5. People might say that that these symmetries …
        3.5.1. Do not correlate directly with the MM1 meta-model.

~ ~ ~

This subsection discusses categories of conservation laws and provides examples of possible non-traditional conservation laws.

We think that work above points to various categories of conservation laws. We think that work above points to possibilities for non-traditional conservation laws. Table 2.12.5 pertains.

**Table 2.12.5** Categories of conservation laws, plus possible non-traditional conservation laws or symmetries

1. FRERAN SPATIM-related $\sigma = +1$ conservation laws.
    1.1. The symmetries (1;3)> pertain.
        1.1.1. In this table, we use notation that decomposes the symbol (1;3)> in to four components. (See Table 2.12.1.) Respectively, the components are …
            1.1.1.1. (1×1; …, …, …)>.
            1.1.1.2. (…; 1×3, …, …)>.
            1.1.1.3. (…; …, 1×3, …)>.
            1.1.1.4. (…; …, …, 1×3)>.
    1.2. People might say that these conservation laws correlate with mathematical-modeling possibilities for coordination between energy-momentum representations of fields (and/or particles) and space-time representations of wave functions.
    1.3. Conservation of energy correlates with (1×1; …, …, …)>.
        1.3.1. Here, energy involves a 1-vector construct.
    1.4. Conservation of momentum correlates with (…; 1×3, …, …)>.
        1.4.1. Here, momentum involves a 3-vector construct.
    1.5. Conservation of angular momentum correlates with (…; …, 1×3, …)>.
        1.5.1. Here angular momentum involves a 3-vector construct.
    1.6. Conservation of relative velocity correlates with (…; …, …, 1×3)>.
        1.6.1. Here, relative velocity features a 3-vector construct.
    1.7. Conservation of RESENE correlates with (…; …, …, 1×3)>.
        1.7.1. People might say that conservation of RESENE correlates with a construct (rest energy) that is invariant with respect to observer.
2. Quantum-property-transfer-related conservation laws.
    2.1. People might say that these conservation laws correlate with aspects regarding interaction vertices.
    2.2. Conservation of charge (regarding transfers of charges between elementary particles and not regarding transfers of charges between, say, atoms) …
        2.2.1. Correlates with transfers of charge via interactions intermediated by non-zero-charge 2W and 2O bosons.
            2.2.1.1. 2W and 2O bosons transfer charge in integer multiples of $|q_e|/3$.
                2.2.1.1.1. For example, a $W^-$, in effect, takes −3 units of $|q_e|/3$ from an electron, leaving a MM1MS1-neutrino with zero charge.
        2.2.2. Pertains to all interaction vertices.
            2.2.2.1. For a vertex that does not involve a 2W or 2O boson, no transfer of charge takes place.

3. Extensions to SPATIM-related symmetries.
   3.1. Table 2.12.4 shows examples of symmetries that might correlate with new conservation laws.
      3.1.1. For example, conservation of generation …
         3.1.1.1. Correlates with interaction vertices in which both …
            3.1.1.1.1. An elementary fermion absorbs or emits an elementary boson.
            3.1.1.1.2. The interaction cannot change the generation of the fermion.
         3.1.1.2. Pertains to some vertices and not to other vertices. (See Table 2.13.4 and Table 2.13.12.)
      3.1.2. For example, people might gain useful perspective regarding the topic of quantum chromodynamics (or, QCD) by considering the extent to which various (a`;b`)< symmetries correlate with conservation laws. (See Section 2.13.)

Table 2.12.6 pertains.

**Table 2.12.6** Possible correlation between conservation of generation and variation by generation regarding interaction strength

1. We assume the following.
   1.1. For any type of FRERAN interaction between an elementary boson and elementary fermions that differ (essentially only) by generation, …
      1.1.1. The type of interaction exhibits conservation of fermion generation if and only if the strength of such interaction varies with the generation of the fermion.
2. For example, for an MM1MS1-photon, …
   2.1. Interaction strengths are the same (based on charge) for electrons, muons, and tauons.
   2.2. Conservation of generation need not pertain.
3. For example, for a graviton, …
   3.1. Interaction strengths vary (with mass) for electrons, muons, and tauons.
   3.2. Conservation of generation pertains.
4. For example, for COMPAR interactions, …
   4.1. Conservation of fermion generation need not pertain regarding some interactions involving pairs of quarks and pairs of 2W-subfamily bosons.

People might say that Table 2.12.7 provides an opportunity for research. Regarding 022G2&, conservation of charge pertains. Regarding 244G4&, conservation of generation pertains.

**Table 2.12.7** Possible correlation between measurement of a property of an elementary fermion by an elementary boson and conservation of that property

1. To what extent does the following (or a variant of the following) correlation pertain?
   1.1. An interaction vertex in which an elementary boson essentially measures an internal property of an elementary fermion exhibits conservation of that property.

Table 2.12.8 discusses a thought experiment.

**Table 2.12.8** A relationship possibly useful for thinking about boost-related symmetry or conservation of relative velocity

1. Consider a set of at least two observers. Consider a non-zero-mass object. Consider a frame of reference.
2. For each of the observers in the set, …
   2.1. A 3-vector v characterizes the velocity of the observer relative to the object.
      2.1.1. Each component of the vector has units of velocity.
3. Compute a sum of the squares of the various v. Divide by the number of observers.
   3.1. The result is a number with units of square of velocity.
   3.2. Denote that number by $<v^2>$.
4. Compute an average of the velocities.
   4.1. Denote that average by a 3-vector $<v>$.
5. Denote the dot product of that 3-vector with itself by $<v>\cdot<v>$.
   5.1. $<v>\cdot<v>$ is a number with units of square of velocity.
6. The next result pertains.
   6.1. $<v^2> - <v>\cdot<v> \leq c^2$.
7. To the extent at least one observer has non-zero rest energy (or, non-zero rest mass), the next result pertains.

$$<v^2> - <v>\cdot<v> < c^2 \tag{2.103}$$

~ ~ ~

This subsection shows traditional and possible correlations between generalized notions of symmetries and FRERAN SPATIM-related $\sigma = +1$ conservation laws.

Table 2.12.5 shows four aspects of FRERAN SPATIM-related $\sigma = +1$ conservation laws. (See the decomposition, in Table 2.12.1, of (1;3)> symmetries into four components.) Table 2.12.9 pertains. People might say that people can gain insight about overlaps and differences between classical-physics models and quantum-physics models by comparing the first item in the table with subsequent items in the table.

**Table 2.12.9** Concepts regarding symmetries and FRERAN SPATIM-related $\sigma = +1$ conservation laws

1. People might say that, regarding classical-physics models, …
   1.1. People correlate conservation of energy with invariance (with respect to adding a constant to a coordinate for time) of models.
   1.2. People correlate conservation of momentum with invariance (with respect to adding constants to three coordinates for position) of models.
   1.3. People correlate conservation of angular momentum with invariance (with respect to adding constants to coordinates for rotational orientation) of models.
   1.4. People might correlate relative-velocity (or, boost-related) invariances (with respect to adding constants to vectors for relative velocities of various observers) with aspects of models. (See Table 2.12.8.)
2. People might say that, regarding quantum-physics models, people might envision possibilities for a set of four invariances.
   2.1. Each invariance would correlate, respectively, with quantum aspects of a FRERAN SPATIM-related $\sigma = +1$ conservation law.
   2.2. The first two invariances correlate with complementary-variable aspects of uncertainty.
3. The first member of such a set (of four invariances) would be …
   3.1. Invariance under time translation.
      3.1.1. The complementary variables are time and energy.
4. The second member of such a set (of four invariances) would be …
   4.1. Invariance under space translation.
      4.1.1. The complementary variables are position and momentum.
5. Possibly, the third member of such a set (of four invariances) would …
   5.1. Correlate with the result that $<(2J)^2> - (<2J>)^2$ is, for a traditional quantum base state, a nonnegative integer.
      5.1.1. Here, J denotes the quantum operator people correlate with a term like total angular momentum divided by ħ. Here, $(<2J>)^2$ is a vector dot product.

            5.1.1.1. People may use the formula J = L + S, in which …
                5.1.1.1.1. L correlates with orbital angular momentum divided by ħ.
                5.1.1.1.2. S correlates with MM1MS1 uses of the symbol S.
        5.1.2. People might say that this relationship correlates with a concept somewhat related to uncertainty.
            5.1.2.1. People might use the term self-uncertainty.
            5.1.2.2. People might say that, from a quantum mechanical point of view, total angular momentum does not have a complementary variable.

6. Possibly, the fourth member of such a set (of four invariances) would …
    6.1. Correlate with conservation of RESENE.
    6.2. Correlate with concepts Table 2.12.8 presents.
        6.2.1. People might say that the relationship $<v^2> - <v>\cdot<v> < c^2$ correlates with a concept somewhat related to uncertainty.
            6.2.1.1. People might use the term mutual certainty.
            6.2.1.2. People might say that relative velocity does not have complementary variable.

7. For an elementary particle, …
    7.1. People might say that the following statements correlate with the respective four invariances.
        7.1.1. Conservation of energy correlates with energy, which is an observer-specific quantity.
        7.1.2. Conservation of momentum correlates with momentum, which is an observer-specific quantity.
        7.1.3. Conservation of angular momentum correlates with the next expression, which (for S being a quantum operator and $(<2S>)^2$ being a vector dot product) is an observer-independent quantity.

$$<(2S)^2> - (<2S>)^2 \tag{2.104}$$

        7.1.4. Conservation of RESENE correlates with RESENE, which is an observer-independent quantity.

~ ~ ~

This subsection contrasts discussion above with some traditional interpretations of (1;3)> symmetries.

Some traditional discussion of what we call (1;3)> symmetries features a different interpretation of the (…; …, …, 1×3)> component. A traditional interpretation may include the terms CPT-symmetry and Lorentz invariance. (This contrasts with the term Lorentz invariance symmetry, which people might correlate with (1;3)> symmetries.) A traditional interpretation of Lorentz invariance may correlate each of the 3 generators with a different term from a list of three terms - charge inversion symmetry, position inversion symmetry, and time inversion symmetry. People may extend such discussion to include three approximate conservation laws. People use the terms invariance under charge conjugation, conservation of parity, and invariance under time reversal. Alternative names for these approximate conservation laws are, respectively, are C-symmetry (with C for charge), P-symmetry (with P for parity), and T-symmetry (with T for time).

~ ~ ~

This subsection provides a non-traditional interpretation of some aspects of C-symmetry, P-symmetry, and T-symmetry.

We use the term CPT-related symmetries to denote symmetries that include C-symmetry, P-symmetry, CP-symmetry, T-symmetry, and so forth.

Table 2.12.10 defines and interprets oscillator swap symmetries. People might say that (at least as shown in the table) these possibly physics-relevant symmetries correlate with swaps of values of various N(..) and not with swaps of oscillators. People might say that some aspects (such as portions of item 8) of this table are speculative.

**Table 2.12.10** Oscillator swap symmetries

1. For an oscillator pair E[even]-and-E[even − 1] or an oscillator pair P[even − 1]-and-P[even], an oscillator swap involves the following.
   1.1. Swap the values of N(..) for the two oscillators.
      1.1.1. For example, for a case of pair E2-and-E1, …
         1.1.1.1. Make the swap N(E2) ↔ N(E1).
2. For elementary particles for which #E ≤ 2 and #P ≤ 2, the following provide notations for each of the only two possibly relevant oscillator swaps.
   2.1. Each of CQE-swap and T-swap denotes the swap correlating with the E2-and-E1 oscillator pair.
   2.2. Each of CQP-swap and P-swap denotes the swap correlating with the P1-and-P2 oscillator pair.
3. The following concepts pertain.
   3.1. For quarks specifically (and, thus, all #P ≤ 2 particles generally), …
      3.1.1. C-symmetry (in traditional physics) correlates with the combination of CQE-swap and CQP-swap.
   3.2. P-symmetry correlates with P-swap.

- 3.3. CPT-symmetry correlates with the combination of all four of CQE-swap, CQP-swap, P-swap, and T-swap.
- 3.4. T-symmetry correlates with T-swap.
- 3.5. CPT-symmetry pertains.

4. People might say that, based on the previous item, …
   - 4.1. The term time reversal is misleading.
   - 4.2. The term time inversion might be misleading.
5. To extend the discussion to include all elementary particles, …
   - 5.1. To maintain the relevance of G-family solutions, …
     - 5.1.1. The notion of P-swap extends to (also) include oscillator pairs P3-and-P4, P5-and-P6, and P7-and-P8.
       - 5.1.1.1. For example, without P3-and-P4 swap, solutions correlating with the 2G24& particle become irrelevant.
   - 5.2. To maintain the relevance of Y-family solutions, …
     - 5.2.1. The notion of T-swap extends to (also) include oscillator pair E4-and-E3.
       - 5.2.1.1. For example, without E4-and-E3 swap, differences between 4Y-subfamily matter-correlated solutions and 4Y-subfamily antimatter-correlated solutions break down.
   - 5.3. To maintain CPT-symmetry, …
     - 5.3.1. CQE-swap extends to equal T-swap.
     - 5.3.2. CQP-swap extends to equal P-swap.
6. We can define C-swap to equal CQE-swap plus CQP-swap.
   - 6.1. C-swap equals T-swap plus P-swap.
   - 6.2. C-symmetry correlates with C-swap.
7. The G-family splits into sets. (See Section 3.2 and Section 3.3. In this table, we feature %68even solutions and deemphasize %68odd solutions.)
   - 7.1. The 1-swap G-family set …
     - 7.1.1. Features particles for which the sub-list % has one of element.
     - 7.1.2. Includes 2G2& and 4G4&.
       - 7.1.2.1. Thereby, related symmetries include symmetries related to electromagnetism (perhaps, other than as correlates with traditional elementary-particle magnetic dipole moment) and gravity.
   - 7.2. The 2-swap G-family set …
     - 7.2.1. Features particles for which the sub-list % has two elements.
     - 7.2.2. Includes 2G24& and 2G68&.
       - 7.2.2.1. Thereby, related symmetries may include symmetries related to traditional elementary-particle magnetic dipole moment.

- 7.3. The 3-swap G-family set …
  - 7.3.1. Features particles for which the sub-list % has three elements.
  - 7.3.2. Includes 4G268& and 2G468&.
- 7.4. The 4-swap G-family set …
  - 7.4.1. Features particles for which the sub-list % has four elements.
  - 7.4.2. Includes 4G2468&.

8. Perhaps, …
   - 8.1. Physics does not correlate electromagnetism with the breaking of CPT-related symmetries.
   - 8.2. Physics does not correlate gravitation with the breaking of CPT-related symmetries.
   - 8.3. For ENS48 models and for ENS6 models, …
     - 8.3.1. Within an ensemble, interactions mediated by G-family bosons adhere to all CPT-related symmetries.
     - 8.3.2. Across ensembles, interactions mediated by 2-swap, 3-swap, and 4-swap G-family bosons adhere to CPT-symmetry.
     - 8.3.3. Across ensembles, interactions mediated by 2-swap, 3-swap, and 4-swap G-family bosons might appear (with respect to each of the two relevant ensembles) not adhere to some CPT-related symmetries.
       - 8.3.3.1. Such apparent violations might correlate with apparent violations of at least one of C-swap symmetry, P-swap symmetry, or T-swap symmetry.
     - 8.3.4. Existence of apparent violations of at least one of C-swap, P-swap, or T-swap symmetry might suffice (assuming symmetry under C-swap plus P-swap plus T-swap) to imply …
       - 8.3.4.1. Existence of apparent violations of each of C-swap, P-swap, and T-swap symmetry.
     - 8.3.5. Existence of apparent violations of at least one of C-swap, P-swap, or T-swap symmetry might correlate with …
       - 8.3.5.1. Existence of apparent violations of each of P-symmetry, C-symmetry, T-symmetry, and CP-symmetry.
9. People might say that work in this monograph correlates with the term approximate (in the traditional physics notions that each of the following is an approximate symmetry) …
   - 9.1. Conservation of parity.
   - 9.2. Invariance under charge conjugation.
   - 9.3. Invariance under time reversal. (Here, we use a traditional term.)
   - 9.4. CP-symmetry.

~ ~ ~

This subsection contrasts discussion above with traditional interpretations of conservation laws pertaining to charge.

In traditional physics, people correlate the conservation-of-charge conservation law with a symmetry people associate with the term Gauge invariance.

This monograph correlates conservation of charge with a lack of interactions that can change overall net charge.

# Section 2.13 FRERAN and COMPAR applications of LADDER models

Section 2.13 discusses LADDER models that correlate with internal properties of elementary particles, motions of particles, and interactions between elementary particles. We define INSSYM7, an instance-related symmetry. We provide perspective about transforming INTERN LADDER solutions into SPATIM LADDER solutions. We transform INTERN LADDER solutions so as to include SPATIM symmetries and INSSYM7 symmetry. People might say that, for some interpretations, INSSYM7 symmetry correlates with the possibility the universe includes multiple ensembles, with each ensemble including a set of elementary particles that includes at least the Standard Model set of elementary particles. People might say that one ensemble correlates with ordinary matter, five ensembles correlate with dark matter, some ensembles possibly correlate with dark-energy stuff, and some elementary particles do not correlate with an ensemble. People might say that we correlate G-family solutions with various symmetries, with various force laws, and with the concept of the span on an instance of a force. People might say that COMPAR-related symmetries and solutions correlate with phenomena correlating with QCD (or, quantum chromodynamics) and with aspects of composite particles. We discuss possibilities for states of matter (other than as found in hadrons) that involve quarks.

~ ~ ~

This subsection provides perspective about this section.

This section discusses symmetries that pertain to solutions that correlate with fields that correlate with elementary particles. One such symmetry is an instance-related symmetry we call INSSYM7.

Table 2.13.1 lists two concepts that people might correlate with INSSYM7.

**Table 2.13.1** Two concepts possibly correlating with INSSYM7

1. INSSYM7 pertains to mathematics underlying models this monograph shows.
2. INSSYM7 pertains to symmetry correlating with #ENS, which is the number of ensembles. (See Table 2.1.2.)

~ ~ ~

This subsection provides perspective about elementary particles and composite particles.

[Physics:] Some elementary particles, such as electrons and photons, can move over long distances. People apply the term free-ranging to such particles and to some other particles, such as the Z and W bosons, that, under normal circumstances, have limited ranges.

[Physics:] Some elementary particles have been found only in close association with other particles. Quarks and gluons provide examples. People apply the term non-free-ranging to these elementary particles.

For elementary particles, free-ranging correlates with $\sigma = +1$ and non-free-ranging correlates with $\sigma = -1$. Given that, under normal circumstances, the ranges of W- and H-family particles have similarities to the ranges of elementary bosons for which $\sigma = -1$, people might prefer, in some circumstances, to use terminology based on $\sigma$ rather than use terminology related to free-ranging.

[Physics:] People apply the term free-ranging to some particles, such as composite particles, that contain non-free-ranging particles. Protons and pions are examples of composite particles.

For convenience, in tables, we list composite particles as correlating with $\sigma = +1$.

~ ~ ~

This subsection provides perspective about constructing FRERAN SPATIM LADDER solutions from INTERN LADDER solutions.

Table 2.13.2 pertains. For cases in which $\sigma = +1$, we assume that QE-like considerations correlate with instance symmetry and QP-like considerations correlate with b\`. For cases in which $\sigma = -1$, we assume that QP-like considerations correlate with instance symmetry and QE-like considerations correlate with b\`. (Regarding instance symmetry, see, for example, Table 2.7.1 and Table 2.13.1.)

**Table 2.13.2** Steps for constructing a FRERAN SPATIM LADDER solution from one INTERN LADDER solution

1. We anticipate adding (to the relevant INTERN LADDER solution) …
   1.1. 3 QE-like oscillator pairs.
   1.2. 3 QP-like oscillator pairs.

2. We anticipate that each 3-generator SPATIM symmetry correlates with an SU(2) group that correlates with an instance of an added oscillator pair. (See Table 2.11.1 and Table 2.12.1.)
    2.1. For σ = +1, …
        2.1.1. We assume such an oscillator pair is QP-like.
    2.2. For σ = −1, …
        2.2.1. We assume such an oscillator pair is QE-like.
3. We anticipate …
    3.1. Setting each of the 12 new N(..) to the same value.
    3.2. For solutions not correlating with the QIRD-families, …
        3.2.1. The 12 values of N(..) equal the value of …
            3.2.1.1. N(E0), for cases for which σ = +1.
            3.2.1.2. N(P0), for cases for which σ = −1.
    3.3. For solutions correlating with the QIRD-families, …
        3.3.1. Table 2.13.9 shows results.
4. We add the 6 oscillator pairs.
    4.1. Doing so effects the transition from an INTERN LADDER solution to a FRERAN SPATIM LADDER solution.
    4.2. Work above in this table results in Œ = 0 for the FRERAN SPATIM LADDER solution.
5. Nominally, we assume #b' = 3.

∼ ∼ ∼

This subsection shows, for all families except the G-family, FRERAN SPATIM ground-state LADDER solutions and instance-related symmetries.

For σ = +1 non-G-family ground-state solutions, we follow steps Table 2.13.2 shows.

Table 2.13.3 shows ground-state FRERAN SPATIM LADDER solutions for non-G-family solutions for which σ = +1. (Regarding the symbol ∼, see Table 2.3.11.)

**Table 2.13.3** σ = +1 non-G-family ground-state FRERAN SPATIM LADDER solutions

| E | E | E | E | E | E | E | E | E | P | P | P | P | P | P | P | P | P | P | P | (σ = +1) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 8R | 8L | 6R | 6L | 4R | 4L | 2R | 2L | 0 | 0 | 2L | 2R | 4L | 4R | 6L | 6R | 8L | 8R | AL | AR | Sub-family |
| | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | | | | | 0H0 |
| | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | ∼ | ∼ | 0 | 0 | 0 | 0 | 0 | 0 | | | 1C |
| | | −1 | −1 | −1 | −1 | −1 | −1 | −1 | −1 | ∼ | ∼ | −1 | −1 | −1 | −1 | −1 | −1 | | | 1N |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | | | 2W |
| | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | −1 | 0 | 0 | ∼ | ∼ | 0 | 0 | 0 | 0 | 0 | 0 | 3N |

For each row in Table 2.13.3, the rightmost 3 open oscillator pairs correlate with 3 occurrences of SU(2) symmetries. These six oscillators are QP-like. We interpret each pair as correlating with a contribution to SPATIM symmetries. Thus, for each row in the table, #b' = 3. For each row in the table, the 7 QE-like oscillators E6R through E0 have identical values. INSSYM7 pertains. The term INSSYM abbreviates the term instance-related symmetry. For each row, a` = 1. For each of the rows in the table, (1;3)> symmetries pertain.

Table 2.13.4 restates Table 2.13.3. Table 2.13.4 uses the symbol 'I to point out aspects leading to occurrences of INSSYM7. For each row, the symbol 'I appears exactly 7 times. This table uses the symbol 'S to point out aspects correlating with the b` portion of (1;3)> symmetries. Here, for each row, b` = 3 pertains and the symbol 'S appears 6 times. (Here, use of the letter S correlates with the acronym SPATIM.) For each row, each of the oscillators for which N(..) = 'S does not pertain for the INTERN solution. For the 2W-subfamily, for each of the oscillators E8R and E8L, we assign the symbol 'G. This pair of 'G symbols correlates with an additional 3-generator (SU(2)-related) symmetry. [Physics:] People might say that this 3-generator symmetry correlates with conservation of generation for interactions between leptons and 2W bosons. For example, in an interaction in which the emission of a $W^-$ boson converts a muon into a neutrino, the neutrino is a muon-neutrino and not, for example, an electron-neutrino. People consider muons to be generation-2 leptons. Muon-neutrinos are generation-2 leptons. (See, for example, Section 2.8.)

**Table 2.13.4** A second depiction of σ = +1 non-G-family ground-state FRERAN SPATIM LADDER solutions

| E | E | E | E | E | E | E | E | E | P | P | P | P | P | P | P | P | P | P | P | (σ = +1) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 8R | 8L | 6R | 6L | 4R | 4L | 2R | 2L | 0 | 0 | 2L | 2R | 4L | 4R | 6L | 6R | 8L | 8R | AL | AR | Sub-family |
| | | 'I | 'I | 'I | 'I | 'I | 'I | 'I | 0 | 'S | 'S | 'S | 'S | 'S | 'S | | | | | 0H0 |
| | | 'I | 'I | 'I | 'I | 'I | 'I | 'I | 0 | ~ | ~ | 'S | 'S | 'S | 'S | 'S | 'S | | | 1C |
| | | 'I | 'I | 'I | 'I | 'I | 'I | 'I | −1 | ~ | ~ | 'S | 'S | 'S | 'S | 'S | 'S | | | 1N |
| 'G | 'G | 'I | 'I | 'I | 'I | 'I | 'I | 'I | 0 | 0 | 0 | 'S | 'S | 'S | 'S | 'S | 'S | | | 2W |
| | | 'I | 'I | 'I | 'I | 'I | 'I | 'I | −1 | 0 | 0 | ~ | ~ | 'S | 'S | 'S | 'S | 'S | 'S | 3N |

Table 2.13.5 restates Table 2.13.4. In Table 2.13.5, for each row, each of the three occurrences of "S correlates with the type of use we make of a pair of 'S in Table 2.13.4. For each row, b` = 3. Each "I correlates with the type of use we make of a pair of 'I in tables above. Each ~~ correlates with the type of use we make of a pair of ~ in Table 2.13.4. (See, for example, Table 2.3.11.) Each 00 correlates with the type of use we make of a pair of 0 in tables above. For the 2W row, the occurrence of "G correlates with the type of use we make of a pair of 'G in Table 2.13.4.

**Table 2.13.5** A third depiction of σ = +1 non-G-family ground-state FRERAN SPATIM LADDER solutions

| E | E | E | E | E | E | P | P | P | P | P | P | (σ = +1) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| A9 | 87 | 65 | 43 | 21 | 0 | 0 | 12 | 34 | 56 | 78 | 9A | Subfamily |
| | | "I | "I | "I | 'I | 0 | "S | "S | "S | | | 0H0 |
| | | "I | "I | "I | 'I | 0 | ~~ | "S | "S | "S | | 1C |
| | | "I | "I | "I | 'I | −1 | ~~ | "S | "S | "S | | 1N |
| | "G | "I | "I | "I | 'I | 0 | 00 | "S | "S | "S | | 2W |
| | | "I | "I | "I | 'I | −1 | 00 | ~~ | "S | "S | "S | 3N |

Table 2.13.6 shows instance-related symmetry and FRERAN SPATIM symmetries for solutions for which σ = +1. [Physics:] These solutions correlate with fields for all the free-ranging elementary particles, except for G-family particles, this monograph discusses.

**Table 2.13.6** Instance-related symmetry and FRERAN SPATIM symmetries for σ = +1 non-G-family ground-state ENVIRO LADDER solutions

| (σ = +1) | Instance-related symmetry | FRERAN SPATIM | Solution subfamily |
|---|---|---|---|
| | INSSYM7 | (1;3)> | 0H0 |
| | INSSYM7 | (1;3)> | 1C |
| | INSSYM7 | (1;3)> | 1N |
| | INSSYM7 | (1;3)> | 2W |
| | INSSYM7 | (1;3)> | 3N |

For elementary particles for which σ = −1, Table 2.13.7 pertains.

**Table 2.13.7** Correlation of aspects of FRERAN modeling with aspects of phenomena involving elementary particles for which σ = −1

1. People might say that people have yet to observe FRERAN phenomena for aspects of nature that we correlate with σ = −1.
2. People might say that people use, for aspects of nature that we correlate with σ = −1, traditional models that correlate with symmetries that correlate with special relativity. (See Table 6.3.1.)
    2.1. Examples include models that correlate with Feynman diagrams.
3. People might say that those models correlate with modeling we correlate with σ = +1.
4. We think FRERAN models pertain to some aspects of modeling elementary particles that we correlate with σ = −1.
    4.1. COMPAR models pertain to some other aspects of modeling elementary particles that we correlate with σ = −1.

For elementary particles for which σ = −1, we follow steps Table 2.13.2 shows.

Table 2.13.8 shows results (paralleling results Table 2.13.5 shows) for σ = −1 boson ground-state FRERAN SPATIM LADDER solutions. Here, we use the symbol ..,.. to indicate two occurrences of the .. symbol. [Physics:] People might say that one occurrence of "G correlates with conservation of fermion generation for interactions with non-lepton spin-1/2 elementary fermions. People might say that two occurrences of "G correlates with conservation of fermion generation for interactions with non-lepton spin-3/2 elementary fermions. (See, for example, Table 2.13.12.)

**Table 2.13.8** σ = −1 boson ground-state FRERAN SPATIM LADDER solutions

| E A9 | E 87 | E 65 | E 43 | E 21 | E 0 | P 0 | P 12 | P 34 | P 56 | P 78 | P 9A | (σ = −1) Subfamily |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | "S | "S | "S | 0 | 'I | "I | "I | "I | | | 0O |
| | "S | "S | "S | ..,.. | .. | 'I | "I | "I | "I | "G | | 2Y |
| | "S | "S | "S | 00 | 0 | 'I | "I | "I | "I | "G | | 2O |
| "S | "S | "S | ..,.. | ..,.. | .. | 'I | "I | "I | "I | "G | "G | 4Y |
| "S | "S | "S | 00 | 00 | 0 | 'I | "I | "I | "I | "G | "G | 4O |

Table 2.13.9 shows results for σ = −1 fermion ground-state FRERAN SPATIM LADDER solutions. (See Table 2.13.2.) Here, we use the symbol −1,−1 to indicate two occurrences of the symbol −1. Here, regarding the symbol ~,'I, the symbol ~ pertains to the one oscillator (in the relevant oscillator pair) for which N(..) ≠ 'I. (For each row in the table, there are 4 cases. Here, 4 = 2 × 2, with one factor of 2 correlating with the symbol ~~. For each value of ~~, 2 solutions pertain. For example, for the 1Q-subfamily, for one solution, ~ correlates with oscillator P1 and 'I correlates with oscillator P2. More specifically, 'I = N(P2). For the other solution, ~ correlates with oscillator P2 and 'I correlates with oscillator P1. More specifically, 'I = N(P1).)

**Table 2.13.9** σ = −1 fermion ground-state FRERAN SPATIM LADDER solutions

| E A9 | E 87 | E 65 | E 43 | E 21 | E 0 | P 0 | P 12 | P 34 | P 56 | P 78 | P 9A | (σ = −1) Subfamily |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | "S | "S | "S | ~~ | 0 | 0 | ~,'I | "I | "I | "I | | 1Q |
| | "S | "S | "S | ~~ | −1 | −1 | ~,'I | "I | "I | "I | | 1R |
| "S | "S | "S | ~~ | 00 | 0 | 0 | 00 | ~,'I | "I | "I | "I | 3Q |
| "S | "S | "S | ~~ | −1,−1 | 0 | 0 | −1,−1 | ~,'I | "I | "I | "I | 3I |
| "S | "S | "S | ~~ | 00 | −1 | −1 | 00 | ~,'I | "I | "I | "I | 3R |
| "S | "S | "S | ~~ | −1,−1 | −1 | −1 | −1,−1 | ~,'I | "I | "I | "I | 3D |

Table 2.13.10 shows instance-related symmetry and FRERAN SPATIM symmetries for solutions for which σ = −1. [Physics:] These solutions correlate with fields for all the non-free-ranging elementary particles this monograph discusses. (Starting with discussion pertaining to Table 2.13.26, we discuss, for σ = −1 solutions, COMPAR SPATIM symmetries.)

**Table 2.13.10** Instance-related symmetry and FRERAN SPATIM symmetries for $\sigma = -1$ ground-state FRERAN SPATIM LADDER solutions

| ($\sigma = -1$) | Instance-related symmetry | FRERAN SPATIM | Solution subfamily |
|---|---|---|---|
| | INSSYM7 | (1;3)> | 0O |
| | INSSYM7 | (1;3)> | 2YO |
| | INSSYM7 | (1;3)> | 4YO |
| | INSSYM7 | (1;3)> | 1QR |
| | INSSYM7 | (1;3)> | 3QIRD |

~ ~ ~

This subsection provides perspective about the next few subsections.

Above, for each family (of elementary particles that work in this monograph could correlate with) except the G-family, we show models correlating with INSSYM7 symmetry and (1;3)> FRERAN SPATIM symmetries.

The next few subsections explore, for the G-family, various aspects and possible symmetries.

~ ~ ~

This subsection introduces the concept of channels.

[Physics:] Define a number $\beta$ via the formula $(4/3)(\beta^6)^2$ = (electromagnetic repulsion between two electrons) / (gravitational attraction between the same two electrons). (See Table 3.9.2.) Numerically, $\beta$ × (the rest mass of an electron) may provide a more accurate estimate (and a smaller standard deviation) for the rest mass of a tauon than do experiments (as of 2014). (See Table 3.9.5.) This monograph interprets the exponent 2 as correlating with the two interaction vertices in which a G-family boson participates. This monograph interprets $\beta^6$ as providing a per-channel-per-vertex ratio of vertex strengths - in terms of the strength of electromagnetism (in general) and the strength of gravitation (in general). (See, also, Table 3.9.3, Table 3.9.20, Table 3.9.21, and Table 3.13.4.) This thinking correlates with concepts related to channels. Perhaps, people will think of channels as parallel paths connecting the two vertices in an interaction between two fermions. MM1MS1-photon (or, 2G2&) vertices (and interactions) have four channels. Gravitational (or, 4G4&) vertices (and interactions) have three channels. #P = 6 G-family vertices would correlate with two channels. #P = 8 G-family vertices would correlate with one channel. #P = A (or, #P = [10]) vertices would have zero strength. (Table 3.3.3 shows definitions and uses of the symbols 2G2& and 4G4&.)

This monograph suggests two possibilities for modelling the number of channels. We correlate one possibility with the acronym EACUNI. We correlate one possibility with the acronym SOMMUL. Table 2.13.11 pertains. This monograph does not explore much the possibility that notions of channels pertain to other than G-family elementary particles.

**Table 2.13.11** Two ways to model channels related to G-family elementary particles

1. Aspects common to EACUNI models and SOMMUL models.
    - 1.1. For each G-family elementary particle, …
        - 1.1.1. A symbol of the form nG%& denotes the particle. (See, for example, Table 3.3.3.)
        - 1.1.2. One of more epnG%& solutions exist (with e = #E ≤ 6 and with p = #P ≤ 8).
        - 1.1.3. Either (EACUNI or SOMMUL) set of models correlates with the number of channels.
    - 1.2. Table 3.2.1 provides an example, based on 2G2&.
    - 1.3. Table 3.2.8 provides an example, based on 4G4&.
2. Aspects specific to EACUNI models.
    - 2.1. The acronym EACUNI abbreviates the words each G-family elementary particle correlates with a unique G-family solution.
    - 2.2. For a G-family particle nG%&, …
        - 2.2.1. The solution with the least value of #P correlates with the particle.
            - 2.2.1.1. That solution also has the least value of #E.
        - 2.2.2. Any other solutions do not correlate with the particle.
    - 2.3. Table 2.13.12 shows …
        - 2.3.1. How to represent channels.
        - 2.3.2. How to count the number of channels.
3. Aspects specific to SOMMUL models.
    - 3.1. The acronym SOMMUL abbreviates the words some G-family particles correlate with multiple G-family solutions (or, more than one G-family solution).
    - 3.2. For a G-family particle nG%&, …
        - 3.2.1. Each epnG%& solution (with e = #E ≤ 6 and with p = #P ≤ 8) correlates with the particle.
        - 3.2.2. The number of such solutions equals the number of channels.

Work below (in this section specifically and in this monograph generally) may use EACUNI models and tends to deemphasize SOMMUL models.

~ ~ ~

This subsection starts development of G-family ground-state ENVIRO LADDER solutions.

We use the limits #E ≤ 6 and #P ≤ 8. We use the possible relevance of SU(7) and the possible relevance of SU(17). (See Table 8.2.1.)

In Table 2.13.12, we show 17 QE-like oscillators and 17 QP-like oscillators. We introduce the symbols #E" and #P". We let #E" = #P" = [G] = 16 denote limits regarding oscillators.

We assume that #b' = 3 for each G-family elementary particle. (See Table 2.12.3.)

Table 2.13.12 shows ground-state FRERAN SPATIM LADDER solutions for G-family subfamilies. (Compare with Table 2.8.2.) Each x denotes a closed oscillator pair. For each row in the table, the number of such pairs of closed oscillators equals (1/2) (#P – #E). For each row in the table, based on the rightmost four QE-like columns (that is, column E6-and-E5 through column E0), one might think that, for each solution, INSSYM7 pertains. Per the rightmost three QP-like columns (that is, column PB-and-PC through column PF-and-PG) of Table 2.13.12, #b' = 3 pertains for each row. The symbol 00 symbolizes a 0 for each of the two oscillators to which the column pertains. The table uses EACUNI models. QP-like use of the symbol * correlates with the concept of channels. The number of QP-like uses of the symbol * equals the number of channels. (For each row in Table 2.13.12, the number of QE-like instances of the symbol * equals the number of QP-like instances of the symbol *.) People might say that each * correlates with a closed oscillator pair. (See Table 2.2.8.) The symbol "G denotes QE-like instances of 00 for oscillators for which "I does not pertain in ground-state FRERAN SPATIM LADDER solutions. (See Table 2.8.2.)

**Table 2.13.12** Channels and (bases for) FRERAN SPATIM symmetries for G-family ground-state LADDER solutions (assuming EACUNI models and that #b' = 3 pertain for each solution or subfamily)

| E GF | E ED | E CB | E A9 | E 87 | E 65 | E 43 | E 21 | E 0 | P 0 | P 12 | P 34 | P 56 | P 78 | P 9A | P BC | P DE | P FG | Solution or subfamily |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| x | * | * | * | * | "I | "I | "I | 'I | −1 | 00 | * | * | * | * | @' | @' | @' | 022G2& |
| x | * | * | * | "G | "I | "I | "I | 'I | −1 | 00 | 00 | * | * | * | @' | @' | @' | 24..G |
| x | x | * | * | * | "I | "I | "I | 'I | −2 | 00 | 00 | * | * | * | @' | @' | @' | 042G24& |
| x | * | * | "G | "G | "I | "I | "I | 'I | −1 | 00 | 00 | 00 | * | * | @' | @' | @' | 46..G |
| x | x | * | * | "G | "I | "I | "I | 'I | −2 | 00 | 00 | 00 | * | * | @' | @' | @' | 26..G |
| x | x | x | * | * | "I | "I | "I | 'I | −3 | 00 | 00 | 00 | * | * | @' | @' | @' | 064G246& |
| x | * | "G | "G | "G | "I | "I | "I | 'I | −1 | 00 | 00 | 00 | 00 | * | @' | @' | @' | 68..G |
| x | x | * | "G | "G | "I | "I | "I | 'I | −2 | 00 | 00 | 00 | 00 | * | @' | @' | @' | 48..G |
| x | x | x | * | "G | "I | "I | "I | 'I | −3 | 00 | 00 | 00 | 00 | * | @' | @' | @' | 28..G |
| x | x | x | x | * | "I | "I | "I | 'I | −4 | 00 | 00 | 00 | 00 | * | @' | @' | @' | 084G2468& |

~ ~ ~

This subsection discusses models for interpreting aspects of FRERAN SPATIM symmetries for G-family ground-state LADDER solutions.

Table 2.13.13 discusses some math possibly related to interpretations of INSSYM7.

**Table 2.13.13** Math related to possible interpretations of INSSYM7

1. INSSYM7 correlates with SU(7).
2. The following expressions correlate with subgroups of SU(7).

$$SU(7) \supset SU(5) \times SU(2) \times U(1) \qquad (2.105)$$

$$SU(7) \supset SU(3) \times SU(4) \times U(1) \qquad (2.106)$$

3. The following pertain.
   - 3.1. For SU(7), …
     - 3.1.1. 48 generators pertain.
     - 3.1.2. 48 / 48 = 1.
   - 3.2. For SU(5), …
     - 3.2.1. 24 generators pertain.
     - 3.2.2. 48 / 24 = 2.
   - 3.3. For SU(3), …
     - 3.3.1. 8 generators pertain.
     - 3.3.2. 48 / 8 = 6.
   - 3.4. For no symmetry, …
     - 3.4.1. 48 / 1 = 48.

Table 2.13.14 discusses possible physics-relevance of ensembles. Motivation for this table includes physics data that the first item in the table discusses.

**Table 2.13.14** Possible correlation between observations and instance-related symmetries

1. [Physics:] People say that gravity intermediates interactions between ordinary matter and dark matter. Regarding the recent universe, inferred ratios of dark-matter density of the universe to ordinary-matter density of the universe exceed 5, but not by much. Possibly, inferred ratios (pertaining to earlier times) have not varied much over most of the history of the universe. (See Section 4.3.)
2. Gravitational interactions occur between ordinary matter and dark matter.
3. Perhaps, a QE-like SU(3) symmetry correlates with gravitons (and gravity). (See the 24..G row in Table 2.8.2. There, #E = 2.)
   - 3.1. 244G4& correlates with gravitons. (See Section 3.2 and Section 3.3.)
4. Perhaps, the 8 generators correlating with that symmetry correlate with the 8 generators correlating with an item in Table 2.13.13.
5. Perhaps the number 6 (correlating with that item in Table 2.13.13) correlates with 6 similar units of stuff.
6. Perhaps, dark matter correlates (in essence) with 5 copies of ordinary matter.
7. Perhaps, the universe includes at least 6 ensembles. (See Table 2.1.2.)
   - 7.1. Numbers of physics-relevant ensembles might be (See Table 2.13.13.) either of …

7.1.1. 48.
7.1.2. 6.
7.2. Here, we ignore the following (because each is less than 6). (Elsewhere, we discuss ENS1 models, which feature just 1 physics-relevant ensemble. Generally, we deemphasize the case that would feature exactly 2 physics-relevant ensembles.)
7.2.1. 2.
7.2.2. 1.
8. Perhaps, one of ENS48 models and ENS6 models pertains.
9. For each row in Table 2.8.2 for which #E = 2, 4, or 6, …
9.1. Let #E` = #E.
9.2. Perhaps, the number of generators of SU(#E` + 1) correlates with a number of physics-relevant instances of each solution that correlates with the row.
9.2.1. Above, this table uses #E` = 2 for 244G4&.
10. The notions that 8 instances of 244G4& are physics-relevant and that each instance correlates with 6 ensembles correlate with …
10.1. 48 instances of 022G2& are physics-relevant.
10.2. For 022G2&, …
10.2.1. #E = 0.
10.2.2. We can assume …
10.2.2.1. #E` = 6.
10.2.2.2. ENS48 models pertain.
11. The notions that 1 instance of 244G4& is physics-relevant and that the one instance correlates with 6 ensembles correlate with …
11.1. For gravitons, …
11.1.1. #E` = 2.
11.1.2. Only 1 of the 8 instances of 244G4& is physics-relevant.
11.2. 6 instances of 022G2& are physics-relevant.
11.3. For 022G2&, …
11.3.1. #E = 0.
11.3.2. We can assume …
11.3.2.1. #E` = 6.
11.3.2.2. Only 6 of the possible 48 instances of 022G2& are physics-relevant.
11.3.2.3. ENS6 models pertain.

~ ~ ~

[Physics:] This subsection shows a method for cataloging, relative to ordinary matter, instances (correlating with ENS48 models) of elementary particles and composite particles.

Traditional physics discusses at least three possible types of stuff - ordinary matter (or, baryonic matter), dark matter, and (possibly) dark-energy stuff. (See Section 4.3.)

So far, we interpret, for ENS48 models, INSSYM7 as correlating with the existence of 48 instances of each elementary particle (other than G-family particles) and of each composite particle. We discuss the existence of 48 ensembles. Each ensemble correlates with an instance of 2G2&, of each elementary particle other than non-2G2& G-family particles, and of each composite particle.

One ensemble correlates with ordinary matter. Table 2.13.15 shows a means to catalog ensembles and to hypothetically include (in such a catalog) various instances of G-family particles other than 2G2&. Table 2.13.16 discusses possible concerns regarding these definitions and adds some definitions. Later, we use the term G#XY to symbolize individual symbols (and all such symbols) items 4, 5, and 6 in Table 2.13.15 show.

**Table 2.13.15** Definitions of OME, DME, DEE, and hypothetical definitions of sets of instances of non-2G2& G-family particles (ENS48 models)

1. The acronym OME denotes ordinary-matter ensemble.
    - 1.1. OME abbreviates ordinary-matter ensemble.
    - 1.2. 1 OME exists.
        - 1.2.1. People sometimes use the term baryonic matter instead of the term ordinary matter.
2. The acronym DME denotes dark-matter ensemble or dark-matter ensembles.
    - 2.1. DME abbreviates dark-matter ensemble(s).
    - 2.2. 5 DME exist.
    - 2.3. One instance of 4G4& intermediates interactions within and between each of the one OME and five DME.
3. The acronym DEE denotes dark-energy ensemble or dark-energy ensembles.
    - 3.1. DEE abbreviates dark-energy ensemble(s).
    - 3.2. 42 DEE exist.
    - 3.3. The instance of 4G4& that intermediates interactions within and between each of the one OME and five DME does not interact with DEE.
4. Regarding G-family particles with spans of 2, …
    - 4.1. G2OMOM correlates with interactions within the OME.
    - 4.2. G2OMDM correlates with interactions between the OME and a DME.
    - 4.3. G2DMDM correlates with interactions between a DME and a DME.
        - 4.3.1. The two DME may be different ensembles.
        - 4.3.2. The two DME may be the same ensemble.
    - 4.4. G2DEDE correlates with interactions between a DEE and a DEE.
        - 4.4.1. The two DEE may be different ensembles.
        - 4.4.2. The two DEE may be the same ensemble.
5. Regarding G-family particles with spans of 6, …
    - 5.1. G6OMOM correlates with interactions within the OME.

- 5.2. G6OMDM correlates with interactions between the OME and a DME.
- 5.3. G6DMDM correlates with interactions between a DME and a DME.
  - 5.3.1. The two DME may differ.
  - 5.3.2. The two DME may be the same ensemble.
- 5.4. G6DEDE correlates with interactions between a DEE and a DEE.
  - 5.4.1. The two DEE may differ.
  - 5.4.2. The two DEE may be the same ensemble.

6. Regarding G-family particles with spans of 48, …
   - 6.1. G48OMOM correlates with interactions within the OME.
   - 6.2. G48OMDM correlates with interactions between the OME and a DME.
   - 6.3. G48OMDE correlates with interactions between the OME and a DEE.
   - 6.4. G48DMDM correlates with interactions between a DME and a DME.
     - 6.4.1. The two DME may differ.
     - 6.4.2. The two DME may be the same ensemble.
   - 6.5. G48DMDE correlates with interactions between a DME and a DEE.
   - 6.6. G48DEDE correlates with interactions between a DEE and a DEE.
     - 6.6.1. The two DEE may differ.
     - 6.6.2. The two DEE may be the same ensemble.

Table 2.13.16 extends Table 2.13.15 and notes possible concerns about the catalog that Table 2.13.15 suggests.

**Table 2.13.16** Discussion regarding a possible cataloging method for ensembles and other particles, plus definitions of DME-1, DME-4, and OMDME (ENS48 models)

1. The definition of DME correlates with notions that …
   - 1.1. The OME is one of six ensembles spanned by one instance of 4G4&.
   - 1.2. The term DME correlates with the other five such ensembles.
2. One possible concern could be that, for at least one of 4G268& and 2G468&, the instance (of that particle) that correlates with the OME does not correlate exactly with the five ensembles we designate as DME (and, thus, does correlate with a different set of five non-OME ensembles, of which some least some ensembles would be DEE).
   - 2.1. We acknowledge this possibility.
   - 2.2. This monograph deemphasizes this possibility.
3. Similar concerns could arise regarding alignment of spans between 4G4& and G-family bosons with spans of 2.
   - 3.1. This monograph assumes that each instance of a G-family boson with a span of 2 intermediates interactions only within a set of two ensembles for which one instance of 4G4& intermediates interactions between the same two ensembles.

4. Similar concerns could arise regarding possible non-alignment of spans between two types of G-family bosons, each having span 2. (These concerns would parallel those regarding alignment of spans regarding two types of G-family bosons for which each type has a span of 6.)
   4.1. Again, we acknowledge and deemphasize the possibility.
5. Given discussion above in this table, we define the following.
   5.1. The acronym DME-4 denotes the four DME for which no span-2 G-family boson intermediates interactions between those ensembles and the OME.
   5.2. The acronym DME-1 denotes the one DME with which those instances of G-family bosons with span 2 for which the instances interact with the OME interact with this one DME.
6. The acronym OMDME denotes ordinary-matter plus dark-matter ensemble(s).
   6.1. 6 OMDME exist.
7. Another type of possible concern involves G#XY cataloguing of non-2G2& G-family bosons. For non-2G2& G-family bosons that have been emitted and absorbed, (at least conceptually) the G#XY cataloguing scheme might be useful. But what about, for example, a 4G4& particle that the OME has emitted and nothing has yet to absorb? Presumably, eventually, the particle would correlate with one of G6OMOM and G6OMDM. But, until absorption occurs, (conceptually) one cannot determine which one.
   7.1. For this monograph, this issue pertains only to the topic of densities of the universe of various types of stuff.
      7.1.1. Possibly, regarding known and anticipated data, the issue has little significance.

The second, third, and fourth items in Table 2.13.16 reflect an assumption that people might characterize by the term telescoping or by the term nesting. Table 2.13.17 explains.

**Table 2.13.17** Principles and characteristics of assumptions, regarding interactions mediated by G-family bosons, of telescoping or nesting (ENS48 models)

1. Instances of G-family particles with spans of 2 align.
   1.1. We assume that, if one instance of a 2-ensemble-span G-family boson intermediates interactions between two ensembles, any instance of another 2-ensemble-span G-family boson that interacts with one of those ensembles interacts with the other one of those two ensembles (and not with a third ensemble).
   1.2. This assumption groups ensembles into 24 sets such that each instance of a 2-ensemble-span G-family boson correlates with exactly one set of two ensembles.

2. Instances of G-family particles with spans of 6 align.
    2.1. We assume that, if one instance of a 6-ensemble-span G-family boson intermediates interactions between two ensembles, any instance of another 6-ensemble-span G-family boson that interacts with one of those ensembles interacts with the other one of those two ensembles.
    2.2. This assumption groups ensembles into 8 sets such that each instance of a 6-ensemble-span G-family boson correlates with exactly one set of the six ensembles.
3. Instances of G-family particles with spans of 2 align with instances of G-family particles with spans of 6.
    3.1. We assume that each instance of a 2-ensemble-span G-family boson intermediates interactions with two ensembles within the span of a relevant instance of each 6-ensemble-span G-family boson.
    3.2. The assumption correlates with each of the 24 sets of two ensembles being a subset of just one of the 8 sets of six ensembles.

~ ~ ~

This subsection discusses the possibility that non-G-family elementary bosons mediate interactions that span ensembles.

People might say that, for example, if 244G4& can mediate interactions between fermions in two different ensembles, perhaps, 2W can mediate interactions between fermions in two different ensembles. A reason might be that similar #"G = 1 considerations pertain for each of 244G4& and 2W.

Table 2.13.18 pertains.

**Table 2.13.18** Possible aspects correlating with deemphasizing the notion that non-G-family elementary bosons mediate interactions that span ensembles

1. Practicality.
    1.1. Regarding conditions for which people make or infer measurements, …
        1.1.1. The amount of stuff (from ensembles other than the OME) within the range of a WHO-family boson or a Y-family boson (emitted or absorbed by an OME fermion) could be minimal. (Regarding ranges, see, for example, elementary bosons that Table 2.9.5 lists.)
        1.1.2. People might say that cross-ensemble interactions would …
            1.1.2.1. Be hard to detect.
            1.1.2.2. Have essentially no effect, either locally or on (anything other than the very early) evolution of universe.
2. Theory.

   2.1. The G-family correlates with concepts of polarization modes.
   2.2. Each of the WHO-families and the Y-family does not similarly correlate with concepts of polarization modes.
   2.3. Perhaps, this distinction leads to a relevant difference regarding relevant models.
3. Theory.
   3.1. The G-family correlates with concepts of channels.
   3.2. Each of the WHO-families and the Y-family may not similarly correlate with concepts of channels.
   3.3. Perhaps, this distinction leads to a relevant difference regarding relevant models.

~ ~ ~

This subsection discusses possible numbers of physics-relevant ensembles and possible values of #E`.

Table 2.13.19 pertains.

**Table 2.13.19** Possible values of #E` for G-family bosons for which #E = 0 (ENS1 models, ENS6 models, and ENS48 models)

1. For ENS1 models, …
   1.1. #ENS = 1.
   1.2. Only 1 ensemble (the OME) is physics-relevant.
      1.2.1. No elementary bosons intermediate interactions between two different ensembles.
   1.3. In effect, we can assume …
      1.3.1. #E` = 6 for each G-family boson for which #E = 0.
2. For ENS6 models, …
   2.1. #ENS = 6.
   2.2. 6 ensembles (the OME and the 5 DME) are physics-relevant.
      2.2.1. No elementary bosons intermediate interactions between an OMDME and a DEE.
   2.3. In effect, we can assume …
      2.3.1. #E` = 6 for 222G2&.
      2.3.2. #E` = 6, 4, or 2 for each other G-family boson for which #E = 0.
3. For ENS48 models, …
   3.1. #ENS = 48.
   3.2. All 48 ensembles (the OME, the 5 DME, and the 42 DEE) are physics-relevant.
      3.2.1. At least one G-family elementary boson intermediates interactions between the OME and a DEE.

3.2.1.1. Thus, at least one G-family boson correlates with #E` = 0
3.2.2. None of the G-family bosons for which #E ≠ 0 exhibits #E` = 0.
3.3. We require …
3.3.1. #E` = 6 for 222G2&.
3.3.2. #E` = 0 for at least one G-family boson for which #E = 0.
3.4. In effect, we can assume …
3.4.1. #E` = 6, 4, or 2 for each other G-family boson for which #E = 0.

~ ~ ~

This subsection discusses ENS48 models this monograph features.

For ENS48 models, this monograph features one set of choices for #E` for G-family bosons for which #E = 0. We call this choice ENS48". This monograph deemphasizes other possibilities. (See Table 2.13.19.)

Table 2.13.20 shows assumptions regarding #E` for G-family solutions for which #E = 0. Here, the solution 084G2468& correlates with an elementary particle (4G2468&) that can intermediate interactions between fermions in any 2 of 48 ensembles. In effect, we interpolate between 022G2& and 084G2468&. We correlate this interpolation with the term ENS48" models. With this interpretation, the solution 084G2468& provides the only example of a solution for which the span is 48 ensembles.

**Table 2.13.20** Values of #E` for G-family solutions for which #E = 0 (ENS48" models)

1. For 022G2&, #E` = 6.
2. For 042G24&, #E` = 4.
3. For 064G246&, #E` = 2.
4. For 084G2468&, #E` = 0.

~ ~ ~

This subsection discusses ENS6 models this monograph features.

For ENS6 models, this monograph features one set of choices for #E` for G-family bosons for which #E = 0. We call this choice ENS6'. This monograph deemphasizes other possibilities. (See Table 2.13.19.)

Table 2.13.21 shows assumptions regarding #E` for G-family solutions for which #E = 0. Here, we set (the other) three values to match the value pertaining to 022G2&. We correlate this extrapolation with the term ENS6' models.

**Table 2.13.21** Values of #E` for G-family solutions for which #E = 0 (ENS6' models)

1. For 022G2&, #E` = 6.

2. For 042G24&, #E` = 6.
3. For 064G246&, #E` = 6.
4. For 084G2468&, #E` = 6.

~ ~ ~

This subsection discusses some aspects of ENS48 models.

Table 2.13.22 discusses aspects regarding solutions correlating with various values of #E`. For each value of #E`, the table shows a group that correlates with a relevant symmetry, the number of generators that correlates with the group, the number instances of each G-family solution for which #E` pertains, and the span of such a solution. Here, span denotes the number of ensembles that an instance of a G-family solution connects.

**Table 2.13.22** Instances and spans, by #E`, for G-family solutions (ENS48 models)

1. A #E` = 6 G-family solution correlates with an SU(7) symmetry.
   1.1. SU(7) correlates with 48 generators.
   1.2. 48 / 48 = 1.
   1.3. An instance of a #E` = 6 G-family boson can intermediate interactions between fermions in 1 of 48 ensembles (assuming the specific fermions interact via the boson).
   1.4. A #E` = 6 G-family boson cannot intermediate interactions between the 1 ensemble and any other ensembles.
   1.5. 48 instances of such a boson exist, with each instance pertaining to 1 ensemble.
2. A #E` = 4 G-family solution correlates with an SU(5) symmetry.
   2.1. SU(5) correlates with 24 generators.
   2.2. 48 / 24 = 2.
   2.3. An instance of a #E` = 4 G-family boson can intermediate interactions between fermions in any 2 of 2 ensembles (assuming the specific fermions interact via the boson).
   2.4. A #E` = 4 G-family boson cannot intermediate interactions between those 2 ensembles and any other ensembles.
   2.5. 24 instances of such a boson exist, with each instance pertaining to 1 set of 2 ensembles.
3. A #E` = 2 G-family solution correlates with an SU(3) symmetry.
   3.1. SU(3) correlates with 8 generators.
   3.2. 48 / 8 = 6.
   3.3. An instance of a #E` = 2 G-family boson can intermediate interactions between fermions in any 2 of 6 ensembles (assuming the specific fermions interact via the boson).

3.4. A #E` = 2 G-family boson cannot intermediate interactions between those 6 ensembles and any other ensembles.
3.5. 8 instances of such a boson exist, with each instance pertaining to 1 set of 6 ensembles.
4. The #E` = 0 G-family solution correlates with 1 instance.
4.1. 48 / 1 = 48.
4.2. The #E` = 0 G-family boson can intermediate interactions between fermions in any 2 of 48 ensembles (assuming the specific fermions interact via the boson).
4.3. 1 instance of the boson exists, with that instance pertaining to the 1 set of 48 ensembles.

For ENS48" models (and not for other ENS48 models), Table 2.13.23 summarizes results from above and brings forward elementary-particle names from Table 3.3.4. (See, for example, Table 2.8.3, Table 2.13.12, Table 2.13.20, and Table 2.13.22.) The SPAN column provides the number of ensembles an instance of the particle spans. The INST column shows the number of instances of the particle. The CHN column shows the number of channels. The #"G column shows that number of instances of the symbol "G.

**Table 2.13.23** Spans and instances for G-family particles (ENS48" models)

| G-family particle (%68both) | S | SDI | SPAN | INST | CHN | #"G |
|---|---|---|---|---|---|---|
| 2G2& | 1 | $r^{-2}$ | 1 | 48 | 4 | 0 |
| 4G4& | 2 | $r^{-2}$ | 6 | 8 | 3 | 1 |
| 2G24& | 1 | $r^{-4}$ | 2 | 24 | 3 | 0 |
| 4G26& | 2 | $r^{-4}$ | 6 | 8 | 2 | 1 |
| 2G46& | 1 | $r^{-4}$ | 6 | 8 | 2 | 1 |
| 2G246& | 1 | $r^{-6}$ | 6 | 8 | 2 | 0 |
| 4G48& | 2 | $r^{-4}$ | 2 | 24 | 1 | 2 |
| 2G68& | 1 | $r^{-4}$ | 2 | 24 | 1 | 2 |
| 2G248& | 1 | $r^{-6}$ | 6 | 8 | 1 | 1 |
| 4G268& | 2 | $r^{-6}$ | 6 | 8 | 1 | 1 |
| 2G468& | 1 | $r^{-6}$ | 6 | 8 | 1 | 1 |
| 4G2468& | 2 | $r^{-8}$ | 48 | 1 | 1 | 0 |

~ ~ ~

This subsection discusses some aspects of ENS6 models.

For ENS6' models (and not for other ENS6 models), Table 2.13.24 summarizes results from above and brings forward elementary-particle names from Table 3.3.4. (See, for example, Table 2.8.3, Table 2.13.12, and Table 2.13.21.) The SPAN column provides the number of ensembles an instance of the particle spans. The INST column shows the number of instances of the particle. The CHN column shows the number of channels. The #"G column shows that number of instances of the symbol "G.

**Table 2.13.24** Spans and instances for G-family particles (ENS6' models)

| G-family particle (%68both) | S | SDI | SPAN | INST | CHN | #"G |
|---|---|---|---|---|---|---|
| 2G2& | 1 | $r^{-2}$ | 1 | 6 | 4 | 0 |
| 4G4& | 2 | $r^{-2}$ | 6 | 1 | 3 | 1 |
| 2G24& | 1 | $r^{-4}$ | 1 | 6 | 3 | 0 |
| 4G26& | 2 | $r^{-4}$ | 6 | 1 | 2 | 1 |
| 2G46& | 1 | $r^{-4}$ | 6 | 1 | 2 | 1 |
| 2G246& | 1 | $r^{-6}$ | 1 | 6 | 2 | 0 |
| 4G48& | 2 | $r^{-4}$ | 2 | 3 | 1 | 2 |
| 2G68& | 1 | $r^{-4}$ | 2 | 3 | 1 | 2 |
| 2G248& | 1 | $r^{-6}$ | 6 | 1 | 1 | 1 |
| 4G268& | 2 | $r^{-6}$ | 6 | 1 | 1 | 1 |
| 2G468& | 1 | $r^{-6}$ | 6 | 1 | 1 | 1 |
| 4G2468& | 2 | $r^{-8}$ | 1 | 6 | 1 | 0 |

~ ~ ~

This subsection discusses some aspects of ENS1 models.

For ENS1 models, Table 2.13.25 summarizes results from above and brings forward elementary-particle names from Table 3.3.4. (See, for example, Table 2.8.3 and Table 2.13.12.) The SPAN column provides the number of ensembles an instance of the particle spans. The INST column shows the number of instances of the particle. The CHN column shows the number of channels. The #"G column shows that number of instances of the symbol "G.

**Table 2.13.25** Spans and instances for G-family particles (ENS1 models)

| G-family particle (%68both) | S | SDI | SPAN | INST | CHN | #"G |
|---|---|---|---|---|---|---|
| 2G2& | 1 | $r^{-2}$ | 1 | 1 | 4 | 0 |
| 4G4& | 2 | $r^{-2}$ | 1 | 1 | 3 | 1 |
| 2G24& | 1 | $r^{-4}$ | 1 | 1 | 3 | 0 |
| 4G26& | 2 | $r^{-4}$ | 1 | 1 | 2 | 1 |
| 2G46& | 1 | $r^{-4}$ | 1 | 1 | 2 | 1 |
| 2G246& | 1 | $r^{-6}$ | 1 | 1 | 2 | 0 |
| 4G48& | 2 | $r^{-4}$ | 1 | 1 | 1 | 2 |
| 2G68& | 1 | $r^{-4}$ | 1 | 1 | 1 | 2 |
| 2G248& | 1 | $r^{-6}$ | 1 | 1 | 1 | 1 |
| 4G268& | 2 | $r^{-6}$ | 1 | 1 | 1 | 1 |
| 2G468& | 1 | $r^{-6}$ | 1 | 1 | 1 | 1 |

| G-family particle (%68both) | S | SDI | SPAN | INST | CHN | #"G |
|---|---|---|---|---|---|---|
| 4G2468& | 2 | $r^{-8}$ | 1 | 1 | 1 | 0 |

~ ~ ~

This subsection discusses further uses of the terms ENS48 models, ENS48" models, ENS6 models, and ENS6' models and further uses of concepts related to those terms.

People might say that, generally, some details of subsequent work in this monograph correlate with assuming that ENS48 models are ENS48" models. We think that people can, if desired, modify such work to explore ENS48 models that are not ENS48" models. Modifications would involve changing one or more of the values of #E` that Table 2.13.20 shows.

People might say that, generally, some details of subsequent work in this monograph correlate with assuming that ENS6 models are ENS6' models. We think that people can, if desired, modify such work to explore ENS6 models that are not ENS6' models. Modifications would involve changing one or more of the values of #E` that Table 2.13.21 shows.

~ ~ ~

This subsection shows COMPAR LADDER solutions for elementary particles for which σ = −1.

For each elementary particle for which σ = −1, the term free-ranging does not pertain. People might say that physics of composite particles correlates with models based on special relativity and that such models correlate with (1;3)> symmetries. People might say that people can expect that (1;3)> symmetries pertain for fields correlating with the entirety of a composite particle. We explore the possibility that (a`;b`)< symmetries pertain for fields related to elementary particles (for which σ = −1) that combine to form composite particles.

People might say that, for bosons for which σ = −1, people should explore the notion that INSSYM7 pertains. (See Table 2.13.10.) Starting from ground-state INTERN LADDER solutions, we add enough QP-like harmonic oscillators to bring the total number of QP-like oscillators to 7. We assume a` = 1. We set N(..) for each added oscillator to match N(..) for each previously relevant QP-like oscillator. To maintain Œ = 0, we add the same (as we added QP-like oscillators) number of QE-like oscillators and use, for each of these added oscillators, the value N(..) that we use for added QP-like oscillators. Let b` denote the number of added QE-like pairs of oscillators. The number b` is an integer. We assume that (1;b`)< symmetries pertain.

Table 2.13.26 shows ground-state COMPAR ENVIRO LADDER solutions for elementary bosons for which σ = −1.

**Table 2.13.26** σ = −1 boson ground-state COMPAR ENVIRO LADDER solutions

| E 8R | E 8L | E 6R | E 6L | E 4R | E 4L | E 2R | E 2L | E 0 | P 0 | P 2L | P 2R | P 4L | P 4R | P 6L | P 6R | P 8L | P 8R | (σ = −1) Subfamily |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | | | 0O |
| | | −1 | −1 | −1 | −1 | .. | .. | .. | −1 | −1 | −1 | −1 | −1 | −1 | −1 | | | 2Y |
| | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | | | 2O |
| | | −1 | −1 | .. | .. | .. | .. | .. | −1 | −1 | −1 | −1 | −1 | −1 | −1 | | | 4Y |
| | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | | | 4O |

Table 2.13.27 restates Table 2.13.26. Table 2.13.27 uses the symbol 'I to point out aspects leading to occurrences of INSSYM7. For each row, the symbol 'I appears exactly 7 times. For each row, a\` = 1. The table uses the symbol 'S in ways that correlate with b\`.

**Table 2.13.27** A second depiction of σ = −1 boson ground-state COMPAR ENVIRO LADDER solutions

| E 8R | E 8L | E 6R | E 6L | E 4R | E 4L | E 2R | E 2L | E 0 | P 0 | P 2L | P 2R | P 4L | P 4R | P 6L | P 6R | P 8L | P 8R | (σ = −1) Subfamily |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 'S | 'S | 'S | 'S | 'S | 'S | 0 | 'I | 'I | 'I | 'I | 'I | 'I | 'I | | | 0O |
| | | 'S | 'S | 'S | 'S | .. | .. | .. | 'I | 'I | 'I | 'I | 'I | 'I | 'I | | | 2Y |
| | | 'S | 'S | 'S | 'S | 0 | 0 | 0 | 'I | 'I | 'I | 'I | 'I | 'I | 'I | | | 2O |
| | | 'S | 'S | .. | .. | .. | .. | .. | 'I | 'I | 'I | 'I | 'I | 'I | 'I | | | 4Y |
| | | 'S | 'S | 0 | 0 | 0 | 0 | 0 | 'I | 'I | 'I | 'I | 'I | 'I | 'I | | | 4O |

For elementary fermions for which σ = −1, starting from ground-state INTERN LADDER solutions, we add enough QP-like harmonic oscillators to match considerations related to generations. (That is, for 1QR particles, we add one pair of QP-like oscillators. For 3QIRD particles, we add two pairs of QP-like oscillators. See Table 2.6.4.) We add the same number of QE-like oscillators. Let b\` denote the number of added QE-like pairs of oscillators. The number b\` is an integer. We assume that (0;b\`)< symmetries pertain. (Assuming that (1;b\`)< symmetries pertain would not change results that we show starting with Table 2.13.33. But, assuming a\` = 1 would change results we show in Table 2.13.42.) Here, we think that the value of each of the N(..) for added oscillators need not be relevant, as long as the values are the same. (For example, perhaps N(..) = 0 pertains for each added oscillator.) In tables, for each N(..) for such an added oscillator, we use the notation =.

Table 2.13.28 shows ground-state COMPAR ENVIRO LADDER solutions for fermions for which σ = −1.

**Table 2.13.28** σ = −1 fermion ground-state COMPAR ENVIRO LADDER solutions

| E 8R | E 8L | E 6R | E 6L | E 4R | E 4L | E 2R | E 2L | E 0 | P 0 | P 2L | P 2R | P 4L | P 4R | P 6L | P 6R | P 8L | P 8R | (σ = −1) Subfamily |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | = | = | ~ | ~ | 0 | 0 | ~ | ~ | = | = | | | | | 1Q |
| | | | | = | = | ~ | ~ | −1 | −1 | ~ | ~ | = | = | | | | | 1R |
| = | = | = | = | ~ | ~ | 0 | 0 | 0 | 0 | 0 | 0 | ~ | ~ | = | = | = | = | 3Q |
| = | = | = | = | ~ | ~ | −1 | −1 | 0 | 0 | −1 | −1 | ~ | ~ | = | = | = | = | 3I |
| = | = | = | = | ~ | ~ | 0 | 0 | −1 | −1 | 0 | 0 | ~ | ~ | = | = | = | = | 3R |
| = | = | = | = | ~ | ~ | −1 | −1 | −1 | −1 | −1 | −1 | ~ | ~ | = | = | = | = | 3D |

Table 2.13.29 restates Table 2.13.28. In Table 2.13.29, the symbol 'I does not appear. The table uses the symbol 'G in ways that correlate with the number of generations. For a row in which 'G appears exactly 2 times, an SU(2) symmetry pertains to generations, the number of generators of SU(2) is 3, and there are 3 generations. Regarding a row in which 'G appears exactly 4 times, see Table 2.6.4. Table 2.13.29 uses the symbol 'S in ways that correlate with b`.

**Table 2.13.29** A second depiction of σ = −1 fermion ground-state COMPAR ENVIRO LADDER solutions

| E 8R | E 8L | E 6R | E 6L | E 4R | E 4L | E 2R | E 2L | E 0 | P 0 | P 2L | P 2R | P 4L | P 4R | P 6L | P 6R | P 8L | P 8R | (σ = −1) Subfamily |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | 'S | 'S | ~ | ~ | 0 | 0 | ~ | ~ | 'G | 'G | | | | | 1Q |
| | | | | 'S | 'S | ~ | ~ | −1 | −1 | ~ | ~ | 'G | 'G | | | | | 1R |
| 'S | 'S | 'S | 'S | ~ | ~ | 0 | 0 | 0 | 0 | 0 | 0 | ~ | ~ | 'G | 'G | 'G | 'G | 3Q |
| 'S | 'S | 'S | 'S | ~ | ~ | −1 | −1 | 0 | 0 | −1 | −1 | ~ | ~ | 'G | 'G | 'G | 'G | 3I |
| 'S | 'S | 'S | 'S | ~ | ~ | 0 | 0 | −1 | −1 | 0 | 0 | ~ | ~ | 'G | 'G | 'G | 'G | 3R |
| 'S | 'S | 'S | 'S | ~ | ~ | −1 | −1 | −1 | −1 | −1 | −1 | ~ | ~ | 'G | 'G | 'G | 'G | 3D |

Table 2.13.30 restates Table 2.13.29.

**Table 2.13.30** A third depiction of σ = −1 fermion ground-state COMPAR ENVIRO LADDER solutions

| E A9 | E 87 | E 65 | E 43 | E 21 | E 0 | P 0 | P 12 | P 34 | P 56 | P 78 | P 9A | (σ = −1) Subfamily |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | "S | ~~ | 0 | 0 | ~~ | "G | | | | 1Q |
| | | | "S | ~~ | −1 | −1 | ~~ | "G | | | | 1R |
| | "S | "S | ~~ | 00 | 0 | 0 | 00 | ~~ | "G | "G | | 3Q |
| | "S | "S | ~~ | −1, −1 | 0 | 0 | −1, −1 | ~~ | "G | "G | | 3I |
| | "S | "S | ~~ | 00 | −1 | −1 | 00 | ~~ | "G | "G | | 3R |
| | "S | "S | ~~ | −1, −1 | −1 | −1 | −1, −1 | ~~ | "G | "G | | 3D |

Table 2.13.31 shows instance-related symmetry and COMPAR SPATIM symmetries for solutions for which $\sigma = -1$. [Physics:] These solutions correlate with all the non-free-ranging elementary particles this monograph discusses.

**Table 2.13.31** Instance-related symmetry and COMPAR SPATIM symmetries for $\sigma = -1$ fermion ground-state COMPAR ENVIRO LADDER solutions

| ($\sigma = -1$) | Instance-related symmetry | COMPAR SPATIM | Solution subfamilies |
|---|---|---|---|
| | INSSYM7 | (1;3)< | 0O |
| | INSSYM7 | (0;1)< | 1QR |
| | INSSYM7 | (1;2)< | 2YO |
| | INSSYM7 | (0;2)< | 3QIRD |
| | INSSYM7 | (1;1)< | 4YO |

People might say that, based on Table 2.11.2 rules for combining symmetries and on Table 2.13.31, Table 2.13.32 provides combinations correlating with (1;3)< symmetries. The table lists pairs. The table does not list triplets, quadruplets, and so forth. Assuming a` = 0 pertains for the relevant fermions, of the combinations the table shows, the only combinations that combine to (1;3)< without, in effect, encountering a redundant symmetry, are 1QR+2YO, 3QIRD+4YO, and 0O. [Physics:] People might say that 1Q+2Y correlates with a component of known composite particles.

**Table 2.13.32** Pairs that might combine to correlate with (1;3)<

1. 0O+x, for which x can be any one of the following.
    - 1.1. 0O.
    - 1.2. 1QR.
    - 1.3. 2YO.
    - 1.4. 3QIRD.
    - 1.5. 4YO.
2. 1QR+2YO.
3. 2YO+x, for which x can be any one of the following.
    - 3.1. 3QIRD.
    - 3.2. 4YO.
4. 3QIRD+4YO.

Table 2.13.33 and Table 2.13.34 show and discuss results from combining COMPAR-related symmetries for $\sigma = -1$ ground-state ENVIRO LADDER solutions. ENVIRO LADDER solution-related COMPAR symmetries combine so as to produce items in Table 2.13.33. For example, for 1Q+2Y, (0;1)< (for 1Q) and (1;2)< (for 2Y) combine term-wise to yield (1;3)<.

Table 2.13.33 shows COMPAR SPATIM for combinations of solutions for which $\sigma = -1$ for each solution. We use the term pre-composite to correlate with such combinations of solutions. We use notation that Table 2.3.5 shows. People might say that 0O does not correlate with the term pre-composite.

**Table 2.13.33** COMPAR SPATIM symmetries for σ = −1 combinations of solutions for which σ = −1

| (σ = −1) | Instance-related symmetry | COMPAR SPATIM | Pre-composite |
|---|---|---|---|
| | INSSYM7 | (1;3)< | 0O |
| | INSSYM7 | (1;3)< | 1QR+2Y |
| | INSSYM7 | (1;3)< | 1QR+2O |
| | INSSYM7 | (1;3)< | 3QIRD+4Y |
| | INSSYM7 | (1;3)< | 3QIRD+4O |

Table 2.13.34 shows SPATIM symmetries for combinations of solutions for which σ = +1 for each combination. Here, the term SPATIM symmetries denotes both of COMPAR SPATIM symmetries and FRERAN SPATIM symmetries. Here, n denotes any integer ≥ 2. ([Physics:] For the first row in the table, 2 × 0O would correlate with two occurrences of a particle that is its own antiparticle. For the second row in the table, n = 2 correlates with mesons, n = 3 correlates with baryons, n = 4 correlates with tetraquarks, and n = 5 correlates with pentaquarks.) For n ≥ 2, n instances of (1;3)< combine to yield (1;3)>. (See Table 2.11.2.) Here, we use the term composite to correlate with and extend traditional use of the term composite. ([Physics:] People use the term composite particles to describe mesons and baryons.) People might say that n × 0O does not correlate with the term composite.

**Table 2.13.34** Instance-related symmetry and SPATIM symmetries for σ = +1 combinations of solutions for which σ = −1

| (σ = +1) | Instance-related symmetry | SPATIM | Composite (n ≥ 2) |
|---|---|---|---|
| | INSSYM7 | (1;3)> | n × 0O |
| | INSSYM7 | (1;3)> | n × 1QR+2Y |
| | INSSYM7 | (1;3)> | n × 1QR+2O |
| | INSSYM7 | (1;3)> | n × 3QIRD+4Y |
| | INSSYM7 | (1;3)> | n × 3QIRD+4O |

Perhaps, Table 2.13.35 provides a possible basis for research.

**Table 2.13.35** Speculation regarding relationships among σ = −1, σ = +1, and abilities of QIRD-family elementary particles to interact with YO-family elementary particles

1. To what extent might the following notions correlate with each other?
    - 1.1. The following set of concepts.
        - 1.1.1. The relationship (1;3)< + (1;3)< = (1;3)>.
        - 1.1.2. That (1;3)< correlates with σ = −1 (or, non-free-ranging).

1.1.3. That (1;3)> correlates with σ = +1 (or, free-ranging).

1.2. The possibility that a QIRD-family elementary particle cannot absorb a YO-family particle that it emits.

1.3. The possibility that a sufficiently isolated QIRD-family particle cannot interact with YO-family particles.

~ ~ ~

This subsection shows COMPAR ENVIRO solutions for non-G-family elementary particles for which σ = +1.

[Physics:] People might say that violations of CP-symmetry correlate with weak interactions occurring under circumstances in which the physics of composite particles pertains. Under some such circumstances, people observe effects correlating with generation changes for fermions. Some examples correlate with an asymmetry between zero-charge kaons and their antiparticles. Some examples correlate with decays of neutral B mesons.

Regarding such observations pertaining to kaons, people show Feynman diagrams that depict interactions mediated by two bosons, each from the 2W-subfamily.

We posit correlating these interactions with COMPAR SPATIM solutions and symmetries.

For FRERAN SPATIM LADDER solutions, results Table 2.13.5 states correlate with conservation of fermion generation.

For interactions (such as those people correlate with CP-symmetry violation and we correlate with COMPAR environments) mediated by more than one W-family boson, we think COMPAR SPATIM solutions and symmetries need not correlate completely with FRERAN SPATIM solutions and symmetries. People might say that the behavior of the two (or presumably possibly more than two) W-family bosons are correlated. (See, also, Table 3.10.2.)

Perhaps, for COMPAR solutions, Table 2.13.36 pertains. Compared to Table 2.13.5, Table 2.13.36 shows changes only for the 2W-subfamily. In Table 2.13.36, the QE-like occurrence of "G and the P7-and-P8 occurrence of "S are not present. People might say that the P1-and-P2 occurrence of the symbol "S` correlates with a denaturing of symmetry, namely a lack of reflection symmetry. This lack of reflection symmetry correlates with the topic of parity. The E2-and-E1 occurrence of the symbol "I` correlates with another lack of a reflection symmetry. People might say that the QE-like lack of reflection symmetry correlates with the topic of charge conjugation. People might say that, together, the two lacks correlate with possibilities for violations of CP-symmetry. The lack of a "G correlates with lack of conservation of fermion generation.

**Table 2.13.36** σ = +1 non-G-family ground-state COMPAR LADDER solutions

| E<br>87 | E<br>65 | E<br>43 | E<br>21 | E<br>0 | P<br>0 | P<br>12 | P<br>34 | P<br>56 | P<br>78 | P<br>9A | (σ = +1)<br>Subfamily |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | "I | "I | "I | 'I | 0 | "S | "S | "S | | | 0H0 |

| E | E | E | E | E | P | P | P | P | P | P | (σ = +1) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 87 | 65 | 43 | 21 | 0 | 0 | 12 | 34 | 56 | 78 | 9A | Subfamily |
| | "I | "I | "I | 'I | 0 | ~~ | "S | "S | "S | | 1C |
| | "I | "I | "I | 'I | −1 | ~~ | "S | "S | "S | | 1N |
| | "I | "I | "I` | 'I | 0 | 00("S`) | "S | "S | | | 2W |
| | "I | "I | "I | 'I | −1 | 00 | ~~ | "S | "S | "S | 3N |

Perhaps, Table 2.13.37 provides an alternative possibility regarding the W-family and interactions involving two W-family members. Here, a lack of conservation of generation (regarding spin-1/2 elementary fermions) correlates with #"G = 2. We do not discuss this possibility further.

**Table 2.13.37** Speculative alternative solution for 2W bosons in a COMPAR environment

| E | E | E | E | E | E | P | P | P | P | P | P | (σ = +1) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| A9 | 87 | 65 | 43 | 21 | 0 | 0 | 12 | 34 | 56 | 78 | 9A | Subfamily |
| "G | "G | "I | "I | "I` | 'I | 0 | 00("S`) | "S | "S | "S | "S | 2W |

~ ~ ~

This subsection summarizes COMPAR ENVIRO solutions for elementary particles for which σ = +1.

Table 2.13.38 pertains.

**Table 2.13.38** Assumptions regarding COMPAR LADDER solutions for elementary particles for which σ = +1

1. For elementary particles for which σ = +1, …
    - 1.1. We assume that behavior in COMPAR environments correlates with behavior that correlates with solutions for which FRERAN SPATIM symmetries pertain, except …
        - 1.1.1. Interactions involving more than one W-family boson ...
            - 1.1.1.1. Can correlate with results Table 2.13.36 shows.
            - 1.1.1.2. Need not correlate with results Table 2.13.5 shows.
2. Thus, elementary particles for which σ = +1, we, in effect, assume …
    - 2.1. Except for some interactions mediated by W-family bosons, …
        - 2.1.1. COMPAR SPATIM equals FRERAN SPATIM.
    - 2.2. For some interactions mediated by W-family bosons, each of the following pertains.
        - 2.2.1. COMPAR SPATIM does not equal FRERAN SPATIM.
        - 2.2.2. Violations of CP-symmetry can occur.

~ ~ ~

This subsection summarizes instance-related symmetry and SPATIM symmetries for elementary particles and for composite particles.

Table 2.13.39 shows instance-related symmetry and SPATIM symmetries for elementary particles and for composite particles.

**Table 2.13.39** Instance-related symmetry, FRERAN SPATIM symmetries, and COMPAR SPATIM symmetries for elementary particles and for composite particles

1. Each σ = +1 ground-state FRERAN LADDER solution or COMPAR LADDER solution correlates with …
    - 1.1. An instance-related symmetry of INSSYM7.
        - 1.1.1. For the 2W-subfamily in a COMPAR environment (at least, regarding some interactions intermediated by more than one W-family particle), …
            - 1.1.1.1. The instance-related symmetry lacks reflection symmetry.
    - 1.2. SPATIM symmetries of (1;3)>.
        - 1.2.1. For the 2W-subfamily in a COMPAR environment (at least, regarding some interactions intermediated by more than one W-family particle), …
            - 1.2.1.1. One of the three SU(2)-related symmetries lacks reflection symmetry.
2. Each σ = +1 composite correlates with …
    - 2.1. An instance-related symmetry of INSSYM7.
    - 2.2. SPATIM symmetries of (1;3)>.
3. Each σ = −1 ENVIRO LADDER solution correlates with …
    - 3.1. An instance-related symmetry of INSSYM7.
    - 3.2. (Perhaps hypothetical) FRERAN symmetries of (1;3)>.
    - 3.3. COMPAR symmetries of (a`;b`)< with the following.

$$a` = 1, \text{ for bosons} \tag{2.107}$$

$$a` = 0 \text{ (or, possibly, 1), for fermions} \tag{2.108}$$

$$b` = 3 - S, \text{ for bosons} \tag{2.109}$$

$$b` = S + 1/2, \text{ for fermions} \tag{2.110}$$

~ ~ ~

[Physics:] This subsection discusses possibilities that single-quark composite particles might exist and that hybrid composites might exist.

Possibly, people should consider single-quark or hybrid composite particles. Table 2.13.40 pertains. (See, also, Table 4.11.1.)

**Table 2.13.40** Possibilities regarding single-quark composite particles and other hybrid composite particles

1. Each row in Table 2.13.34 combines n items from a single row in Table 2.13.33. In Table 2.13.34, n must be greater than 1.
2. A question arises.
    2.1. To what extent does nature exhibit composites that combine items from differing rows in Table 2.13.33?
3. For example, …
    3.1. Possibly, nature could include (1 × 0O) + (1 × 1Q+2Y) or could include (1 × 0O) + (1 × 1Q+2O).
        3.1.1. These could be free-ranging composites, some with a charge with absolute value of either (1/3)$|q_e|$ or (2/3)$|q_e|$.

Table 2.13.41 shows some possible examples for which (1;3)> symmetries might pertain. The last two examples in the table could correlate with free-ranging fractionally charged particles. These two examples might pertain to a plasma (or, sea) of quarks, before the formation nucleons. (See Section 4.5.)

**Table 2.13.41** Possibilities for hybrid composite particles

1. (1 × 1Q+2Y) + (1 × 1Q+2O).
2. (1 × 0O) + (1 × 1Q+2Y).
3. (1 × 0O) + (1 × 1Q+2O).

~ ~ ~

This subsection discusses possible limitations regarding interactions among elementary particles for which σ = −1.

Work above correlates with the notion that, assuming a` = 0 for the relevant fermions, 1QR+2YO, 3QIRD+4YO, and 0O are the only combinations of fields (that correlate with elementary particles for which σ = −1) that correlate with COMPAR (1;3)< symmetries without a redundancy in a` and without a redundancy in b`. (See, for example, discussion before Table 2.13.32.)

Possibly, Table 2.13.42 pertains.

**Table 2.13.42** Possible limitations on interactions between elementary fermions for which σ = −1 and elementary bosons for which σ = −1

1. 1QR-subfamily particles …
    1.1. Interact with 2YO-subfamily particles.
    1.2. Do not interact with 4YO-subfamily particles.
2. 3QIRD-subfamily particles …
    2.1. Do not interact with 2YO-subfamily particles.
    2.2. Interact with 4YO-subfamily particles.

~ ~ ~

This subsection discusses possible implications of assuming #b' = 2.

People might say that steps Table 2.13.43 suggests might correlate with opportunities for research.

**Table 2.13.43** Possible steps for research regarding possibilities that #b' =2

1. Assume #b' = 2 (and #b' ≠ 3) for all phenomena for which σ = +1.
2. Try to parallel work this monograph presents in Section 2.11, in Section 2.12, and above in Section 2.13.
3. Perhaps explore possibilities such as the following.
   - 3.1. Instance-related symmetry (or, INSSYM) correlates with concepts people might correlate with or denote by the following.

$$\text{SU(5)} \tag{2.111}$$

INSSYM5

   - 3.1.1. Here, 24 correlates with the number of generators of SU(5).

$$\text{24 mathematically possible ensembles} \tag{2.112}$$

$$\#\text{ENS} = 24 \tag{2.113}$$

ENS24 models

$$\#\text{ENS} = 24 / 8 = 3 \tag{2.114}$$

ENS3 models

   - 3.1.2. Here, 8 correlates with the number of generators of SU(3).

$$\#\text{ENS} = 24 / 24 = 1 \tag{2.115}$$

ENS1 models

   - 3.1.3. Here, 24 correlates with the number of generators of SU(5).
   - 3.2. For each of the cases ENS24 models and ENS3 models, …
     - 3.2.1. The number of DME is 2 (= 3 − 1, in which the 3 comes from above in this table and equals the number of OMDME).
     - 3.2.2. Each ensemble contains significant amounts of SEDMS. (See Section 4.3.)

# Section 2.14 EXTINT LADDER models for fermion generations and color charge

Section 2.14 discusses EXTINT LADDER solutions that correlate with numbers of generations for elementary fermions and with numbers of color charges for elementary fermions existing in COMPAR environments. People might say that MM1MS1 models correlate with 5 color charges for elementary fermions for which $\sigma = -1$ and $S = 3/2$. People might say that MM1MS1 models provide, for spin-1/2 elementary fermions for which $\sigma = +1$, some correlation between numbers of generations and numbers of charges.

~ ~ ~

This subsection provides perspective about internal properties and INTERN LADDER solutions.

People might say that generation is an internal property of elementary fermions. People might say that color charge is an internal property of elementary fermions for which $\sigma = -1$.

INTERN LADDER solutions do not explicitly show information regarding generations.

In this section, we discuss possibilities for EXTINT LADDER solutions that correlate with number of generations and numbers of color charges.

~ ~ ~

This subsection provides perspective about work in this section.

People might say that work in this section correlates with possible theory. (See, for example, Table 2.14.5.) This monograph tends not to directly use EXTINT LADDER solutions in trying to correlate MM1MS1 models with experimental or observational endeavors.

~ ~ ~

This subsection discusses color charge and gluons.

People might say that, in the course of developing traditional physics theory regarding quarks bound in composite particles, people developed the concept of color charge to overcome otherwise evident theoretical problems regarding multiple quarks existing in the same fermion state. Traditional physics provides that no more than one fermion can occupy any one state. The notion of multiple color charges provides a construct for overcoming the problem. For example, two quarks that occupy an otherwise identical state can differ with respect to color charge.

For a quark, S = 1/2. People might say that people have observed quarks only in COMPAR environments. People might say that for quarks in COMPAR environments, the number of color charges (not counting antimatter color charges that pertain to antimatter quarks) is 3.

We know of no discussion regarding color-charge analogs that might pertain regarding possible spin-3/2 elementary fermions.

We use the term color charge (or, color-charge analog) regarding 3QIRD solutions. People might say that, here, MM1MS1 models correlate with 5 color charges.

People might say that gluons change the color charges of individual quarks. People might say that gluons provide interactions (the strong interaction) that bind quarks into hadrons.

People might say that quarks can exist in conditions other than COMPAR environments. For example, the early universe would have been too dense for the formation of hadrons. People might speculate regarding the extent to which quarks not governed by COMPAR considerations exist in neutron stars.

Perhaps, Table 2.14.1 points to opportunities for research. Here, #CC denotes the number of possible color charges. (See, also, Table 2.6.6.)

**Table 2.14.1** Possible concepts regarding color charge

1. For non-lepton elementary fermions, …

$$\#CC = \#E + 1 \qquad (2.116)$$

   1.1. #CC denotes the number of possible color charges.

2. People might say that, for leptons, …

$$\#CC = \#E + 1 = 1 \qquad (2.117)$$

   2.1. No interaction changes that color charge.

   2.2. The lack of an interaction that could change that color charge correlates with …

      2.2.1. A lack of physics-relevance of 0Y solutions.

~ ~ ~

This subsection discusses symmetries for EXTINT solutions for elementary fermions for which σ = −1.

Table 2.14.2 pertains. We start with Table 2.13.30. We replace each occurrence of the symbol "S with the symbol "C. For each QE-like occurrence of the symbol ~~, we replace the QE-like occurrence of ~~ with the symbol 'C,~. Here, we use the symbol −1,−1 to indicate two occurrences of the symbol −1. Here, regarding the symbol 'C,~, the symbol ~ pertains to the one oscillator (in the relevant oscillator pair) for which N(..) ≠ 'C. (For each row in the table, there are 4 cases. Here, 4 = 2 × 2, with one factor of 2 correlating with the symbol ~~. For each value of ~~, 2 solutions pertain. For example, for the 1Q-subfamily, for one solution, ~ correlates with oscillator E2 and 'C correlates with oscillator E1. Here, 'C = N(E1). For the other solution, ~ correlates with oscillator E1 and 'C correlates with oscillator E2. Here, 'C = N(E2).) In general, we use the symbol #'C to denote the number of occurrences of 'C. In general, we count each "C as correlating with two occurrences of 'C. For the 1QR-subfamilies, #'C = 3. People might say that, for the 1QR-subfamilies, #'C is the number of color charges. That is, #'C = #CC. For the 3QIRD-subfamilies, #'C = 5. People might say that, for the 3QIRD-subfamilies, #'C is the number of color-charge analogs. That is, #'C = #CC. Occurrences of "G correlate with numbers of generations.

**Table 2.14.2** A depiction of σ = −1 fermion ground-state EXTINT LADDER solutions

| E A9 | E 87 | E 65 | E 43 | E 21 | E 0 | P 0 | P 12 | P 34 | P 56 | P 78 | P 9A | (σ = −1) Subfamily |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | "C | 'C,∼ | 0 | 0 | ∼∼ | "G | | | | 1Q |
| | | | "C | 'C,∼ | −1 | −1 | ∼∼ | "G | | | | 1R |
| | "C | "C | 'C,∼ | 00 | 0 | 0 | 00 | ∼∼ | "G | "G | | 3Q |
| | "C | "C | 'C,∼ | −1, −1 | 0 | 0 | −1, −1 | ∼∼ | "G | "G | | 3I |
| | "C | "C | 'C,∼ | 00 | −1 | −1 | 00 | ∼∼ | "G | "G | | 3R |
| | "C | "C | 'C,∼ | −1, −1 | −1 | −1 | −1, −1 | ∼∼ | "G | "G | | 3D |

Table 2.14.2 directly extends Table 2.3.14. Table 2.3.14 characterizes ground-state INTERN LADDER solutions for which σ = −1.

∼ ∼ ∼

This subsection discusses symmetries for EXTINT solutions for elementary fermions for which σ = +1.

Table 2.14.3 recasts relevant rows from Table 2.3.11.

**Table 2.14.3** Ground-state fermion INTERN LADDER solutions for σ = +1

| E A9 | E 87 | E 65 | E 43 | E 21 | E 0 | P 0 | P 12 | P 34 | P 56 | P 78 | P 9A | (σ = +1) Subfamily |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | 0 | 0 | ∼∼ | | | | | 1C |
| | | | | | −1 | −1 | ∼∼ | | | | | 1N |
| | | | | | 0 | −1 | 00 | ∼∼ | | | | 3N |

Table 2.14.4 adds (to Table 2.14.3) QE-like occurrences of symbols of the form "G to correlate with results from Table 2.6.4. Each occurrence of "G correlates with two occurrences of N(E0). For each QE-like occurrence of "G, we add a QP-like occurrence of the symbol "X. Each occurrence of "X correlates with two occurrences of a symbol 'X. For each QP-like occurrence of the symbol ∼∼, we replace the QP-like occurrence of ∼∼ with the symbol ∼,'X. Each occurrence of 'X correlates with an occurrence of N(E0). That is, 'X = N(E0). Somewhat paralleling considerations for 'C,∼ in Table 2.14.2, in Table 2.14.4, each ∼,'X correlates with two cases.

**Table 2.14.4** Ground-state fermion EXTINT LADDER solutions for σ = +1, correlating with generations

| E | E | E | E | E | E | P | P | P | P | P | P | (σ = +1) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| A9 | 87 | 65 | 43 | 21 | 0 | 0 | 12 | 34 | 56 | 78 | 9A | Subfamily |
| | | | | "G | 0 | 0 | ~,'X | "X | | | | 1C |
| | | | | "G | −1 | −1 | ~,'X | "X | | | | 1N |
| | | | "G | "G | 0 | −1 | 00 | ~,'X | "X | "X | | 3N |

Table 2.14.4 directly extends Table 2.6.4. In Table 2.14.4, occurrences of "G correlate with numbers of generations.

In Table 2.14.4, 1C correlates with 3 occurrences of 'X. 1N correlates with 3 occurrences of 'X. 3N correlates with 5 occurrences of 'X.

People might say that, for 1C and 1N, the 3 occurrences of 'X correlate with the 3 charges that correlate with the 2W family. 1C correlates with the two charges that people consider to be non-zero. One of these charges is the charge of the $W^+$ boson. One of these charges is the charge of the $W^-$ boson. 1N correlates with the one charge that people consider to be zero. This is the charge of the Z boson.

People might say that, for 3N, the 5 occurrences of 'X correlate with a property that might pertain, if 4W solutions correlated with nature. For MM1MS1 models, 4W solutions do not correlate with nature.

~ ~ ~

This subsection suggests possible opportunities for research regarding, for elementary fermions, various models for and relationships among spin, generations, color charge, charge, and SPATIM symmetries.

Table 2.14.5 pertains.

**Table 2.14.5** Possible opportunities for research regarding, for elementary fermions, relationships among spin, generations, color charge, charge, and SPATIM symmetries

1. For modeling regarding elementary fermions, determine correlations (and lacks of correlations) among ...
   - 1.1. Any or all of the following properties.
     - 1.1.1. Spin.
     - 1.1.2. Numbers of generations.
     - 1.1.3. Numbers of color charges.
     - 1.1.4. Numbers of charges.
     - 1.1.5. Other.
   - 1.2. Any or all of the following modeling techniques.
     - 1.2.1. DIFEQU.
     - 1.2.2. INTERN LADDER.
     - 1.2.3. EXTINT LADDER.

   - 1.2.4. FRERAN SPATIM LADDER.
   - 1.2.5. COMPAR SPATIM LADDER (or, COMPAR ENVIRO LADDER).
   - 1.2.6. FERTRA LADDER.
   - 1.2.7. Other.
- 1.3. Any or all of the properties and any or all of the modelling techniques.

# Section 2.15 The MM1 meta-model and various MM1MS1 models

Section 2.15 discusses using the MM1 meta-model. We define the MM1 meta-model by featuring a list of topics that correlate with choices people can make when using the MM1 meta-model to define models. We provide a process for using the MM1 meta-model to select and work with models. We list some key parameters (such as #ENS) that people can set in order to specify a model. We list some characteristics that models share. We discuss some aspects of ENS48 models, ENS6 models, and ENS1 models. We describe models we discuss in subsequent chapters.

~ ~ ~

This subsection discusses aspects of the MM1 meta-model.

People might say that work above provides the essence of a meta-model. (See Section 2.1 through Section 2.14.)

People might say that Table 2.15.1 notes topics that correlate with limitations this monograph places on the MM1 meta-model.

**Table 2.15.1** Topics that correlate with limitations this monograph places on the MM1 meta-model

1. The extent to which models should be based on math that has discrete solutions.
2. The extent to which models should be based on math regarding isotropic pairs of isotropic quantum harmonic oscillators.
3. Œ = 0.

Table 2.4.1 discusses relationships between the MM1 meta-model and MM1MS1 models.

Below, we discuss aspects (of the MM1 meta-model) correlating with choices people can make when selecting or developing MM1MS1 models. (See, for example, Table 2.15.5, Table 2.15.6 , and Table 2.15.7.)

~ ~ ~

This subsection outlines steps for using the MM1 meta-model to select a model and for using the model.

Table 2.15.2 pertains.

**Table 2.15.2** Steps for using the MM1 meta-model and using a MM1MS1 model

1. Choose each of the following.
    1.1. Information related to some relevant aspects of nature. The information might include one or more of …
        1.1.1. Data.
        1.1.2. Past, ongoing, or planned experiments or observations.
        1.1.3. Various theories or models.
    1.2. A model (based on the MM1 meta-model) that might correlate with those aspects.
        1.2.1. Perhaps, consider limiting the set of topics (to explore) to topics that correlate with steps Table 2.15.5 and Table 2.15.6 show.
        1.2.2. Perhaps, consider making choices that correlate with examples this monograph shows.
        1.2.3. Perhaps, consider topics Table 2.15.7 shows.
2. Try to correlate aspects of the information with aspects of the model.

~ ~ ~

This subsection discusses topics for which people might have one choice or limited choices when using the MM1 meta-model.

Table 2.15.3 notes topics for which people might not be able to make choices when using the MM1 meta-model to develop MM1MS1 models. (Regarding the terms %68even, %68odd, and %68both, see discussion preceding Table 3.3.3 and see Table 3.3.3.)

**Table 2.15.3** Some topics correlating with the MM1 meta-model and possibly with all MM1MS1 models

1. A most comprehensive list of candidate elementary particles.
    1.1. This list includes the G-family %68both set.
2. D* = 3, for DIFEQU aspects correlating directly with cataloging non-zero-mass elementary particles.
    2.1. In this monograph, uses of D* = 2 and uses of D* = 1 correlate with projections of functions (for example, functions for which D* = 3).

Table 2.15.4 notes a topic for which people might not be able to make choices when using the MM1 meta-model to develop one or more types of MM1MS1 models.

**Table 2.15.4** A topic for which people might not be able to make choices when using the MM1 meta-model to develop one or more types of MM1MS1 models

1. For LADDER models, …
    1.1. For physics-relevant solutions other than G-family solutions, …
        1.1.1. Relationships among spin, S, and #P.

~ ~ ~

This subsection illustrates steps that people might take to specify a model.

Table 2.15.5 shows a list of steps that, for some situations, people might want to take to specify MM1MS1 models. For example, people making models that correlate with the nature of dark-energy stuff and dark matter may want to do the second step in the table.

**Table 2.15.5** Steps for specifying a model (an example)

1. Choose aspects of nature to try to model.
2. Choose #ENS from one of the following possibilities.
    2.1. #ENS = 48.
        2.1.1. This choice correlates with ENS48 models (including ENS48" models).
        2.1.2. People might say that this selection is not incompatible with much physics developed before the year 2016.
        2.1.3. People might say that this selection correlates with more data than does the choice of #ENS = 1.
            2.1.3.1. People might say that, for example, #ENS = 48 provides possible insight regarding ratios of densities of dark matter to densities of ordinary matter.
    2.2. #ENS = 6.
        2.2.1. This choice correlates with ENS6 models.
        2.2.2. People might say that this selection is not incompatible with much physics developed before the year 2016.
        2.2.3. People might say that this selection correlates with more data than does the choice of #ENS = 1.
            2.2.3.1. People might say that, for example, #ENS = 6 …
                2.2.3.1.1. Provides possible insight regarding ratios of densities of dark matter to densities of ordinary matter.
                2.2.3.1.2. Requires dark-energy stuff to correlate with SEDES. (See, for example, Table 4.3.2 and Table 4.3.13.)
    2.3. #ENS = 1.
        2.3.1. This choice correlates with ENS1 models.

- 2.3.2. People might say that this selection correlates with much physics developed before the year 2016.
  - 2.3.2.1. For example, this choice correlates with exactly one instance of Standard Model particles.
- 2.3.3. People might say that this selection …
  - 2.3.3.1. Requires dark-energy stuff to correlate with SEDES. (See, for example, Table 4.3.2 and Table 4.3.13.)
  - 2.3.3.2. Requires dark matter stuff to correlate with SEDMS. (See, for example, Table 4.3.2 and Table 4.3.14.)

3. Choose a set of solutions to consider including in the model.
   - 3.1. The following items show possibilities for limiting the choices.
     - 3.1.1. Assumptions regarding lack of physics-relevance for some solutions. (See Table 2.15.8.)
     - 3.1.2. The possibility that %68odd G-family solutions are not physics-relevant.

Table 2.15.6 shows some further steps that, for some situations, people might want to take to specify MM1MS1 models. For example, people making models that correlate with aspects of nature that people traditionally model via general relativity may want to do the first three steps in the table.

**Table 2.15.6** Possible further steps for specifying a model (an example)

1. List solutions correlating with elementary particles (for which σ = +1) for which to choose the extent, for each SPATIM solution, to which to assume that …
   - 1.1. #b" = 2 (and, therefore, #b" ≠ 3).
     - 1.1.1. Perhaps, only G-family solutions appear on the list.
       - 1.1.1.1. For example, to the extent phenomena people model via general relativity are relevant, people might include in the list 244G4&.
2. List solutions correlating with elementary particles (for which σ = +1) for which to choose the extent, for each solution, to which to assume that …
   - 2.1. #b' = 3.
     - 2.1.1. People might say that this choice correlates with being able to use, on at least not too large a scale, models that correlate with special relativity.
   - 2.2. #b' = 2.
     - 2.2.1. People might say that this choice implies that #b" = 2 also pertains for the specific solution.
3. Choose, for each solution for which σ = +1 and at least one of #b' and #b" equals 2, …

    3.1. To what extent and how to modify relevant results above (such as results Table 2.13.12 shows). (See Table 2.13.43.)
4. Choose, for each of the 3QIRD-subfamilies, a number of generations, #GEN(3/2). (See Table 2.6.4.)
5. Choose a value of #E` for each G-family solution for which #E = 0. (See, for example, Table 2.13.20.)

~ ~ ~

This subsection provides a list of topics about which people might make choices when using the MM1 meta-model to develop one or more models.

Table 2.15.7 notes topics about which people might make choices when using the MM1 meta-model to develop one or more models. People might say that we define and limit, via this monograph, the MM1 meta-model by choices such as choices Table 2.15.1 shows. People might say that, for many purposes, work including Table 2.15.5 and Table 2.15.6 provides a more tractable list than does Table 2.15.7.

**Table 2.15.7** Some topics correlating with the MM1 meta-model and with choices available when developing a MM1MS1 model

1. Aspects of nature to try to model.
2. Limits regarding #E, #P.
3. The extents to which ground-state solutions correlate with aspects with which the model should correlate.
4. Types of models to explore and the extents to which results might correlate with nature. Examples of types of models include …
    4.1. INTERN, EXTINT, ENVIRO, and FERTRA.
        4.1.1. (ENVIRO:) SPATIM, FRERAN, and COMPAR.
    4.2. LADDER and DIFEQU.
    4.3. CORMAT and CORPHY.
5. Relevance of and interpretations regarding various groupings of oscillators. For example, …
    5.1. QE-like and/or QP-like.
    5.2. Various oscillator pairs.
    5.3. Other sets of oscillators.
6. Relationships between MM1MS1 models and other models.
7. Physics-relevance of work and/or solutions. (See, for example, Table 2.15.8).
8. Families and subfamilies of solutions to include (or exclude).
9. Representations specific to various families.
10. Appropriateness of use, in models, of even numbers of QE-like or QP-like harmonic oscillators.
11. Ranges and interpretations of N(P0) for ground states.
12. Applicability of traditional and/or non-traditional solutions.

13. Needs for normalization (regarding DIFEQU solutions).
14. Correlations (for numbers such as S) between solutions and physics quantities.
15. Roles and significances of $\sigma$ (regarding FRERAN solutions, COMPAR solutions, and possibly other solutions).
16. Correlations between LADDER solutions and DIFEQU solutions.
17. Correlations between solutions and fields.
18. Correlations between solutions and particles.
19. Correlations between field solutions and particle solutions.
20. Possibilities that multiple elementary particles correlate (beyond correlations we discuss) with a solution.
21. Limits regarding parameters (such as S).
22. The extent to which to consider DIFEQU solutions (that might correlate directly with fields and/or particles) for which $D^* \neq 3$.
23. The extent of relevance for projections from solutions for which $D^* = 3$ to mathematical constructs for which $D^* = 2$ or $D^* = 1$.
24. Other uses for solutions which $D^* = 2$ or $D^* = 1$.
25. Correlations between relevant values of $\eta$ and the notions of fields and particles.
26. Correlations between phenomena that might correlate with the E0-and-P0 oscillator pair and applications of DIFEQU math for which $D^* = 2$.
27. Significance of aspects of solutions for which $D^* + 2\nu = 0$.
28. The extent to which some integers related to $D^* + 2\nu = 0$ solutions correlate with approximate relative masses of at least the Z, W, and Higgs bosons.
29. The extent to which, for fermion elementary particles and their fields, $\min(D, 2(2S + 1))$ provides a factor relevant to calculating a maximum number of candidate solutions.
30. The extent to which, for fermion elementary particles, generations provides a factor relevant to calculating a maximum number of candidate solutions.
31. Modeling that correlates with a value for #GEN(3/2).
32. Roles for models that involve more than one QE-like coordinate.
33. The extent to which $\sigma = -1$ correlates with phenomena and/or models.
34. The extent of correlations between FERTRA representations and fermion INTERN representations.
35. The extent of correlations between SPATIM symmetries, INSSYM7 symmetry, uses of symmetries in models, and nature.
36. Algebra for combining various (a`;b`)< and/or (a`;b`)>.
37. Correlations between various components of (1;3)> symmetries and conservation laws.
38. Possible new conservation laws, such as conservation of fermion generation for some vertices.

39. Relationships between conservation of angular momentum and quantization related to angular momentum.
40. Relationships between FRERAN boost-related symmetries and various concepts (such as, limitation on velocities and conservation of rest energy).
41. Possible significance of oscillator swap symmetries.
42. Interpretations of INSSYM7.
43. Physics-relevant values for #ENS.
44. Values of #E` for G-family solutions for which #E = 0.
45. Construction and interpretations of various sets of solutions (for example, FRERAN LADDER solutions).
46. Relationships between various sets of solutions.
47. Choices of G-family solutions, for example …
    - 47.1. EACUNI set or SOMMUL set.
    - 47.2. %68even or %68both.
    - 47.3. For ENS48 models, definitions and roles of OME, DME, DEE, various G#XY. (See Table 2.13.15.)
        - 47.3.1. Here, # = 2, 6, or 48; X = OM, DM, or DE; and Y = OM, DM, or DE.
    - 47.4. For ENS48 models, relationships between OMS, DMS, DES, and other stuff and OME, DME, DEE, and various G#XY. (See Table 4.3.2 and Table 2.13.15.)
    - 47.5. For ENS6 models, considerations similar to those (just above) for ENS48 models.
48. Correlations between oscillators (in representations for non-G-family elementary particles) and various G-family forces.
    - 48.1. For example, between P0 and 4G4&.
49. For various particles and/or fields, choices of #b'.
50. For various particles and/or fields, choices of #b".
51. Metrics to consider as pertaining to energy-momentum space coordinates and/or to space-time coordinates.
52. The extents to which to consider that MM1MS1-currents should be treated as tensors (with rank ≥ 2) and not just as vectors …
    - 52.1. Especially, for cases in which a relevant #b' = 2 and/or a relevant #b" = 2.
    - 52.2. Possibly, especially, regarding currents that interact with 4G4&.
53. Details regarding models pertaining to currents that correlate with the symbol 3v (for example, currents that correlate with 2G24&). (See Table 6.1.3.)
54. Details regarding interactions between elementary bosons and zero-mass elementary particles.
55. Details regarding modeling interactions between individual G-family bosons and an object (for example, an astrophysical object) that contains many elementary particles and/or composite particles.

56. Details regarding modeling interactions between (classical-physics-like) fields based on G-family bosons and an object (for example, an astrophysical object) that contains many elementary particles and/or composite particles.
57. Strengths for interactions mediated by G-family bosons other than 2G2& and 4G4&.
58. Strengths for interactions mediated by non-G-family bosons (perhaps, other than W-family bosons).
59. Possible significance of 17 oscillators and/or SU(17).
60. Roles and mechanisms regarding G-family channels.
61. Possibilities of other families having channels.
62. Aspects correlating #"G with conservation of fermion generation.
63. The extent to which INSSYM7 might be incompatible with COMPAR solutions, symmetries, and interpretations.
64. Limits on applicability, for elementary particles for which $\sigma = -1$, for FRERAN solutions and for COMPAR solutions.
65. Treatments regarding quark-and-boson states other than quark-and-gluon composites.
66. Aspects regarding color-charge analogs for spin-3/2 fermions and 4Y-related bosons.
67. Possibly, other.

~ ~ ~

This subsection notes some criteria people might use to decide to assume that solutions are not physics-relevant.

People might say that Table 2.15.8 pertains.

**Table 2.15.8** Assumptions regarding lack of physics-relevance for some solutions

1. Particle (that would correlate with solution) shown not to exist in nature.
2. Lack of an appropriate D.
    - 2.1. For a specific solution, the number of dimensions, D, correlating with work similar to work that Table 2.5.16 summarizes would be undefined or negative.
    - 2.2. Examples include …
        - 2.2.1. The 3C-subfamily. (See Table 2.3.1.)
        - 2.2.2. The 4W-subfamily. (See Table 2.3.1.)
3. Inappropriate redundancy.
    - 3.1. For a specific solution, this solution and at least one other similar solution would correlate with one known elementary particle or one candidate elementary particle.
    - 3.2. Examples include …

3.2.1. The 3C-subfamily. (See Table 2.3.1.)
3.2.2. G-family solutions for which #P ≥ A = [10]. (See Table 2.3.13.)

4. Inappropriate square of rest mass.
    4.1. For a subfamily of solutions, at least one solution within the subfamily would correlate with an elementary particle for which applicable models would calculate an inappropriate square of the mass.
        4.1.1. For example, models for rest mass calculate, for a supposedly non-zero mass particle, a rest mass of 0.
        4.1.2. For example, models for rest mass calculate, for a particle, a negative number for the square of the rest mass.
    4.2. Examples may include …
        4.2.1. The 4W-subfamily. (See Table 2.3.1 and Table 3.8.2.)
5. Non-excitable.
    5.1. For a specific solution, math correlates with a lack of excitations of the ground state.
    5.2. Examples include …
        5.2.1. The 0Y-subfamily. (See Table 2.3.3.)
6. Too large a value of S.
    6.1. S would be > 2.
        6.1.1. [Physics:] The limit of S ≤ 2 for elementary particles comes from field theory. (See Table 2.3.10.)
7. Redundancy specific to a relevant model.
    7.1. This pertains to G-family solutions and some models.
        7.1.1. This correlates with two choices regarding models. (See, for example, Table 3.2.1.)
            7.1.1.1. We denote one choice by the term EACUNI set (of solutions), as in …
                7.1.1.1.1. For each G-family elementary particle, exactly one solution correlates with the particle.
            7.1.1.2. We denote one choice by the term SOMMUL set (of solutions), as in …
                7.1.1.2.1. For each of some G-family elementary particles, more than one solution correlates with the particle.
                    7.1.1.2.1.1. That number of solutions equals the number of channels that correlate with the particle. (See Section 2.13.)
        7.1.2. People might say that the set of G-family elementary particles does not depend on this choice regarding models.
    7.2. For example, …
        7.2.1. People might say that people can assume that …

7.2.1.1. Out of an entire set of epnG%& solutions for which all the values of n are identical and all the values of % are identical (and for which p = #P is not greater than 8), …

7.2.1.1.1. The solution with the least value of p (and, it turns out, the least value of e) is physics-relevant.

7.2.1.1.2. Each solution with a less-than-minimal value of p (and, it turns out, a less-than-minimal value of e) is not physics-relevant.

7.2.2. People might say that this approach correlates with a definition of channel that correlates with Table 2.13.12.

7.2.3. For such approaches, we use the term EACUNI models (or EACUNI-set models).

7.3. However, …

7.3.1. People might say that people might assume that …

7.3.1.1. Out of an entire set of epnG%& solutions for which all the values of n are identical and all the values of % are identical (and for which p = #P is not greater than 8), …

7.3.1.1.1. Each solution is physics-relevant.

7.3.2. People might say that each solution correlates with a channel.

7.3.3. For such approaches, we use the term SOMMUL models (or, SOMMUL-set models).

People might say that Table 2.15.9 pertains. (Regarding the terms %68even, %68odd, and %68both, see discussion preceding Table 3.3.3 and see Table 3.3.3. See, also, Table 3.13.2.)

**Table 2.15.9** Possible lack of physics-relevance for some solutions, plus definitions of the terms %68even, %68both, and %68odd

1. Inappropriate appearance of 6 ∈ % or 8 ∈ %.

1.1. This pertains to G-family solutions. (See Table 3.13.2.)

1.2. Each G-family solution correlates with a symbol of the form epnG%&.

1.2.1. % denotes a sub-list that can include up to four even integers.

1.2.2. The even integers are 2, 4, 6, and 8.

1.2.3. The spin correlating with a G-family solution correlates with 2S equaling (the absolute value of) an arithmetic combination that features, for each element of %, plus or minus that element.

1.3. The criterion S ≤ 2 disqualifies from being physics-relevant …

1.3.1. % = 6.

1.3.2. % = 8.

- 1.4. People might say that, for a % for which 6 ∈ % or for which 8 ∈ % to be physics-relevant, …
  - 1.4.1. Both 6 ∈ % and 8 ∈ % must pertain.
  - 1.4.2. The arithmetic contribution to 2S must include one of …
    - 1.4.2.1. $\pm 2 = \pm|-6 + 8|$.
    - 1.4.2.2. $\pm 2 = \pm|+6 - 8|$.
- 1.5. For the choice that, for each physics-relevant G-family solution, either none or both of 6 and 8 must appear in %, …
  - 1.5.1. We use the term %68even.
    - 1.5.1.1. Here the word even denotes that the total number of appearances and 6 and/or 8 in each solution is 0 or 2.
- 1.6. For those solutions for which exactly one of 6 and 8 appears in %, …
  - 1.6.1. We use the term %68odd.
- 1.7. Possibly, …
  - 1.7.1. All %68odd solutions are not physics-relevant.
- 1.8. We use the term %68both to denote the union of the %68even set and the %68odd set.

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This subsection suggests considerations and results that may be relevant for many models correlating with the MM1 meta-model.

Table 2.15.10 pertains. Here and elsewhere, we call attention to a gravitational somewhat analog to electromagnetism's magnetic field. We do so, in part, because we are uncertain as to the extent traditional physics models take into account effects that would correlate with such an analog. (See, for example, Section 4.7, Section 4.8, and Section 4.9.)

**Table 2.15.10** Characteristics shared by many models that correlate with the MM1 meta-model

1. For all families except the G- and Y-families, …
   - 1.1. Aspects related to energy and momentum correlate with the E0-and-P0 oscillator pair.
   - 1.2. For (at least) spin-1/2 and spin-1 elementary particles, …
     - 1.2.1. Aspects related to charge correlate with correlate with the E2-and-E1 oscillator pair and with the P1-and-P2 oscillator pair.
   - 1.3. Aspects related to spin correlate with #P.
2. For interactions mediated by G-family bosons, …
   - 2.1. Aspects related to charge (and charge-related MM1MS1-currents) of the interacting (generally, non-G-family) particles correlate with (regarding the G-family) the P1-and-P2 oscillator pair.

- 2.2. Aspects related to energy-and-momentum MM1MS1-currents of the interacting (generally, non-G-family) particles correlate with (regarding the G-family) the P3-and-P4 oscillator pair.
  - 2.2.1. Regarding a gravitational (or 4G4&-mediated) interaction between two objects that move parallel (not antiparallel) to each other (and, therefore produce energy-and-momentum currents that parallel {not antiparallel} each other), …
    - 2.2.1.1. People might say that a gravitational somewhat analog to electromagnetism's magnetic field provides for attraction between the two currents. (See, for example, Section 4.7.)
- 2.3. Aspects related to magnetic dipole moments of the interacting (generally, non-G-family) elementary particles correlate with (regarding the G-family) the P1-and-P2 and P3-and-P4 oscillator pairs.
- 2.4. Aspects related to spins of the interacting (generally, non-G-family) particles correlate with (regarding the G-family) the P5-and-P6 and P7-and-P8 oscillator pairs.

~ ~ ~

This subsection provides perspective about some models we discuss in this monograph.

Table 2.15.11 pertains. Because the OME might contain DMS (or, SEDMS or dark matter stuff) and/or DES (or, SEDES or dark-energy stuff) and because each DME might contain DES, the table uses the phrase (that) most significantly correlates with. (For definitions of DMS and DES, see Table 4.3.2. For possible examples of SEDMS, see Table 4.3.14. For possible examples of SEDES, see Table 4.3.13. Compare with definitions of OME, DME, and DEE, per Table 2.13.15.)

**Table 2.15.11** #ENS-related models we discuss in this monograph

1. We discuss ENS48 models (including ENS48" models).
   - 1.1. We think these models can correlate with known data and might correlate with future physics.
   - 1.2. Here, each ensemble that most significantly correlates with dark-energy stuff has similarities to each ensemble that most significantly correlates with dark matter and to the ensemble that most significantly correlates with ordinary matter.
2. We discuss ENS6 models (including ENS6' models).
   - 2.1. We think these models can correlate with known data and might correlate with future physics.

   2.2. Here, dark-energy stuff correlates with aspects of each of the 6 ensembles that most significantly correlate with dark matter (5 ensembles) or most significantly correlate with ordinary matter (1 ensemble).
3. We discuss ENS1 models.
   3.1. We think these models can correlate with known data and might correlate with future physics.
   3.2. Here, dark-energy stuff correlates with aspects of the one physics-relevant ensemble.
   3.3. Here, dark matter correlates with aspects of the one physics-relevant ensemble.

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