# Preface

Noether's first and second theorems are formulated in a very general setting of reducible degenerate Grassmann-graded Lagrangian theory of even and odd variables on graded bundles.

Lagrangian theory generally is characterized by a hierarchy of nontrivial Noether and higher-stage Noether identities and the corresponding gauge and higher-stage gauge symmetries which characterize the degeneracy of a Lagrangian system. By analogy with Noether identities of differential operators, they are described in the homology terms. In these terms, Noether's inverse and direct second theorems associate to the Koszul–Tate graded chain complex of Noether and higher-stage Noether identities the gauge cochain sequence whose ascent gauge operator provides gauge and higher-stage gauge symmetries of Grassmann-graded Lagrangian theory. If these symmetries are algebraically closed, an ascent gauge operator is generalized to a nilpotent BRST operator which brings a gauge cochain sequence into a BRST complex and provides the BRST extension of original Lagrangian theory.

In the present book, the calculus of variations and Lagrangian formalism are phrased in algebraic terms of a variational bicomplex on an infinite order jet manifold that enables one to extend this formalism to Grassmann-graded Lagrangian systems of even and odd variables on graded bundles. Cohomology of a graded variational bicomplex provides the global solutions of the direct and inverse problems of the calculus of variations.

In this framework, Noether's direct first theorem is formulated as a straightforward corollary of the global variational formula. It associates to any Lagrangian symmetry the conserved symmetry current whose total differential vanishes on-shell. Proved in a very general setting, so-called Noether's third theorem states that a conserved symmetry current along any gauge symmetry is reduced to a superpotential, i.e., it is a total differential on-shell. This also is the case of covariant Hamiltonian formalism on smooth fibre bundles seen as the particular Lagrangian one on phase Legendre bundles.

Lagrangian formalism on smooth fibre bundles and graded bundles provides the comprehensive formulations both of classical field theory and nonrelativistic mechanics.

Non-autonomous nonrelativistic mechanics is adequately formulated as particular Lagrangian and Hamiltonian theory on a configuration bundle over the time axis. Conserved symmetry currents of Noether's first theorem in mechanics are integrals of motion, but the converse need not be true. In Hamiltonian mechanics, Noether's inverse first theorem states that all integrals of motion come from symmetries. In particular, this is the case of energy functions with respect to different reference frames.

The book presents a number of physically relevant models: superintegrable Hamiltonian systems, the global Kepler problem, Yang–Mills gauge theory on principal bundles, SUSY gauge theory, gauge gravitation theory on natural bundles, topological Chern–Simons field theory and topological BF theory, exemplifying a reducible degenerate Lagrangian system.

Our book addresses to a wide audience of theoreticians, mathematical physicists and mathematicians. With respect to mathematical prerequisites, the reader is expected to be familiar with the basics of differential geometry of fibre bundles. We have tried to give the necessary mathematical background, thus making our exposition self-contained. For the sake of convenience of the reader, a number of relevant mathematical topics are compiled in appendixes.

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