

Research on the Monopolist' Repurchase and Remanufacture Decisions Based on Heterogeneous Consumer Considering Buy-Back Cost

Zhao-fang Mao, Xin-Xin Li and Wei Liu

Abstract This paper explores the repurchase and remanufacture decision of the monopolist who produce a durable product that can be used for two periods. It turns out that the remanufacture decision is profitable only when the production cost, consumers' opinion on the remanufactured product subject to some conditions. And the revenue under remanufacture decision is increasing as the cost of new product increases. We also find that, it brings market volume increase effect instead of cannibalization, which makes the remanufacture behavior beneficial to the monopolist.

Keywords Cannibalization · Durable product · Remanufacture · Repurchase cost

1 Introduction

For a long time, remanufacturing has been popular globally. Because in some way it is beneficial to reducing the natural resource usage and alleviate the environment burden. It is estimated that the total annual sales in the US is \$53 billion in 1997, covering more than 72,000 remanufacturing firms. Also some famous companies, such as BMW, IBM, DEC et al. have already proved that remanufacturing is profitable.

However, it is difficult to make the remanufacture decision for the insufficiency of guidance that the managers can follow. All the managers seem afraid that the remanufactured product may bring the cannibalization effect since those product has the similar or the same function as the new product. "Under which conditions will the remanufacture decision brings more benefit than the cannibalization effect?" More management acknowledges are needed for those managers to make differentiated and effective remanufacturing decisions.

Therefore, the goal of this paper is to provide some useful guidance for the manufacturer to make remanufacturing decisions. We will explore profitable conditions for the repurchase and remanufacture decision by taking the following factors

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into consideration: (1) The cost of remanufactured product is lower than the new product. (2) Remanufactured product usually have lower valuation from the consumers. (3) The repurchased price of the used product is a decision variable, which will have an impact on the supply of the remanufactured product in the second period.

There have been extensive researches on the remanufactured product. One of the most important direction is that the probability of the remanufacturing. Ayres and Ferrer [1] evaluates the economics of remanufacturing and points out some problematic issues specific to remanufacturing that have a substantial impact on profitability. Ferrer and Ayres [2] analyzes the macroeconomic impact of remanufacturing and confirm the intuition that it promotes demand for labor and reduces the consumption of raw materials. DeboL and Van Wassenhove [3] makes research on the pricing strategy and combined technology choice of the remanufactured product and new product. Ferguson and Toktay [4] analyzes the remanufacture decision of the monopolist when there is a third party that is remanufacturing. Heese et al. [5] explores the effect of the remanufacture decision on the revenue. Guide and Li [6] points out that the heterogeneity of the consumers makes a great difference to the monopolist's remanufacture decision. Another important stream is about studies on the pricing decisions of the monopolist. Ding et al. [7] studies the pricing decision of the monopolist under different market share from the perspective of the consumer. Liu and Tan [8] studies the remanufacturing strategy for monopolist based on the consumers' type. Other related paper includes [9–13].

However, by summarizing the papers mentioned, it can be concluded that the buy-back cost of used product from the consumer is not taken into the model as a decision variable. In their model, they regard the volume of returned product as a given variable, which has nothing to do with the buy-back price. However, in reality, the volume of the returned products is greatly impacted by the price of the monopolist announced. So in this paper, the writer try to answer whether the remanufacture decision is still profitable when the buy-back cost is regarded as a decision variable. And how the best solution changes when the given variables changes.

2 Assumptions and Consumer Utility Analyse

2.1 *Summary of Nominations*

The nominations in the paper is summarized as Table 1.

2.2 *Model Assumptions*

Assuming that the monopolist manufactures a durable product, which can be used for two periods. Consumers who purchases the product in the first period can choose to sell the used product to the monopolist or remain it to the second period.

Table 1 Nominations in model

Parameters	Definition
c_n	Manufacturing cost of new product
c_r	Manufacturing cost of remanufactured product
δ	Consumer value discount of used product at the end of first period
δ_r	Consumer value discount of remanufactured product
p_1	Sales price of the new product
p_r	Sales price of the remanufactured product
s	The monopolist’s repurchased price of the used product

The salvage of the product at the end of the first period is $\delta\theta$ and is 0 at the end of the second period.

As for the monopolist, the whole process can be divided into three parts. In the first period, the monopolist announces the price of the product. Before the second period begins, the monopolist decides whether to repurchase the used products from the consumer and remanufacture. If the monopolist does make the remanufacture decision, he will announce the repurchase price s and the price of the remanufactured product p_r . If not, the monopolist announces the price for the new product. As for the consumer, they are heterogeneous with respect to their willingness to pay for the new product. For simplicity, we assume it is θ , uniformly distributed between 0 and 1. The consumers arrive at the markets in two periods, N_1 and N_2 respectively in the first and second period. The consumers buy one product at most and they decide whether to buy according to the utility model. In the first period, if the consumer’s willingness to pay is larger than the price, the consumer will buy the product. At the beginning of the second period, the consumer will sell their used products to the monopolist on the condition that s is larger. As the second period begins, the consumer will make a decision between the new and remanufactures products after comparing the utility they can obtain. The timeline is as Fig. 1 represents.

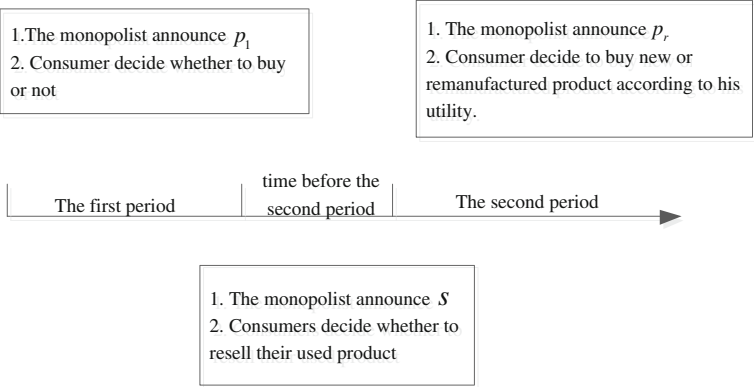


Fig. 1 Timeline of the model

2.3 Consumer Utility Analysis

The utility of buying the product in the first period is $U_1 = \theta - p_1$. When $U_1 = \theta - p_1 > 0$, the consumer chooses to buy the product. Consequently the demand in the first period is

$$d_1 = (1 - p_1)N_1 \quad (1)$$

At the end of the first period, the monopolist announces the repurchase price s . Consumers' restrictive value to the product is $\delta\theta$. When $\delta\theta < s$, the consumer chooses to sell their used product to the monopolist. Therefore, the collecting volume of the used products at the end of the first period is

$$q_r(s) = N_1 \int_{p_1}^s f(\delta\theta) = N_1 \left(\frac{s}{\delta} - p_1 \right), \quad \delta p_1 < s < \delta \quad (2)$$

At the second period, if the consumer chooses to buy the new product, the utility is $U_{2N} = \theta - p_2$. On the contrary, for the remanufactured product, the utility is $U_{2R} = \delta_r\theta - p_r$. When $U_{2N} > U_{2R}$ and $U_{2N} > 0$, the consumer choose to buy the new product. That is to say $\theta - p_2 > \delta_r\theta - p_r$. It is easily to say that if $p_r > \delta_r p_2$, the utility of buying new product is always larger than buying remanufactured product, which will be of no sense in this paper. Therefore, we constrict the discussion under the situation when $p_r < \delta_r p_2$. The demand of new product in the second period is

$$d_{2n} = \left(1 - \frac{p_2 - p_r}{1 - \delta_r} \right) N_2 \quad (3)$$

When $U_{2R} > U_{2N}$ and $U_{2R} > 0$, consumer choose the remanufactured product. Then the demand of the remanufactured product in the second period is

$$d_{2r} = \frac{p_2 - p_r}{1 - \delta_r} - \frac{p_r}{\delta_r} = \frac{\delta_r p_2 - p_r}{(1 - \delta_r)\delta_r} N_2 \quad (4)$$

For consumers whose restrictive value less than $\frac{p_r}{\delta_r}$ will buy nothing.

3 Model and Analysis

3.1 The Benchmark Case When the Monopolist of No Remanufacturing

If the monopolist decide to not remanufacture, the demand of the new products in the first period is $d_1^N = (1 - p_1)N_1$. Demand of the second period is $d_2^N = (1 - p_1)N_2$.

The profit formula of the monopolist is

$$\max_{p_1} \Pi_N^*(p_1) = (p_1 - c_n)(d_1^N + d_2^N) \quad (5)$$

So the solution is:

$$p_1^* = \frac{c_n + 1}{2}, p_2^* = \frac{c_n + 1}{2}; d_1^* = \frac{(1 - c_n)N_1}{2}, d_2^* = \frac{(1 - c_n)N_2}{2}$$

$$\Pi_N^* = \frac{(1 - c_n)^2}{4}(N_1 + N_2)$$

Proposition 1 *When the monopolist did not remanufacture, the best solution is $p_1^* = p_2^* = \frac{c_n + 1}{2}$. The revenue is $\Pi_N^* = \frac{(1 - c_n)^2}{4}(N_1 + N_2)$.*

3.2 When Monopolist Decide to Repurchase and Remanufacture

According to the former demand analysis, it is easily to obtain that the demand of the new and remanufactured products is as the formula (1)–(4) shows. Then the revenue formula of the two period is

$$\max_{p_1, s, p_r} \Pi_T^r(p_1, s, p_r) = (p_1 - c_n)d_1 - (s + c_r) \cdot q_r(s) + (p_1 - c_n)d_{2n} + p_r \cdot \min(q_r(s), d_{2r}) \quad (6)$$

According to the relationship of the demand and supply of the remanufactured products, it could be divided into two situations, $d_{2r} \leq q_r(s)$ and $d_{2r} \geq q_r(s)$. In the following analysis, the condition of $d_{2r} \leq q_r(s)$ is taken as an example. When $d_{2r} \leq q_r(s)$, that is to say $p_r \geq \delta_r p_2 - (\frac{s}{\delta} - p_1) \frac{N_1}{N_2} (1 - \delta_r) \delta_r$.

Under this condition, the revenue of the monopolist is

$$\max_{p_1, s, p_r} \Pi_T^r(p_1, s, p_r) = (p_1 - c_n)d_1 - (s + c_r) \cdot q_r(s) + (p_1 - c_n)d_{2n} + p_r \cdot d_{2r} \quad (7)$$

$$\begin{aligned} \text{s.t. } & p_r < \delta_r p_2 \\ & \delta p_1 < s \leq \delta \\ & p_r \geq \delta_r p_2 - (\frac{s}{\delta} - p_1) \frac{N_1}{N_2} (1 - \delta_r) \delta_r \end{aligned}$$

The first constraint $p_r < \delta_r p_2$ guarantees that the price of the remanufactured product won't be too high to have no market. The second one $\delta p_1 < s < \delta$ assures that the repurchased price is set among the bound of the consumers' restrictive

value for the used products. The third one $p_r \geq \delta_r p_2 - (\frac{s}{\delta} - p_1) \frac{N_1}{N_2} (1 - \delta_r) \delta_r$ make sure that the demand of the remanufactures product is no large than the supply of the used products at the beginning of the second period.

When the variables are substituted into the revenue formula (7), the Hessian matrix is

$$H = \begin{bmatrix} -2N_1 - \frac{2N_2}{1-\delta_r} & N_1 & \frac{2N_2}{1-\delta_r} \\ N_1 & -\frac{2N_1}{\delta} & 0 \\ \frac{2N_2}{1-\delta_r} & 0 & -\frac{2N_2}{(1-\delta_r)\delta_r} \end{bmatrix}$$

It could be easily proved that the Hessian matrix is negative. So the revenue formula is negative and there exists the best solution.

In the following solution, we focus on the Lagrange Multipliers and on the orthogonality conditions of the Karush-Kuhn-Tucker theorem. Parameters in the equation meet the following inequalities.

$$0 < \delta_r < 1 \quad 0 < \delta < 1 \quad \delta_r > \delta \quad c_n > c_r$$

The Lagrange multipliers and the Karush-Kuhn-Tucker optimality conditions are:

$$\begin{aligned} L(p_r, s, p_1) &= (p_1 - c_n)(1 - p_1)N_1 - (s + c_r) \cdot N_1(\frac{s}{\delta} - p_1) + (p_2 - c_n) \left(1 - \frac{p_2 - p_r}{1 - \delta_r}\right) N_2 + p_r \cdot \frac{\delta_r p_2 - p_r}{(1 - \delta_r)\delta_r} N_2 \\ &\quad + \lambda_1(\delta_r p_2 - p_r) + \lambda_2(s - \delta p_1) + \lambda_3[p_r - \delta_r p_2 + (\frac{s}{\delta} - p_1) \frac{N_1}{N_2} (1 - \delta_r) \delta_r] + \lambda_4(\delta - s) \\ \frac{\partial L(p_r, s, p_1)}{\partial p_1} &= N_1(1 + s + c_n + c_r - 2p_1) + N_2 \frac{1 + c_n - 2p_1 + 2p_r}{1 - \delta_r} + \delta_r \lambda_1 - \delta \lambda_2 - (\delta_r + \frac{N_1(1 - \delta_r)\delta_r}{N_2}) \lambda_3 = 0 \\ \frac{\partial L(p_r, s, p_1)}{\partial p_r} &= \frac{N_2(-c_n + p_1)}{1 - \delta_r} - \frac{N_2 p_r}{(1 - \delta_r)\delta_r} + \frac{N_2(-p_r + p_1 \delta_r)}{(1 - \delta_r)\delta_r} - \lambda_1 + \lambda_3 = 0 \\ \frac{\partial L(p_r, s, p_1)}{\partial s} &= -\frac{N_1(s + c_r)}{\delta} - N_1(\frac{s}{\delta} - p_1) + \lambda_2 + \frac{N_1(1 - \delta_r)\delta_r \lambda_3}{\delta N_2} - \lambda_4 = 0 \\ \lambda_1(\delta_r p_1 - p_r) &= 0 \\ \lambda_2(s - \delta p_1) &= 0 \\ \lambda_3[p_r - \delta_r p_1 + (\frac{s}{\delta} - p_1) \frac{N_1}{N_2} (1 - \delta_r) \delta_r] &= 0 \\ \lambda_4(\delta - s) &= 0 \end{aligned}$$

$\lambda_1, \lambda_2, \lambda_3, \lambda_4$ are either zero or positive, as one of the most important property of Lagrange multipliers. If $\lambda_2 > 0$, then $s - \delta p_1 = 0$, which means no consumer will sell their used products to the monopolist and there would no remanufactured product in the second period. Similarly, if $\lambda_1 > 0$, then $\delta_r p_2 - p_r = 0$, which means all the consumer choose to buy the new product in the second period. In this condition, there will be no difference with the remanufacture decision. Therefore, there must be $\lambda_1 = \lambda_2 = 0$. And it could be divided into four scenarios to examine.

By analyzing, the best solution exists under the condition $\lambda_4 = 0, \lambda_3 > 0$, which is

$$\begin{aligned}
 & (2(1+c_n)(-N_2^2+2(1+c_n)N_1^2(-1+\delta_r)\delta_r+N_1N_2(-\delta_{c_r}-2(\delta+\delta_r-\delta_r^2)+c_n(-2\delta+(-2+\delta)\delta_r+2\delta_r^2))) \\
 p_1 = & \frac{\delta_r\{-2\delta(1+c_n)N_2^2+N_1^2(-1+\delta_r)(\delta+\delta_{c_n}+2c_r+2\delta_r)\}}{A} \\
 p_r = & \frac{N_1N_2[-3\delta-c_r(2+\delta-2\delta_r)-2\delta_r+\delta\delta_r+2\delta_r^2+\delta_{c_n}(-3+2\delta_r)]}{A} \\
 s = & \frac{-\delta\{-2(1+c_n)N_1^2(-1+\delta_r)\delta_r+B+N_1N_2[\delta+(-2+\delta)c_r+2\delta_r-2\delta_r^2+C]\}}{A} \\
 A = & (-4\delta N_2^2+4N_1^2(-1+\delta_r)\delta_r+N_1N_2((-4+\delta)\delta-4\delta_r+4\delta_r^2)) \\
 B = & N_2^2[\delta-2c_r+c_n(\delta+2\delta_r)] \\
 C = & c_n[\delta-(-4+\delta)\delta_r-2\delta_r^2]
 \end{aligned}$$

Because $\lambda_3 > 0$, there must be $p_r^* = \delta_r p_2 - (\frac{s}{\delta} - p_1) \frac{N_1}{N_2} (1 - \delta_r) \delta_r = 0$, which means the supply and demand of the remanufactured products are the equal. Then $p_r < \delta_r p_2$, since $(\frac{s}{\delta} - p_1) \frac{N_1}{N_2} (1 - \delta_r) \delta_r > 0$. Therefore, the best solution should be subjected to the constraints $s > \delta p_1$ and $s < \delta$.

So the best solution subject to

$$\begin{aligned}
 c_n & \geq \bar{c}_n, c_n \leq \bar{\bar{c}}_n \\
 \bar{c}_n & = -\frac{\delta+2c_r}{\delta-2\delta_r} \\
 \bar{\bar{c}}_n & = \frac{-(3\delta+2c_r)N_2^2+2N_1^2(-1+\delta_r)\delta_r+N_1N_2((-3+\delta)\delta+(-2+\delta)c_r-2\delta_r+2\delta_r^2)}{(-N_2+N_1(-1+\delta_r))(2N_1\delta_r+N_2(\delta+2\delta_r))}
 \end{aligned}$$

The optimized demand is as follows:

$$\begin{aligned}
 & N_1\{1+[2\delta(1+c_n)N_2^2-2(1+c_n)N_1^2(-1+\delta_r)\delta_r+ \\
 d_1 = & \frac{N_1N_2\{\delta_{c_r}+c_n[2\delta-(-2+\delta)\delta_r-2\delta_r^2]+2(\delta+\delta_r-\delta_r^2)\}}{A} \\
 & N_2(2\delta(-1+c_n)N_2^2+N_1^2\delta_r[-2-\delta-(-2+\delta)c_n-2c_r+2\delta_r]+ \\
 d_2 = & \frac{N_1N_2\{-2\delta+\delta^2+c_r(\delta-2\delta_r)-2\delta_r-\delta\delta_r+2\delta_r^2+2c_n[\delta-(-1+\delta)\delta_r]\}}{A} \\
 d_r = q_u = & \frac{N_1N_2(N_1+N_2)(\delta+2c_r+c_n(\delta-2\delta_r))}{A} \\
 A = & (-4\delta N_2^2+4N_1^2(-1+\delta_r)\delta_r+N_1N_2((-4+\delta)\delta-4\delta_r+4\delta_r^2))
 \end{aligned}$$

Proposition 2 Only under the condition when $\delta+c_r<\delta_r$ and $c_n \in [\bar{c}_n, \bar{\bar{c}}_n]$, the remanufacture decision makes sense.

Proof If $\delta + c_r > \delta_r$, then the inequality $-\frac{\delta+2c_r}{\delta-2\delta_r} > 1$ and $c_n \geq \bar{c}_n = -\frac{\delta+2c_r}{\delta-2\delta_r} > 1$ will hold. Since the manufacturing cost of product is between $(0, 1)$. So there will be no solution. Therefore, the converse to be true.

At this time, the monopolist best solution is:

$$\begin{aligned}
 p_1 &= \frac{(2(1+c_n)(-N_2^2+2(1+c_n)N_1^2(-1+\delta_r)\delta_r+N_1N_2(-\delta c_r-2(\delta+\delta_r-\delta_r^2)+c_n(-2\delta+(-2+\delta)\delta_r+2\delta_r^2)))}{A} \\
 &\quad \delta_r\{-2\delta(1+c_n)N_2^2+N_1^2(-1+\delta_r)(\delta+\delta c_n+2c_r+2\delta_r)+ \\
 p_r &= \frac{N_1N_2[-3\delta-c_r(2+\delta-2\delta_r)-2\delta_r+\delta\delta_r+2\delta_r^2+\delta c_n(-3+2\delta_r)]}{A} \\
 s &= \frac{-\delta\{-2(1+c_n)N_1^2(-1+\delta_r)\delta_r+B+N_1N_2[\delta+(-2+\delta)c_r+2\delta_r-2\delta_r^2+C]\}}{A} \\
 A &= (-4\delta N_2^2+4N_1^2(-1+\delta_r)\delta_r+N_1N_2((-4+\delta)\delta-4\delta_r+4\delta_r^2)) \\
 B &= N_2^2[\delta-2c_r+c_n(\delta+2\delta_r)] \\
 C &= c_n[\delta-(-4+\delta)\delta_r-2\delta_r^2]
 \end{aligned}$$

The optimized revenue is

$$\frac{\left[-((N_1+N_2)(\delta(-1+c_n)^2N_2^2-(-1+c_n)^2N_1^2(-1+\delta_r)\delta_r+N_1N_2(\delta+\delta c_r+c_r^2+\delta_r-\delta_r^2+c_n^2(\delta-(-1+\delta)\delta_r)+c_n((-2+\delta)\delta+c_r(\delta-2\delta_r)-(2+\delta)\delta_r+2\delta_r^2)))) \right]}{(-4\delta N_2^2+4N_1^2(-1+\delta_r)\delta_r+N_1N_2((-4+\delta)\delta-4\delta_r+4\delta_r^2))}$$

Proposition 2.2 proves that remanufacturing is profitable only when the consumers remain a high value to the remanufactured product, which is bigger than the total value of the salvage of the used product and the remanufacture cost. It tells us the management sight that when the monopolist is making the remanufacture decisions, he should not only consider the remanufacture cost, but also the popularity of the remanufactured product among the consumers. Therefore, the monopolist can disseminate the remanufactured product among the consumers to implement repurchase and remanufacture decisions.

Comparing the revenue of proposition 1 and 2, we could obtain:

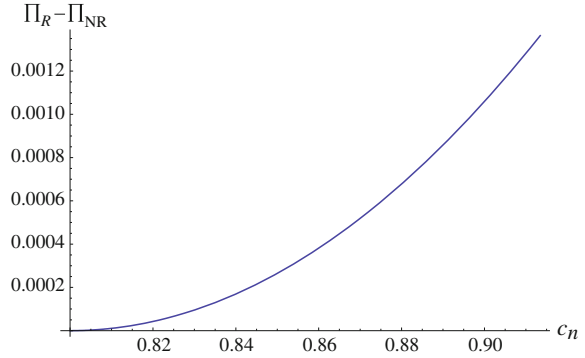
Lemma 1 *If $\delta + c_r < \delta_r$ and $c_n \in [\bar{c}_n, \bar{\bar{c}}_n]$, then remanufacturing is always more profitable. And the revenue advantage is bigger as the manufacturing cost of new product increases.*

Proof

$$\Pi_R - \Pi_{NR} = -\frac{N_1N_2(N_1+N_2)(\delta+2c_r+c_n(\delta-2\delta_r))^2}{4(-4\delta N_2^2+4N_1^2(-1+\delta_r)\delta_r+N_1N_2((-4+\delta)\delta-4\delta_r+4\delta_r^2))} > 0$$

Let $N_1 = 1, N_2 = 1; \delta = 0.4, \delta_r = 0.7; c_r = 0.2, c_n \in [0.8, 0.91]$, we can also see this property as Fig. 2 shows.

Fig. 2 The revenue gap property as c_n changes



Lemma 2 *If the monopolist chooses to remanufacture, it could enlarge the market quota and the increase effect is becoming more obvious as the cost of new product increases.*

Proof Let

$$d_g = d'_1 + d'_2 + d'_r - (d_1 + d_2) = \frac{N_1 N_2 (N_1 + N_2) (\delta + 2c_r + c_n (\delta - 2\delta_r)) (2 + \delta - 2\delta_r)}{-8\delta N_2^2 + 8N_1^2 (-1 + \delta_r) \delta_r + 2N_1 N_2 ((-4 + \delta) \delta - 4\delta_r + 4\delta_r^2)} \quad \text{When}$$

$c_n \in [\bar{c}_n, \bar{\bar{c}}_n]$, it is easily to prove that $d_g > 0$. So the total demand is always bigger for the new and remanufactured product than new products only. And $\frac{\partial d_g}{\partial c_n} = \frac{N_1 N_2 (N_1 + N_2) (\delta - 2\delta_r) (2 + \delta - 2\delta_r)}{-8\delta N_2^2 + 8N_1^2 (-1 + \delta_r) \delta_r + 2N_1 N_2 ((-4 + \delta) \delta - 4\delta_r + 4\delta_r^2)} > 0$, so the lemma could be proved.

Solving the condition under the condition of $d_{2r} \geq q_r(s)$, the same solution could be obtain.

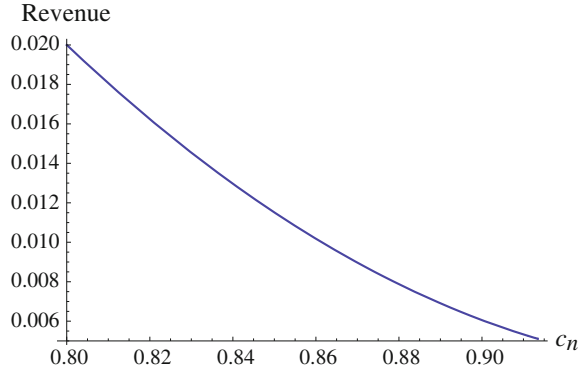
4 Solution Analysis

Let $N_1 = 1, N_2 = 1; \delta = 0.4, \delta_r = 0.7; c_r = 0.2, c_n \in [0.8, 0.91]$ we could get the property of the best solution p_1^*, p_r^*, s^* and the revenue.

From Fig. 3 we can see that the revenue is decreasing as the manufacturing cost of new product increase. It is easily understood that when all the other parameters remain unchanged, as the cost becomes larger, the profit of each product is decreasing. Since consumers' valuation is the same as before, the sales price did not change.

Furthermore, we try to understand the revenue from the perspective of new product and remanufactured product prospectively just as Fig. 4 represents. From the graph, we can see that. Their margin profit are both decreasing as the manufacturing cost of new product increases. However, the decreasing rate of remanufactured product is slower than the new product. It give us the management sight that for products with large manufacturing cost, remanufacture decision is more beneficial and profitable.

Fig. 3 The revenue property as c_n changes



Proposition 3 Under the situation $\delta + c_r < \delta_r$ and $c_n \in [\bar{c}_n, \bar{\bar{c}}_n]$, the monopolist best solution p_1^*, p_r^*, s^* increase as the cost of new product increase.

Proof The first derivative of p_1^*, p_r^*, s^* to c_n is as follows:

$$\frac{\partial p_1^*}{\partial c_n} = \frac{-2\delta N_2^2 + 2N_1^2(-1 + \delta_r)\delta_r + N_1N_2[-2\delta + (-2 + \delta)\delta_r + 2\delta_r^2]}{-4\delta N_2^2 + 4N_1^2(-1 + \delta_r)\delta_r + N_1N_2[(-4 + \delta)\delta - 4\delta_r + 4\delta_r^2]}$$

$$A = -4\delta N_2^2 + 4N_1^2(-1 + \delta_r)\delta_r + N_1N_2[(-4 + \delta)\delta - 4\delta_r + 4\delta_r^2]$$

It is easily to see that $A < 0$, the third item in the numerator can be simplified to $N_1N_2[-2\delta + (-2 + \delta)\delta_r + 2\delta_r^2] = N_1N_2[2\delta_r(-1 + \delta_r) + \delta(\delta_r - 2)] < 0$, so the numerator is negative. Therefore $\frac{\partial p_1^*}{\partial c_n} > 0$ holds.

Fig. 4 Margin profit property as c_n changes

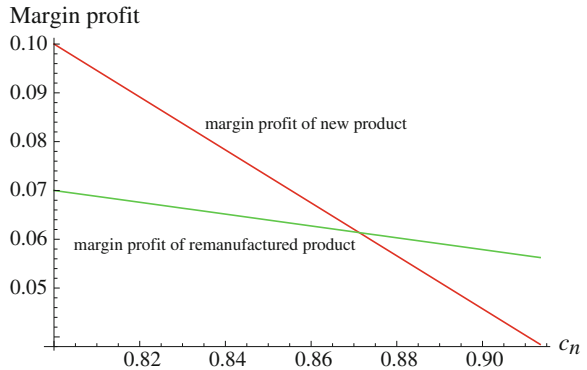
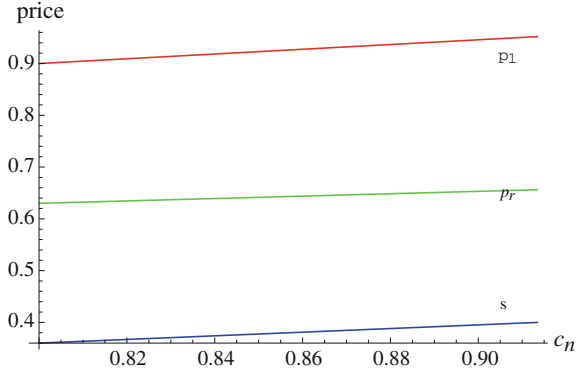


Fig. 5 Optimized solution property as c_n changes

$$\frac{\partial p_r^*}{\partial c_n} = \frac{\delta(N_1 + 2N_2)[-N_2 + N_1(-1 + \delta_r)]\delta_r}{A} > 0$$

$$\frac{\partial s^*}{\partial c_n} = \frac{[-N_2 + N_1(-1 + \delta_r)](2N_1\delta_r + N_2(\delta + 2\delta_r))}{A} > 0$$

Just like the number analysis shows in Fig. 5.

Proposition 4 Under the situation $\delta + c_r < \delta_r$ and $c_n \in [\bar{c}_n, \bar{\bar{c}}_n]$, the monopolist best solution p_1^*, p_r^*, s decrease as the cost of remanufactured product increase.

Proof

$$\frac{\partial p_1^*}{\partial c_r} = \frac{\delta_r N_1 N_2}{-4\delta N_2^2 + 4N_1^2(-1 + \delta_r)\delta_r + N_1 N_2[(-4 + \delta)\delta - 4\delta_r + 4\delta_r^2]} < 0$$

$$\frac{\partial p_r^*}{\partial c_r} = \frac{N_1[-N_2(2 + \delta) - 2\delta_r] + 2N_1(-1 + \delta_r)\delta_r}{A} < 0$$

$$\frac{\partial s^*}{\partial c_r} = \frac{\delta[(\delta - 2)N_1 - 2N_2]N_2}{A} < 0$$

5 Conclusion

After taking the buy-back cost of the used product, the problem becomes more difficult and interesting. We can see that the remanufacture decision is profitable only when the production cost, consumers' opinion on the remanufactured product subject to some conditions. And the revenue under remanufacture decision is increasing as the cost of new product increases. We also find that, there is market volume increase effect instead of cannibalization, which make the remanufacture behavior beneficial to the monopolist.

However, our focus on the supply side of the remanufactured product inspires many questions for further research. First, when the monopolist repurchases the used product, the buy-back price could be different for products of diverse usage condition. Second, consumers without buying the product in the first period can remain in the market until the second period. Third, the capacity constraints on the new product can also be an interesting issue. Future study on these problems may provide interesting new insights for firms that consider remanufacturing.

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Proceedings of the 22nd International Conference on
Industrial Engineering and Engineering Management
2015

Innovation and Practice in Industrial Engineering and
Management (Volume 2)

Qi, E.; Shen, J.; Dou, R. (Eds.)

2016, XIV, 888 p. 180 illus., 96 illus. in color., Hardcover

ISBN: 978-94-6239-176-5

A product of Atlantis Press